PHYSICAL REVIEW VOLUME 80, NUMBER 2 OCTOBER 15, 1950

Variation of Amplitude-Dependent Internal Friction in Single Crystals of Copper with Frequency and Temperature*

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(Received May 16, 1950)

The amplitude-dependent internal friction which originates in the motion of dislocations in single crystals of copper is studied as a function of frequency and temperature. Quantities are introduced which express the dependence of internal friction and of Young's modulus on strain amplitude and it is shown that these quantities are significant measures of the properties of a crystal. Measurements made between −60°C and +33°C show that the observed internal friction can be expressed as the product of a function of temperature and a function of amplitude alone. The data also indicate that the internal friction and elastic modulus are frequency independent when the structure sensitivity of the material is taken into account. The results are considered in terms of two viewpoints: a mechanism of *relaxation* by which dissipation is controlled through a rate process, and *simple hysteresis*, by which the stress-strain loop is independent of the rate of traversal. It is shown that the latter mechanism gives much better agreement with the experimental facts. Finally, simple hysteresis is interpreted in terms of the dislocation theory.

I. INTRODUCTION

THE internal friction of solid materials is usually studied by observing the behavior of a properly shaped specimen body which is vibrating in one of its normal modes. A coefficient of internal friction, Δ, can be defined by the formula

$$\Delta = w^d/2w^v, \tag{1}$$

where w^d is the energy dissipated per unit volume per cycle of vibration and w^v is the total vibrational energy per unit volume. There are numerous sources of dissipation in solids;[1] among these is the internal stress produced by cold-work. In a study of single crystals of copper and zinc in forced longitudinal vibration in the frequency range 30 to 100 kc/sec., Read[2] reports a characteristic amplitude-dependent internal friction which he relates to the internal stress by observing that the dissipation is increased by cold-work and decreased to a minimum by annealing at a high temperature. It is found that the internal friction increases and Young's modulus of the material decreases with increasing strain amplitude of vibration in both zinc and copper crystals. There is, however, a striking difference between the behavior of these two metals; in contrast to the behavior of zinc, the internal friction of copper crystals does not depend on the previous vibrational history of the material, nor is a persistent temporal variation of this quantity observed, following heat treatment or cold-work.

The internal stress associated with cold-work has long been related to the presence of linear lattice imperfections known as dislocations.[3] It is the stress-induced motion of these dislocations that is responsible for the dissipation observed by Read. Although great interest has recently been shown in the internal friction due to dislocations in zinc crystals[4] (probably because zinc contains only a single parallel set of slip planes) the dissipation in copper involves a considerably simpler situation. Inasmuch as the previous vibrational history has no effect on the copper crystals, it is reasonable to suppose that the vibrations cannot alter the distribution and number of dislocations; therefore, the dissipative mechanism is simply the *oscillatory motion of dislocations*, already present in the lattice, about their equilibrium positions. The present research is a contribution toward an understanding of this process. The significant factors here investigated are the effects of temperature and frequency on the manner in which both internal friction and elastic modulus vary with the strain amplitude.

II. EXPERIMENTAL METHOD

The method, which permits the measurement of both the internal friction and Young's modulus, E, of the specimen, is essentially that developed by Cooke and Brown[5] and Read,[2] so modified as to permit observations at various temperatures and on specimens in which the variation of Δ and E with strain amplitude is appreciable even at small strain amplitudes. The specimen, in the form of a right circular cylinder 5.3 mm in diameter and a few cm long, forms one part of a composite piezoelectric oscillator constructed by cementing to one end of the specimen an X-cut cylinder of crystalline quartz of identical cross section. This composite oscillator forms one arm of an alternating-current bridge, and measurements of the equivalent impedance of the composite oscillator near one of its resonant fre-

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* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

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[2] T. A. Read, Phys. Rev. **58**, 371 (1940); Trans. A.I.M.E. **143**, 30 (1941).

[3] An excellent review of dislocation theory by A. H. Cottrell appears in Chapter 2 of: B. Chalmers, *Progress in Metal Physics I* (Interscience Publishers Inc., New York 1949).

[4] T. A. Read and E. P. T. Tyndall, J. App. Phys. **17**, 713 (1946); I. H. Swift and J. E. Richardson, J. App. Phys. **18**, 417 (1947); C. A. Wert, J. App. Phys. **20**, 29 (1949).

[5] W. T. Cooke and W. F. Brown, Phys. Rev. **50**, 1158, 1165 (1936).

quencies yield the elastic and dissipative properties of the specimen.

It is found, as previously remarked, that Δ and E of the specimen material depend on the vibrational strain amplitude, which varies along the specimen cylinder. Therefore, the specimen cylinder is effectively inhomogeneous. The well-known results of the analysis in the case of homogeneous specimens are presented in the following section. It is then shown that under certain assumptions, the validity of which are demonstrated experimentally, a simple extension of these results correctly describes the inhomogeneous specimen.

Homogeneous Specimen; E and Δ Constant

The impedance, Z, of the composite oscillator near one of its resonant frequencies, f_0, can be written in the form $1/Z = i\omega C' + 1/Z_m$, where ω is the angular frequency of the applied potential difference and the form of Z_m is precisely that of an electrical series resonant circuit, $Z_m = R + iX$. The resonant frequencies, for which $X = 0$, are the solutions for f_0 of the equation

$$M_1 f_1 \tan(\pi f_0/f_1) + M_2 f_2 \tan(\pi f_0/f_2) = 0, \tag{2}$$

where M_i = mass of a cylinder,

$$f_i = (E_i/\rho_i)^{\frac{1}{2}}(2L_i)^{-1}, \tag{3}$$

E_i = Young's modulus, ρ_i = density of a cylinder, L_i = length of a cylinder, and the subscripts 1 and 2 refer to the specimen and quartz cylinders respectively. The resistance and reactance near resonance are given by the expressions

$$R = Kf_0(M_1\Delta_1 + M_2\Delta_2), \tag{4}$$

$$X = 2\pi KM\delta f, \tag{5}$$

where K is a constant which must be obtained by experiment, $M = M_1 + M_2$, and δf is the departure of the frequency of the applied potential difference from the resonant frequency, f_0. The quantities Δ_1 and Δ_2, called the "decrements" of the specimen and quartz cylinders respectively, are defined by the formula

$$\Delta_i = W_i^d/2W_i^v, \tag{6}$$

where W_i^d denotes the *total* energy dissipated per cycle of vibration and W_i^v the total vibrational energy, in a cylinder.

In the experiments here reported the lengths of the quartz and specimen cylinders are so adjusted that, when vibrating in the neighborhood of a resonant frequency, very nearly integral numbers of half-waves of vibration appear in both the quartz and specimen cylinders. Under these circumstances the variation of the *maximum* strain amplitude in the specimen cylinder with frequency is given by ϵ_{1m} where

$$\epsilon_{1m}^2 = 2\times 10^7 K(\rho_1/E_1)\mathcal{E}^2/(R^2+X^2) \tag{7}$$

and $\mathcal{E}$ is the r.m.s. voltage applied to the quartz. This formula is obtained by equating the electrical energy supplied to the oscillator in a cycle of vibration to the energy dissipated in the oscillator during the same time interval.

At any frequency near resonance, the composite oscillator can be replaced by a capacity and resistance connected in parallel. This effective resistance, $\mathcal{R}$, of the composite oscillator is given by:

$$\mathcal{R} = (R^2+X^2)/R. \tag{8}$$

The quantity $\mathcal{R}$ is measured experimentally; at exact resonance $\mathcal{R} = R$. When the reactance, X, is eliminated from Eq. (8) with the aid of Eq. (5), the following expression for the factor K results:

$$K^2 = [(\mathcal{R}/R) - 1]\frac{R^2}{[4\pi^2 M^2(\delta f)^2]}. \tag{9}$$

Accordingly, a measurement of $\mathcal{R}$ at a known frequency departure, δf, from resonance yields the value of the quantity K; a separate measurement of the properties of the quartz crystal alone yields f_2 and Δ_2 and this information together with measured values of f_0 and R yields the elastic and dissipative properties of the specimen, after application of Eqs. (2), (3), and (4).

Homogeneous Specimen; E and Δ Variable with Vibration Amplitude

Modification of the foregoing theory necessary to obtain a description of the behavior of an inhomogeneous specimen is most easily obtained with the aid of an auxiliary concept; namely, that of a *fictitious specimen* which is homogeneous, but whose elastic and dissipative coefficients vary with the *maximum* strain amplitude in the specimen cylinder. For such a specimen the observed quantities are the resistance at

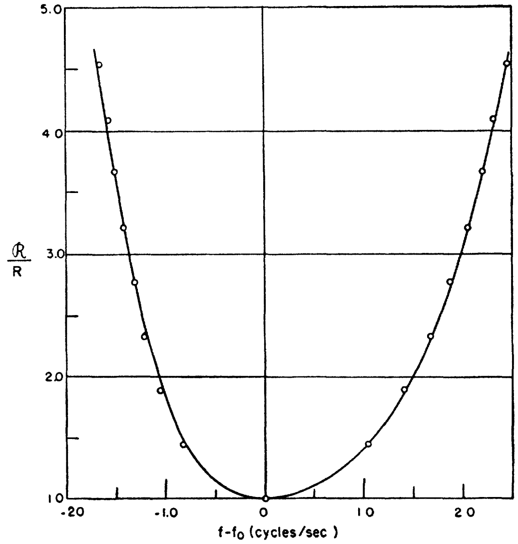

FIG. 1. Comparison of the observed resonance curve with that computed from the theory of the fictitious specimen.

resonance, R, and the resonant frequency, measured as a function of applied voltage. Quantities E_1 and Δ_1, which characterize the fictitious specimen at each amplitude of vibration, are calculated from the measured quantities f_0 and R exactly as in the earlier case, once the value of K is known. The maximum strain amplitude at resonance is related to $\mathcal{E}$ and R by Eq. (7), with X set equal to zero. The relation $\delta E_1/E_1 = 2\delta f_1/f_1$ connects the fractional changes, in the quantities f_1 and E_1, between zero amplitude and the strain amplitude of a given measurement.

In order to obtain the quantity K, Eq. (9) must be reinterpreted, for the variation, δf, in frequency near resonance is accompanied by a rapid decrease in the amplitude of vibration. Corresponding to a decrease in amplitude there is a change in Δ_1 and E_1 and hence in R and f_0. Let the applied potential difference, at which the measurement to determine K is made, be denoted by $\mathcal{E}$ and the corresponding (off resonance) resistance be denoted by $\mathcal{R}$. Then the value to be inserted in Eq. (9) in place of R is the resistance R' which would be measured if the composite oscillator were vibrating at resonance with the actual off resonance amplitude; that is, if the oscillator were vibrating at resonance under a reduced voltage, say $\mathcal{E}'$. When the strain amplitude off resonance under a potential $\mathcal{E}$, obtained from Eq. (7) after elimination of the quantity (R^2+X^2) by means of Eq. (8), is equated to the strain amplitude at resonance under a potential $\mathcal{E}'$ the following expression is obtained

$$R'^{\frac{1}{2}}/\mathcal{E}' = \mathcal{R}^{\frac{1}{2}}/\mathcal{E}. \tag{10}$$

This expression makes it possible to obtain $\mathcal{E}'$ and R' from the observed variation of resistance at resonance with applied voltage. It is necessary merely to plot the ratio, square root of the resistance at resonance divided by voltage at resonance, obtained from the data, as a function of voltage at resonance; $\mathcal{E}'$ is then the value of voltage at resonance at which this ratio equals $\mathcal{R}^{\frac{1}{2}}/\mathcal{E}$.

Now consider the quantity δf in Eq. (9); it must now be re-interpreted as the frequency departure, not from the observed resonant frequency of the oscillator, but from the resonant frequency that the oscillator would have if the elastic modulus of the specimen retained the value characteristic of the vibration amplitude at resonance under an applied voltage $\mathcal{E}'$. Since $\mathcal{E}'$ is already determined, the desired frequency is obtained directly from the experimental curve of resonant frequency *versus* applied voltage.

The Resonance Curve

The behavior of the fictitious specimen is characterized by its "resonance curve," here defined as the variation of $\mathcal{R}$ with frequency in the neighborhood of resonance. In accordance with Eq. (9) and the argument of the preceding section, a single determination of K, corresponding to a single value of δf, together with the data for variation of resonant frequency and of resistance at resonance with applied voltage, suffice uniquely to determine the entire course of the resonance curve characteristic of the fictitious specimen. Such a curve, corresponding to an applied voltage $\mathcal{E}=0.3$ volt, is shown in Fig. 1 together with the *observed* variation of $\mathcal{R}$ with frequency for an actual specimen. Here R is the value of $\mathcal{R}$ at resonance and the ordinates of the curve are the ratios $\mathcal{R}/R$. The value of K employed in the computation is obtained from a resonance curve taken at an applied potential sufficiently small that Δ_1 and E_1 are very nearly constant over the entire range of strain amplitude present in the specimen, so that only the theory of the homogeneous oscillator is involved. Accordingly the close agreement between theory and observation demonstrated in Fig. 1 is obtained without the benefit of a single adjustable parameter.

The concordance exhibited in Fig. 1 means that the behavior of the actual specimen cannot be distinguished from that of a fictitious specimen whose elastic and dissipative coefficients vary with the maximum strain amplitude in the specimen cylinder in precisely the same manner as the quantities denoted by the symbols Δ_1 and E_1 are observed to vary when these are computed, from observations with the formulae of the theory. The sole remaining step in the argument is to relate the quanties Δ_1 and E_1 so defined to the physical properties of the actual specimen.

Relation of E_1 and Δ_1 to Properties of the Specimen Material

The form of the resonance curve of the fictitious specimen is a consequence of the fact that near resonance it vibrates in a single normal mode. In the actual specimen, the excitation of higher harmonics is to be expected whenever the dissipative coefficient varies with the strain amplitude, since such a behavior can only arise from a nonlinear dissipative term in the differential equation of motion of the bar. Accordingly, the physical significance of the agreement between the behavior of the fictitious specimen and the actual specimen is that, under the circumstances of measurement, higher harmonics appear to a negligible extent and hence that the specimen cylinder is, in fact, vibrating very nearly in a single normal mode.

The stress, σ, and strain, ϵ, in the fictitious specimen can be written as

$$\sigma = \sigma_{1m} \sin(\pi x/L_1) \cos\omega t = \sigma_m \cos\omega t, \tag{11}$$

$$\epsilon = (A \cos\omega t + B \sin\omega t) \sin(\pi x/L_1), \tag{12}$$

where ω is the angular frequency of the applied voltage. Here A and B are constants, for a given value of σ_{1m}, which are related to E_1 and Δ_1 by the formulas

$$1/E_1 = A/\sigma_{1m}, \tag{13}$$

$$\Delta_1 = \pi E_1 B/\sigma_{1m}. \tag{14}$$

Consider next an elementary length of the *actual* specimen cylinder located at a point x on the axis.

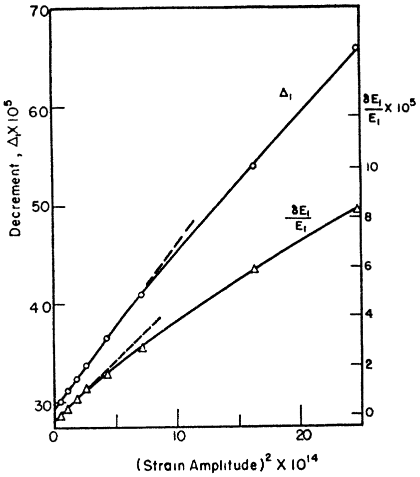

FIG. 2. Both Δ_1 and $\delta E_1/E_1$ are proportional to the square of the strain amplitude at low strain amplitudes.

When higher modes are neglected, the stress and strain at this point can be written as

$$\sigma=\sigma_m \cos\omega t, \tag{15}$$

$$\epsilon=a \cos\omega t+b \sin\omega t. \tag{16}$$

Here a and b are functions of σ_m and therefore of x. The local elastic modulus, E, of the material can be *defined* by

$$1/E=a/\sigma_m \tag{17}$$

and it follows from the definition of Δ, Eq. (1), that

$$\Delta=\pi Eb/\sigma_m. \tag{18}$$

Now, whatever the nature of the variations of a and b with x, these quantities can be represented in the interval $0<x<L_1$, by Fourier series of the forms

$$a=\sum_n a_n \sin(n\pi x/L_1) \tag{19}$$

and similarly for b, with the usual Fourier inversions. The requirement that the vibration of the actual specimen is, very nearly, in a normal mode is equivalent to replacing a and b in Eq. (16) by the first terms only of the Fourier expansions, and it then follows by comparison of the resulting expression for ϵ with Eq. (12) that

$$A=a_1 \quad \text{and} \quad B=b_1. \tag{20}$$

Lastly, A and B are evaluated in terms of E_1 and Δ_1 by Eqs. (13) and (14) and a and b in terms of E and Δ by Eqs. (17) and (18). Accordingly, Eqs. (20) are equivalent to the formulas

$$\Delta_1=(2/L_1)\int_0^{L_1} \Delta \sin^2(\pi x/L_1)dx, \tag{21}$$

and

$$1/E_1=(2/L_1)\int_0^{L_1} (1/E) \sin^2(\pi x/L_1)dx, \tag{22}$$

where the assumption, justified experimentally, that the spatial variation of E is small of the order of 0.01 percent of its value, has been made.

III. EXPERIMENTAL DETAILS

Construction of the Composite Oscillator

The adhesive employed in the construction of oscillators on which measurements at room temperature are to be made is a thin film of phenyl salicylate (salol), a substance which melts at 43°C and supercools in the liquid state to room temperature. Because of this latter property it is possible to freeze the salol at room temperatures and in this way to prevent a differential thermal contraction at the quartz-specimen interface during cooling.

Strain due to differential thermal expansion at the interface has thus far defeated attempts to apply the composite-oscillator method of measuring the variation of internal friction with temperature.[6] In the present experiments this difficulty has been resolved by the use of a liquid adhesive in oscillators on which measurements below room temperature are to be made; the adhesive is a thin film of Dow Corning 50 centistoke silicone, DC200. This substance remains fluid to temperatures near that of solid CO_2.

A special apparatus is required to assemble an oscillator in which the adhesive is a liquid. The assembly procedure is as follows. Initially the component cylinders rest each upon four ball bearings which lie in parallel horizontal grooves cut in the base of the apparatus. The upper ends of a pair of fine wire loops, on which the oscillator is later supported, are cemented to a pair of parallel horizontal rods mounted above and parallel to the axis of the specimen. These rods are geared together, so that the loops can be shortened evenly by a rotational torque applied to one of them only. The lower parts of the loops are positioned very

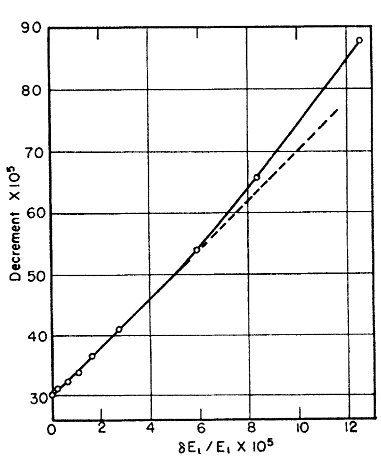

FIG. 3. Combination of the curves of Fig. 2 shows that the decrement is a linear function of $\delta E_1/E_1$ at small amplitudes.

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[6] See C. A. Wert, reference 4; T. A. Read, private communication.

accurately beneath the respective centers of gravity of the two cylinders. A thin film of adhesive covers the end of the quartz cylinder proximal to the specimen. The two cylinders are forced together by a spring loaded pin, which is released from its retracted position by rotating a screw. Lastly, the pin is again retracted and the oscillator is elevated by rotating the geared rods, until the quartz lies in proper position between the electrodes. The latter are mounted in a frame constructed of Lucite. An oscillator so constructed and suspended, remains intact for several days.

The Cryostat

The cryostatic space is a horizontal glass tube two inches in diameter and three feet long with metal flanges and gasketed plates on the ends to permit evacuation. The working fluid of the cryostat is acetone which is circulated in coils through a slush of solid CO_2 in trichloroethylene and around the cryostatic space. Temperature is measured with a calibrated copper-constantan thermocouple.

Preparation of Specimens

Single crystals of copper are grown in graphite crucibles by the Bridgman method, in an oxygen-free atmosphere. Specimens are annealed at 600°C for four hours in vacuum to remove strains induced by cutting and lapping them to the desired length. Prior to certain measurements the crystals are subjected to a measurable amount of cold-work produced by application of a known longitudinal compressive stress.

Specimen Material

The specimen material is commercial copper rod. Spectrographic analysis reveals the presence of the following percentages of impurities: Al and Fe, 0.01 to 0.001; Pb, 0.0001; Ag, 0.003; traces of Ca, Mg, and Si.

IV. EXPERIMENTAL RESULTS

The data reported here are based on an extended series of experiments on five different copper crystals. The orientations of the cylinder axes of these crystals with respect to the crystal lattice are various, hence their lengths, which correspond to the same resonant frequency, vary between 3.5 and 5 cm.

Characteristics of the Variation of Young's Modulus and Decrement with Strain Amplitude

The purpose of this section is to point out certain general aspects of the variation of E_1 and Δ_1 with strain amplitude which characterize the behavior of single crystal specimens, within the ranges explored in the present experiments.

The Quantity Δ_1^0

This quantity is defined as the limiting value of Δ_1 as the strain amplitude approaches zero. All measured values of Δ_1^0 contain unspecifiable contributions ascribable to the adhesive and to the state of the surface of the specimen. The former are less than 10^{-5} when the adhesive is salol, and close to 10^{-4} when the liquid adhesive is used. Variations of Δ_1^0 between 10^{-5} and 5×10^{-5} can be produced by etching the surface of the specimen. Experiment proves, however, that the magnitudes of both of these spurious contributions to the internal friction are *independent of strain amplitude.*

TABLE I. Dissipative characteristics of specimens *A* and *B* at room temperature.

	$\Delta_1^0\times10^5$	$C_1\times10^{-9}$	r_1
Specimen *A*	14.0	0.26	3.4
Specimen *B*	29.4	1.61	4.0

The Quantity C_1

When Δ_1 and $\delta E_1/E_1$ are plotted as functions of ϵ_{1m}^2, graphs of the type shown in Fig. 2 are obtained. It thus appears that, at low amplitudes, both quantities vary linearly with the square of the maximum strain amplitude. The slope of the line which represents the variation of Δ_1 with ϵ_{1m}^2 is here denoted by C_1; thus

$$C_1=(d\Delta_1/d\epsilon_{1m}^2)_{\epsilon_{1m}=0}. \tag{23}$$

The quantity C_1 is the most significant for the description of the dissipative property of single crystals vibrating at low strain amplitudes, for the following reasons:

(*a*) C_1, which by definition is a measure of a property of the specimen cylinder, is closely related to the measure of the corresponding property of the specimen *material.* Thus, if

$$\Delta=\Delta^0+C\epsilon_m^2, \tag{24}$$

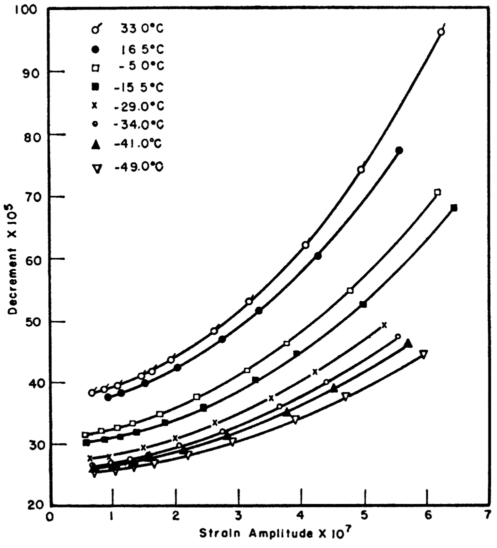

FIG. 4. Variation of internal friction with strain amplitude and temperature.

TABLE II. Variation of dissipative properties with temperature.

T, °A	$C_1\times10^{-9}$	r_1	T, °A	$C_1\times10^{-9}$	r_1
	Specimen A			Specimen B	
298.0	0.260	3.7	306.0	1.58	4.0
293.5	0.250	3.5	289.5	1.44	3.9
286.0	0.235	3.5	268.0	1.17	3.95
270.0	0.200	3.5	257.5	1.04	3.8
260.0	0.180	3.4	244.0	0.90	3.9
248.0	0.158	—	239.0	0.87	4.1
240.0	0.145	3.4	232.0	0.75	3.95
233.0	0.137	3.5	224.0	0.685	3.8
222.0	0.120	3.4			
215.5	0.110	3.3			

where $\epsilon_m=\sigma_m/E$ is the local strain amplitude, then from Eq. (21) it follows that $\Delta_1{}^0=\Delta^0$ and $C_1=\frac{3}{4}C$.

(b) C_1, unlike Δ_1, is free of error introduced by the adhesive and the state of the specimen surface; it is a measure of the dissipative property of the crystal lattice itself. Furthermore, it is an extraordinarily sensitive measure of this property; various values of C_1 observed in the course of these experiments differ by the factor 4000, while the corresponding values of $\Delta_1{}^0$ differ only by the factor 140.

The Quantity r_1

Corresponding values of Δ_1 and $\delta E_1/E_1$ obtained from Fig. 2, are plotted as ordinates and abscissae, respectively, in Fig. 3. The slope of the straight line obtained at small strain amplitudes is here denoted by r_1; thus

$$r_1=[d\Delta_1/d(\delta E_1/E_1)]_{\epsilon_{1m}=0}. \tag{25}$$

An analysis similar to the one which related C_1 to C shows that $r_1=r$, where r is the property of the material defined by an equation similar to Eq. (25) in which the subscript "1" is deleted throughout.

The values of r_1 observed in these experiments lie between 1.5 and 4.4. The quantity r_1 varies approximately monotonically with C_1; it is always less than three for annealed crystals and always greater than three for crystals that have been subjected to cold-work.

Variation of Internal Friction with Temperature

Complete studies were made of the variation of internal friction with strain amplitude in two specimens, over the temperature interval +33°C to −60°C. Both specimens were cut from the same crystal cylinder, but the two were subjected to different amounts of cold work in order to obtain specimens characterized by widely different dissipative properties. The latter are given in Table I which summarizes measurements made at room temperature on composite oscillators constructed with salol adhesive.

The observed variation of Δ_1 with temperature and maximum strain amplitude, for specimen B, is shown in Fig. 4. Computed values of C_1 and r_1 for the two specimens are given in Table II.

TABLE III. Results of measurements of internal friction at two frequencies.

	39 kc/sec.			78 kc/sec.		
No.	$\Delta_1{}^0\times10^5$	$C_1\times10^{-9}$	r_1	$\Delta_1{}^0\times10^5$	$C_1\times10^{-9}$	r_1
1	0.69	0.00074	—	0.81	0.00128	—
2	1.2	0.003	1.8	1.6	0.004	1.6
3	9.10	0.018	2.9	9.85	0.015	2.6
4	31.0	0.090	—	30.8	0.064	—
5	14.8	0.104	3.0	13.9	0.080	3.05
6	14.0	0.26	3.4	18.6	0.388	3.7
7	12.7	0.90	3.3	16.6	1.34	3.5
8	50.7	0.92	3.5	36.7	0.50	3.4
9	29.4	1.61	4.0	28.1	1.16	4.4
10	69.4	2.20	3.5	119.0	4.3	3.9

An analysis of curves such as those of Fig. 4 reveals the following: If due allowance be made for the uncertainty in the value of $\Delta_1{}^0$ by the addition of a suitable constant amount to the ordinates of each curve, then the entire family of curves for each specimen may be represented over the *entire range* of variation of temperature and strain amplitude by a function $F(T,\epsilon_{1m})$ which has the form

$$\Delta_1=F(T,\epsilon_{1m})=F_1(T)F_2(\epsilon_{1m}), \tag{26}$$

where

$$F_1(T)=Pe^{-\theta/T}, \tag{27}$$

and the value of P is chosen to be the same for both specimens. The value of θ has the same value, 700°A, for both specimens within the accuracy of these experiments, as is evident from Fig. 5, in which the natural logarithm of $F_1(T)$ is plotted as a function of $1/T$. It is clear from the form of Eq. (26) that the quantity C_1 differs from $F_1(T)$ by a constant factor.

It is worth noting specifically that the quantity r_1' is *independent of temperature.*

Variation of Internal Friction with Frequency

Measurements of the quantities $\Delta_1{}^0$, C_1, and r_1 were made upon various specimens at the fundamental frequency (39 kc/sec.) and the second harmonic frequency (78 kc/sec.). All such measurements were made at room temperature, on composite oscillators constructed with salol adhesive. Data are recorded in Table III; the first three measurements represent annealed specimens, the

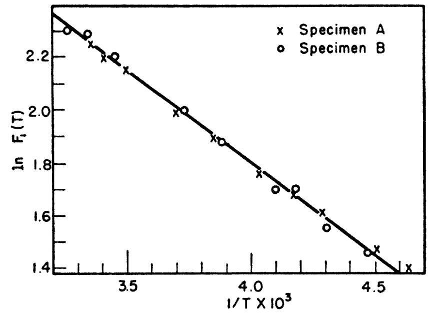

FIG. 5. The logarithm of $F_1(T)$ varies inversely with the absolute temperature.

remaining entries are for specimens which had been subjected to cold-work. The maximum compressive stress employed was 365 p.s.i.

The following general remarks summarize the results of this table. (*a*) Although both the quantities Δ_1^0 and C_1 increase with the degree of cold-work, there is no simple relationship between them. (*b*) In all but one case the changes in Δ_1^0 and C_1 between the fundamental and second harmonic are in the same direction. (*c*) Both C_1 and Δ_1^0 vary with frequency in an apparently random manner. It is particularly noteworthy that the specimens *A* and *B* which, as shown in the last section, obey almost identical laws of temperature dependence, vary with frequency in opposite directions. The data for these two specimens appear in Table III as numbers 6 and 9, respectively. (*d*) Since the experimental error in almost all measurements of r_1 is about ± 0.2, it appears that r_1 is independent of frequency within the accuracy of these experiments, over the entire range of values of C_1 and Δ_1^0 studied. Figure 6 illustrates this result for one of the measurements (No. 8) quoted in the table.

It is well known that the state of internal stress is very sensitive to previous history and to flaws in the crystal which result from accidents of growth; this "structure sensitivity" is reflected in the measured values of internal friction. It is in these terms that the apparently random results for the relative values of internal friction at the first two modes of vibration may be explained. In the fundamental mode of vibration, the maximum stress is located at the center of the specimen cylinder and the measurements obtained are primarily a result of the behavior of this part of the lattice. At the second harmonic stress loops are at $\frac{1}{4}$ and $\frac{3}{4}$ the length of the cylinder. Since the two measurements involve different parts of a highly structure-sensitive crystal lattice, it is not surprising that the frequency dependence of C_1 and of Δ_1^0 cannot be deduced from them. This argument does not apply to the quantity r_1 which is not very sensitive to history since it varies very slowly with C_1. The observation that r_1 is independent of frequency may therefore be regarded as one of the important results of these experiments.

V. DISCUSSION

A phenomenological discussion of two general dissipative mechanisms is presented in this section; the experimental results make it possible to select one of these to represent the behavior of the crystals investigated. The significance of the mechanism selected is then discussed in terms of dislocations. Since the dissipative and elastic properties of the specimen as a whole have been related to the properties of the material in an earlier section, it is necessary to consider only the behavior of an element of material under a stress, σ, which varies sinusoidally in time and produces a strain, ϵ.

In the presence of a source of dissipation (in the present case, dislocations) the strain may be written as the sum of two terms

$$\epsilon = \epsilon_1 + \epsilon_2,$$

where ϵ_1 is the purely elastic strain and ϵ_2, the contribution resulting from the presence of the dissipative process. Clearly,

$$\epsilon_1 = \sigma / E_t, \tag{28}$$

where E_t is the *true* Young's modulus of the material, to be distinguished from the measured value, E. If σ is given by Eq. (15), then ϵ, or at least its fundamental Fourier component, is given by Eq. (16), and E and Δ by Eqs. (17) and (18), respectively. The measured elastic modulus E will be equal to E_t if and only if the strain ϵ_2 is purely dissipative, i.e., contributes only to the factor b in Eq. (16) so that $a = \sigma_m / E_t$.

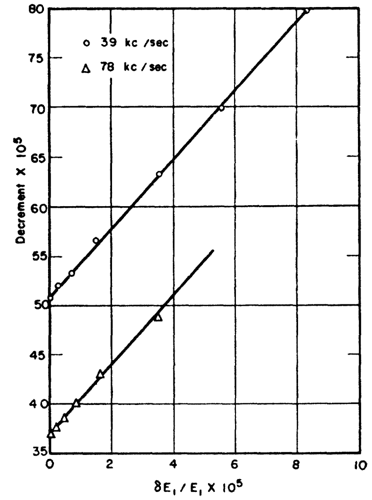

FIG. 6. The quantity r_1 is independent of frequency.

It is useful to distinguish two types of dissipative mechanisms. The first, a mechanism of *relaxation* is defined in terms of a stress-strain curve which depends on the rate of application of the stress. It is therefore characterized by the occurrence of creep at constant stress, or in vibration, by a stress-strain ellipse whose parameters depend on the frequency. The second mechanism, *simple hysteresis*, is defined in terms of a stress-strain loop that is independent of the rate of traversal. Under cyclic stress the strain may differ, for the same value of stress, during loading and unloading, but all values are independent of the frequency of the cycle. Inasmuch as all dissipative mechanisms studied to date fall into the category of relaxation (e.g., anelasticity[1] is a special case of relaxation) it seems natural to start by considering this mechanism.

The Mechanism of Relaxation

One example of relaxation is the behavior known as *pure plastic flow* in which the fundamental equation for ϵ_2 is

$$d\epsilon_2/dt = (1/\tau) f(\sigma), \tag{29}$$

where τ has the dimensions of time. This will generally be the equation of a rate process involving a Boltzmann factor which may be absorbed into the quantity τ, as follows:

$$\tau=\tau_0 e^{H/RT}, \tag{30}$$

where H is the *heat of activation* of the process. In view of the strong temperature dependence of the measured internal friction, the introduction of a heat of activation is not unreasonable. A special case of Eq. (29) is the Newtonian viscosity law, for which the equation becomes linear. It is readily shown that for pure plastic flow the strain ϵ_2 is purely dissipative so that the measured Young's modulus is the true modulus, and therefore is independent of strain amplitude and of the value of the decrement. Furthermore the decrement varies inversely as the frequency and inversely as τ, therefore as $\exp(-H/RT)$.

The concept of pure plastic flow is readily generalized as follows. Suppose that Eq. (29) is obeyed *microscopically* with the internal stress σ_i substituted for the externally applied stress, σ. The value of σ_i is dependent not only on σ but also on previous history, including the instantaneous value of the non-elastic strain ϵ_2 that has already occurred. In this case the relaxation equation becomes

$$d\epsilon_2/dt=(1/\tau)F(\sigma, \epsilon_2). \tag{31}$$

It can be shown that in this more general case the strain will have a component in phase with σ, so that an amplitude dependent Young's modulus will in general be observed.

The observed temperature dependence of internal friction as given by Eq. (27), suggests that the decrement varies very nearly inversely as τ, corresponding to the case of pure plastic flow, with a heat of activation equal to 1400 cal./mole. On the other hand, it is difficult to reconcile this temperature dependence with the experimental result that $\delta E_1/E_1$ is of the same order of magnitude as the decrement; i.e., that r is of the order unity.

The relaxation interpretation also fails to explain the frequency and temperature independence of r. As is shown in the Appendix, the expected variation is proportional to $\omega\tau$, in conflict with the experimental results.

It is possible to generalize to more complex relaxation mechanisms than that of Eq. (31), but such a generalization does not seem to lead to a resolution of the conflict between the theoretical predictions and the experimental results.

Mechanism of Simple Hysteresis

From the definition of simple hysteresis it is immediately predicted that all observable quantities are independent of frequency, if this is the mechanism that applies to the dissipative process under investigation. The experimentally observed frequency independence of the quantity r_1 is in agreement with this prediction. The fact that the ratio of C_1 at the fundamental to its value at the second harmonic is observed to lie on both sides of unity indicates that C_1 may actually be independent of frequency, when the explanation presented for the randomness of these data is taken into account.

The assumption that only fundamental Fourier components of strain are important means that the best possible ellipse is matched to the actual stress-strain loop. The effective modulus under periodic conditions, obtained from the slope of the major axis of this ellipse, is clearly less than the true modulus.

The slope of the stress-strain loop differs in the parts of the cycle where stress is increasing (loading) from its value when stress is being released (unloading); for example

$$\begin{aligned} d\epsilon_2/d\sigma &= g_1(\sigma)(\text{loading}),\\ &= g_2(\sigma)(\text{unloading}). \end{aligned}$$

Fourier analysis of this strain and the application of Eqs. (17) and (18) leads to the result

$$\Delta=2E\int_0^{\pi/2}[g_1(\sigma_m\cos\theta)-g_2(\sigma_m\cos\theta)]\times\cos\theta\,\sin\theta d\theta, \tag{32}$$

$$\frac{1}{E}-\frac{1}{E_t}=\frac{2}{\pi}\int_0^{\pi/2}[g_1(\sigma_m\cos\theta)+g_2(\sigma_m\cos\theta)]\sin^2\theta d\theta. \tag{33}$$

The experiments indicate that $g_1(\sigma)$ and $g_2(\sigma)$ are expressible in power series in which odd powers of σ do not appear. The constant terms leads to a decrement at zero amplitude and a deviation of the measured elastic modulus at zero amplitude from the true modulus; it is from the term in σ^2 that the quantities C and r result. If the function $g_i(\sigma)$ is written

$$g_i(\sigma)=\alpha_i+\beta_i\sigma_i^2 \quad (i=1, 2) \tag{34}$$

then

$$\Delta=E(\alpha_1-\alpha_2)+\tfrac{1}{2}E^3(\beta_1-\beta_2)\epsilon_m^2, \tag{35}$$

$$(E_t-E)/E=\tfrac{1}{2}E(\alpha_1+\alpha_2)+\tfrac{1}{8}E^3(\beta_1+\beta_2)\epsilon_m^2, \tag{36}$$

$$r=4[(\beta_1-\beta_2)/(\beta_1+\beta_2)], \tag{37}$$

where $\epsilon_m=\sigma_m/E$ is the strain amplitude. The most striking result is the prediction that r must lie between 0 and 4, in excellent agreement with experimental results. Only one observation (No. 9 of Table III, 78 kc) shows a value of r greater than 4.0 and the experimental error in this single case was sufficiently large to permit a true value of $r=4$. Numerous other data observed at room temperature and 39 kc, which are not tabulated here, always show values of r in the predicted range and closest to 4.0 for the most highly cold-worked crystals. This agreement with experiment indicates that the dissipative mechanism may be described, phenomenologically, in terms of simple hysteresis.

The following is a possible explanation for the observed dissipative mechanism in terms of the motion of dislocations. The activation energy for the release of dislocations from points at which they are restrained is sufficiently high so that at the high frequencies of these experiments negligible relaxation occurs. Instead, a distribution exists such that at each value of the stress a certain fraction of the dislocations will *snap loose* and move rapidly to another potential minimum, thereby contributing to the non-elastic strain. In terms of this description it is not surprising that corresponding measurements at lower frequencies[1] are of a different nature; e.g., no amplitude-dependent effects are observed. The strain caused by the time-dependent release of dislocations, a relaxation mechanism, can be expected to predominate over the time independent strain of simple hysteresis at sufficiently low frequencies. The present interpetation provides a link between the behavior of dislocations in two ranges of conditions which have hitherto been investigated independently of each other.

The large observed temperature dependence of internal friction may be explained in terms of dislocations when an auxiliary concept is introduced. Cottrell[7] has shown that a force of attraction exists between a solute (impurity) atom and certain preferred sites along a dislocation. Since the dislocations are anchored by impurity atoms located at these sites, the number of dislocations torn loose by an applied stress, and therefore the internal friction, is determined by the extent to which these favorable sites are occupied.[8] The extent of occupation, in turn, is determined by a Boltzmann factor $\exp(-V/kT)$, where V is the interaction energy between solute atom and dislocation, and is therefore a sensitive function of temperature.

The author wishes to express his appreciation to Dr. T. I. Taylor for the spectrographic analysis, to Dr. T. A. Read for valuable information about the apparatus and his own experiments, to Dr. C. Zener for helpful conversations, and to Dr. S. L. Quimby for his guidance throughout the course of this work.

APPENDIX

Calculation of the Frequency and Temperature Dependence of r from a Mechanism of Relaxation.

The observed temperature dependence of internal friction shows that if the dissipative mechanism is relaxation the behavior does not deviate greatly from pure plastic flow (Eq. (29)), so that the more general Eq. (31) can be expanded in a Taylor series

$$d\epsilon_2/dt=(1/\tau)\{F(\sigma,0)+\epsilon_2(\partial F/\partial\epsilon_2)_{\epsilon_2=0}+\cdots\}.$$

The fact that the observed decrement varies parabolically with strain amplitude at low amplitudes means that the above equation relating ϵ_2 and its time derivative to σ has only linear and cubic terms; thus

$$d\epsilon_2/dt=(1/\tau)\{c_1\sigma+c_2\sigma^3+\epsilon_2(c_3+c_4\sigma^2)\}.$$

If now the stress σ is sinusoidal, the internal friction and elastic modulus are obtained from the components of $d\epsilon_2/dt$ in phase and out of phase with the stress and from these quantities it is found that r is proportional to $\omega\tau$.

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[7] A. H. Cottrell, *Rep. Conf. on Strength of Solids* (Phys. Soc. London, 1948), p. 30.

[8] The unusually high internal friction of spectroscopically pure copper observed by J. W. Marx and J. S. Koehler, Phys. Rev. **75**, 1309A (1949), supports this interpretation.