On Spin-Orbit Interactions and Nucleon-Nucleon Scattering

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It is shown that one way of explaining the high energy proton-proton scattering data is by means of a spin-orbit coupling introduced into the nuclear interaction. The radial dependence of this coupling should be such that it has a strongly singular behavior at $r=0$ and a small tail. It seems feasible to retain the charge independence of nuclear forces for all energies explored so far. A possible connection with M. Mayer's interpretation of the shell model of heavy nuclei is indicated.

I. INTRODUCTION

THE experiments recently performed in Berkeley on the scattering of high energy neutrons and protons by protons show the following striking qualitative features, even when due account is taken of the present experimental uncertainties.

n-p Scattering

(*a*) The total cross section is "small." By this is meant that any *n–p* interaction which is made to fit the low energy data also describes the high energy scattering to a good approximation, provided the latter were essentially only due to *S*-wave scattering. In this way, one may, for example, account for about 90 percent of the 90-Mev *n–p* cross section,[1] while the 260-Mev data[2] still show a similar trend.

(*b*) The differential cross section $\sigma(\theta)$ in the center-of-mass system has a tendency toward symmetry around 90°. It is at present not quite clear to what extent this is true, especially as few data are available below about 40° scattering angle. Yet qualitatively this symmetry is rather unexpectedly marked from the point of view of any of the customary "symmetrical," "charged," or "neutral" phenomenological approaches.

p-p Scattering

(*a*) At 350 Mev the cross section is quite large. In fact $\sigma(90°)$ is from two to four times larger than the corresponding quantity for *n–p* scattering at the lower energy of 260 Mev.

(*b*) At 350 Mev $\sigma(\theta)$ is rather isotropic from 90° down to angles of approximately 20°. This contrasts with the marked anisotropy of the 260-Mev *n–p* data over the same angular region.

(*c*) At 30 Mev $\sigma(\theta)$, after a rapid decrease away from small angles due to Coulomb effects, rises again to reach a rather flat behavior around 90°.

These aspects of the nucleon-nucleon scattering are particularly interesting in that at first sight they would seem to indicate a sizable difference between the *n–p* and *p–p* interaction. This is in contradistinction to the well-known low energy charge independence, according to which the *n–p* and *p–p* forces are the same in identical states; i.e., the ^1S, ^3P, $^1D\cdots$ *n–p* forces should be the same as those for the *p–p* system. (Of course at these low energies only *S* scattering occurs.)

To see this, we first consider the theory of the *n–p* scattering given by Christian and Hart.[1] Using a mixture of central and tensor forces adapted to the low energy data, they propose an exchange dependence of such a kind that the interaction is zero (or very small) in states of odd orbital angular momentum *L*. This proposal, made by Serber, will in the following be referred to as the "even-theory." In this way the *P* phases, next largest to the *S* phases, are suppressed. This leads to the desired effects both of small σ as well as to symmetry around 90°, there being no interference between waves of even and odd *L*. As central forces give (for high energies) a negligible contribution at 90°, the large angle scattering is here entirely due to the tensor force acting in even triplet states. Now these states drop out in the *p–p* interaction, so that it is clear that the even-theory applied to *p–p* scattering would give a $\sigma(90°)$ of an order of magnitude smaller than that for the *n–p* case, in conflict with the experimental findings mentioned above. Moreover, the even-theory would not yield the observed *p–p* angular distribution at 30 Mev. In fact, due to the interference of singlet *S*- and *D*-waves, it would lead to a $\sigma(\theta)$ rising away from 90°.

An interpretation of the *p–p* data has been given by Christian and Noyes.[3] Renouncing charge independence, they introduce a strongly singular tensor force for the *p–p* system. This non-central coupling enables them to mask the drop in $\sigma(\theta)$ toward 90° at 30 Mev. Furthermore, the *p–p* tensor force is more strongly singular than is the Yukawa-type tensor force used in the above *n–p* interpretation. Hence, more high frequency Fourier components are present so that the possibility of relatively large momentum transfers exists. In this way the high 90° *p–p* scattering can be accounted for.

It may directly be remarked here that, whatever interpretation one is aiming at, the presence of a

[1] R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950). This paper contains the experimental results for 40- and 90-Mev scattering.

[2] Communicated to us by Dr. Christian to whom we are greatly indebted for this, as well as other, information on the work performed in Berkeley. We also wish to thank Professor H. Feshbach for informing us of his work on the potential constants of the *n–p* interaction.

[3] Most of the data are taken from R. Christian and H. P. Noyes report, Phys. Rev. **79**, 85 (1950).

strongly singular force which is more effective for p–p than for n–p scattering seems to be a fact which is hard to escape on the basis of the present data. Another way of introducing such a singular force between protons has been suggested and analyzed by Jastrow.[4] It consists in assuming the nuclear p–p attraction to go over into a very strong repulsion at distances of about $0.6\cdot 10^{-13}$ cm. As a consequence, high Fourier components are again provided to account for the high cross section at 350 Mev, while a resulting decrease in the ^1D phase at 30 Mev tends to flatten the cross section for this energy around 90°.

It is the aim of this paper to present an alternative qualitative interpretation of the scattering data; *viz.*, by the introduction of a spin-orbit coupling of the type familiar from atomic interactions:

$$V(r)\cdot(\mathbf{L}\cdot\mathbf{S}), \quad \mathbf{S}=\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)} \tag{1}$$

where S is twice the spin operator of the two-body system and L its relative orbital angular momentum vector. Again, $V(r)$ has to be strongly singular (Eq. (18)).

As will be shown in the following, it would seem to be feasible to obtain a charge independent phenomenological description of nuclear forces for all energies explored so far. This is possible, despite the dissimilarity in the observed cross sections. Indeed, there are extra states present for the n–p system that do not occur in the case of similar particles. Thus, forces in the ^3S, ^1P, $^3D\cdots$ states may, at sufficiently high energy have considerable influence on n–p scattering, without affecting the corresponding p–p case. Second, owing to the Pauli principle, states of interest for p–p scattering are weighted differently in the n–p cross section. It will turn out in the subsequent sections that these two circumstances still give sufficient leeway to allow at least qualitatively for charge independence. In this connection it should also be noted that Jastrow's hard core model can be reconciled in principle with charge independence by appropriately choosing the exchange dependence of the forces involved.[4]

The condition of charge independence for all energies is a suggestive requirement in view of the close similarity of n–p and p–p interactions in the ^1S states, deduced from low energy scattering. Such a generalization is a reasonable extrapolation from the little that is known of nuclear forces. However this may be, the main interest in a phenomenological approach to nuclear forces lies in testing heuristic principles which might be a guide in the search for a satisfactory field theory. It must be remembered though that charge independence cannot be considered as imposed at the outset, either from the available experimental evidence or from first principles, which are now notoriously inadequate.

It should be mentioned that all forces which will be considered in the present investigation are of a type compatible with present field theories. Thus, the existence of a spin-orbit coupling (1) is to be expected as a Thomas effect accompanying static forces. However, it will be seen in the next section that we actually need a stronger coupling than that predicted as a Thomas phenomenon. Such stronger LS interactions are not irreconcilable with present theoretical views.[5] It may furthermore be remarked that a strong LS coupling might be of help in explaining the spin-orbit interactions in heavier nuclei postulated by M. Mayer[6] (Section IV(a)).

It has been pointed out by Wigner and Eisenbud[7] that LS coupling is not the only conceivable spin-orbit interaction in the nuclear two-body problem. In fact, one can have also interactions proportional to

$$\mathbf{L}\cdot(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}), \quad \text{or} \quad \mathbf{L}\cdot(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)}) \tag{2}$$

which, however, can be effective only between nucleons in different charge states. Thus, they cannot contribute to the p–p interaction which we have seen to be the primary cause for exploring modifications of the more customarily examined interactions.

Our program will now be as follows. First we shall examine the main qualitative features of an LS interaction (Section II), then we shall show that by superimposing this coupling on an interaction like that used in the even theory of n–p forces, one can understand the qualitative features of p–p scattering (Section III). Thereupon, we will investigate the charge independent features of this interaction and will see that, within the experimental uncertainties, the introduction of the additional LS coupling in the n–p interaction does not spoil the qualitative agreement with experiment obtained by Christian and Hart using the even-theory solely (Section IV). In the course of this part of the work we shall need some formulas which are generalizations of the standard Faxén-Holtsmark results to the presence of LS interactions. These formulas will be derived in Appendix I.

[4] R. Jastrow, Phys. Rev. **79**, 389 (1950). This possibility is also being investigated by N. Kroll.

[5] Thus, the vector meson theory with both vector and tensor coupling can give large LS interactions. See, e.g., B. Holmberg, Kungl. Fysiogr. Sällskapets Förh., Lundies **14**, Nr. 22 (1944). L. Rosenfeld, Kgl. Dansk Vid. Selsk. Math.-Fys. Medd. **23**, Nr. 13 (1945). Moreover, such effects can be expected from a theory consistently taking account of the reactive phenomena.

[6] M. Goeppert-Mayer, Phys. Rev. **75**, 1969 (1949); **78**, 16, 22 (1950); Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949). It should of course not be forgotten that one cannot, from a spin-orbit interaction in the many-body problem, uniquely infer back to such an interaction between pairs of nucleons. Thus, for example, also tensor forces in the two-particle coupling may lead to spin-orbit effects in heavy nuclei, see A. M. Feingold and E. Wigner, Phys. Rev. **79**, 22 (1950).

[7] L. Eisenbud and E. Wigner, Proc. Nat. Acad. Wash. **27**, 281 (1941). It is readily seen that the two interactions (2) are effectively equivalent.

II. QUALITATIVE FEATURES OF SCATTERING CROSS SECTIONS

Consider the two-nucleon Schroedinger equation in the center-of-mass system:

$$\begin{aligned}[(\hbar^2/M)\Delta+E-O^{(1)}(A_1+A_2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}+A_3S_{12})\\ \times(\hbar^2/Mr_0^2)\phi_1(r/r_0)\\ -O^{(2)}A_4\phi_2(r/r_0)\mathbf{L}\cdot\mathbf{S}(\hbar^2/Mr_0^2)]\psi=0, \quad (3)\\ S_{12}=3r^{-2}(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{r})(\boldsymbol{\sigma}^{(2)}\cdot\mathbf{r})-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}.\end{aligned}$$

Here $O^{(1,2)}$ express the exchange character of the forces involved; we take this, as well as the radial dependence, to be the same for central and tensor forces. We omit the Coulomb force between protons as this is irrelevant for the energies and angles with which we will be concerned.

Let

$$P_s=\tfrac{1}{2}[1+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}],\quad P_\tau=\tfrac{1}{2}[1+\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}],$$
$$P_x=\text{space exchange operator.}$$

Then we decompose $O^{(1)}$ as follows:

$$O^{(1)}=\tfrac{1}{2}(1-P_s)(a_sP_\tau+b_s)+\tfrac{1}{2}(1+P_s)(-a_tP_\tau+b_t), \quad (4)$$

$a_s, \cdots, b_t$ depend on the specific exchange type of the forces. In (4) the charge independence is manifest. We normalize these quantities such that

$$a_s+b_s=1,\quad a_t+b_t=-3. \quad (5)$$

Then for the ^1S state, $O^{(1)}=1$, for the ^3S state, $O^{(1)}=-3$. In particular we have for the even-theory

$$a_s=b_s=\tfrac{1}{2},\quad a_t=b_t=-\tfrac{3}{2}. \quad (6)$$

A decomposition of the type (4) need not be made for $O^{(2)}$, as the LS coupling vanishes in singlet states. Taking into account the exclusion principle we may therefore write

$$O^{(2)}=-\alpha P_\tau+\beta=\alpha P_x+\beta. \quad (7)$$

Introducing r_0 as unit of length and putting $r=xr_0$, (3) becomes

$$\begin{aligned}[\Delta+k^2-O^{(1)}(A_1+A_2\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}+A_3S_{12})\phi_1(x)\\ -O^{(2)}A_4\phi_2(x)\mathbf{LS}]\psi=0,\\ k=(ME_0r_0^2/2\hbar^2)^{\frac{1}{2}};\end{aligned}$$

E_0 is the energy of the incident particle in the laboratory system.

Now it is clear that spin-orbit effects vanish in S states, so that the low energy data, due to $A_{1,2,3}$ can be taken over without modification.[8] We have therefore only to consider triplet states, $L>0$ in dealing with the A_4 term.

As a first orientation we consider its effect in the Born approximation in which no interference occurs with the scattering due to central and tensor forces. That this is so follows from

$$(\mathbf{L}\cdot\mathbf{S})_{\text{Av}}=0,\quad \{(\mathbf{L}\cdot\mathbf{S})(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{r})(\boldsymbol{\sigma}^{(2)}\cdot\mathbf{r})\}_{\text{Av}}=0, \quad (8)$$

where Av means an averaging over the initial and a summation over the final spin states. In our units the LS-contribution to the n–p cross section is

$$d\sigma_{LS}^{np}=(r_0/4\pi)^2A_4^2\left|\int\exp(-i\mathbf{k}'\cdot\mathbf{x})(\alpha P_x+\beta)\times\phi_2(x)\mathbf{L}\cdot\mathbf{S}\exp(i\mathbf{k}\cdot\mathbf{x})d\mathbf{x}\right|^2_{\text{Av}}d\Omega, \quad (9)$$

where $\mathbf{k}$, $\mathbf{k}'$ are the initial and final wave vectors, respectively. Since $\mathbf{L}=\mathbf{x}\times\mathbf{p}$, it follows that the matrix element must be proportional to $\mathbf{k}\times\mathbf{k}'$ and hence

$$d\sigma_{LS}\sim|\mathbf{k}\times\mathbf{k}'|^2=k^4\sin^2\theta,$$

where θ is the scattering angle. This property of the LS interaction is, of course, significant in that it shows the tendency to give preference to 90° scattering. Our particular interest in this respect is focussed on the p–p cross section, however, and we must now find what the counterpart of (9) is for that case.

For this purpose we need merely replace $\exp(i\mathbf{k}\cdot\mathbf{x})$ in (9) by $(1-P_x)\exp(i\mathbf{k}\cdot\mathbf{x})$ to obtain the appropriate antisymmetrization as well as normalization. Hence, for p–p scattering

$$d\sigma_{LS}^{pp}=(r_0/4\pi)^2A_4^2\left|\int\exp(-i\mathbf{k}'\cdot\mathbf{x})(-\alpha+\beta)\times(1-P_x)\phi_2(x)\mathbf{L}\cdot\mathbf{S}\exp(i\mathbf{k}\cdot\mathbf{x})d\mathbf{x}\right|^2_{\text{Av}}d\Omega. \quad (10)$$

In (9) and (10) P_x may alternatively be regarded as an operator reversing the direction of $\mathbf{k}$. As the integrals are clearly odd functions of $\mathbf{k}$ for the case that $\mathbf{k}$ and $\mathbf{k}'$ are perpendicular to each other, one may, in considering the 90° scattering, replace P_x by -1. Hence,

$$\sigma_{LS}^{p-p}(90°)=4\sigma_{LS}^{n-p}(90°), \quad (11)$$

which is suggestive in view of the experimentally found ratio of 90° p–p and n–p scattering. It should be noted, however, that the connection (11) can be of help only if the charge dependence of the LS interaction is *not* that of the even-theory. Indeed, according to (6) we would have in that case $\alpha=\beta$ and σ_{LS}^{p-p} would vanish identically (not only in the Born approximation, but also rigorously).

To compare in more detail the n–p with the p–p cross section, we shall take as a simple model a mixture of central and tensor forces with the charge dependence (6) of the even-theory and an LS force with an as yet unspecified, though different, charge dependence. Then at high energies:

$$\sigma^{n-p}(90°)=\sigma_{\text{even}}^{s}(90°)+\sigma_{\text{even}}^{tr}(90°)+\sigma_{LS}^{n-p}(90°),$$
$$\sigma^{p-p}(90°)=4\sigma_{\text{even}}^{s}(90°)+4\sigma_{LS}^{n-p}(90°),$$

[8] We shall disregard here the influence of the A_4 term on the small $^3D_1$ portion of the deuteron ground state wave function. See also Section IV(a) and footnote 16.

where "s" and "tr" denote the singlet and triplet contributions, respectively. As already mentioned,

$$\sigma_{\text{even}}{}^{s}(90°)\ll\sigma_{\text{even}}{}^{tr}(90°)$$

so that the factor 4 occurring in the singlet p–p contribution is no great help. It is, however, possible to exploit the LS ratio to advantage as we shall now see.

III. p–p SCATTERING

In line with the model which we employ, it is possible to use the singlet contributions to the cross section as calculated by Christian and Noyes,[3] but of course not the triplet part, which in the present scheme is entirely due to the LS interaction.

(a) Scattering at $\sim$30 Mev

For this energy it suffices to take into account only the ^1S-, ^1D- and ^3P-waves. In reference 3 curves are given describing the separate SD contribution for various shapes of the singlet potential.[9] We shall here not be concerned with the details of this radial dependence. We note that it is possible on the one hand to fit the experimental points at angles $\sim$20–30°, while then on the other hand, the SD curves drop steadily instead of following the observed rise toward a rather flat relative maximum at 90°. We shall now show that this additional hump, having an estimated magnitude of about 2 mb at 90° can be described as a spin-orbit effect.

Indeed, as follows from the expression of $\sigma(\theta)$ given in Appendix I, the partial cross section for ^3P scattering due to LS coupling is given by

$$\begin{aligned}\frac{2r_0^2}{k^2}\Big\{&\frac{17}{4}\sin^2\delta(^3P_2)+\frac{9}{4}\sin^2\delta(^3P_1)+\frac{1}{2}\sin^2\delta(^3P_0)\\&+\frac{9}{2}\cos(\delta(^3P_2)-\delta(^3P_1))\sin\delta(^3P_2)\sin\delta(^3P_1)\\&+2\cos(\delta(^3P_2)-\delta(^3P_0))\sin\delta(^3P_2)\sin\delta(^3P_0)\\&-\sin^2\theta\Big[\frac{21}{8}\sin^2\delta(^3P_2)+\frac{9}{8}\sin^2\delta(^3P_1)\\&+\frac{27}{4}\cos(\delta(^3P_2)-\delta(^3P_1))\sin\delta(^3P_2)\sin\delta(^3P_1)\\&+3\cos(\delta(^3P_2)-\delta(^3P_0))\sin\delta(^3P_2)\sin\delta(^3P_0)\Big]\Big\}.\end{aligned}\qquad(12)$$

Here $\delta(3Pj)$ denotes the phase shifts for ^3P states with total angular momentum j. It should be noted that we are interested in a large angle effect so that Coulomb interference is not relevant. In this connection it is, moreover, worth mentioning that in the Born approximation the Coulomb—LS—interference actually vanishes, owing to the first of the relations (8). As we will see, only small P phases will come into play for the energy under consideration. In the appropriate approximation, (12) then becomes

$$\begin{aligned}\sim\frac{2r_0^2}{k^2}\Big\{&\frac{17}{4}\delta^2(^3P_2)+\frac{9}{4}\delta^2(^3P_1)+\frac{1}{2}\delta^2(^3P_0)\\&+\frac{1}{2}\delta(^3P_2)\delta(^3P_1)+2\delta(^3P_2)\delta(^3P_0)\\&-\sin^2\theta\Big[\frac{21}{8}\delta^2(^3P_2)+\frac{9}{8}\delta^2(^3P_1)\\&+\frac{27}{4}\delta(^3P_2)\delta(^3P_1)+3\delta(^3P_2)\delta(^3P_0)\Big]\Big\}.\end{aligned}\qquad(13)$$

To simplify this further we make use of the Born approximation in which the scattering phases are proportional to the potential strengths. Now we have for the eigenvalues of $\mathbf{L}\cdot\mathbf{S}$:

$$\mathbf{L}\cdot\mathbf{S}=j(j+1)-l(l+1)-2\qquad(14)$$

and hence

$$\delta(^3P_2):\delta(^3P_1):\delta(^3P_0)=1:-1:-2.\qquad(14\text{a})$$

Inserting this into (13), the ^3P-cross section becomes

$$\sim(18r_0^2/k^2)\delta^2(^3P_2)\cdot\sin^2\theta,\qquad(15)$$

having an angular dependence which is just that suggested by our interpretation of the 30-Mev data. To get the desired 2-mb cross section at 90° we need a phase shift

$$\delta(^3P_2)\sim3.7°.\qquad(16)$$

Any $\phi_2(r)$ in (3) which yields (16) will therefore reproduce the desired effect. This evidently leaves much arbitrariness which, however, is considerably reduced by a simultaneous inspection of the 30- and the 350-Mev data.

(b) Scattering at 350 Mev

Here we consider the Born approximation for the entire potential. Neglecting again all Coulomb effects, we find

$$\begin{aligned}r_0^{-2}\sigma(\theta)=&\tfrac{1}{4}(A_1-3A_2)^2[a_s'Y_1(K_1)+b_s'Y_1(K_2)]^2\\&+\tfrac{3}{4}(A_1+A_2)^2[a_t'Y_1(K_1)+b_t'Y_1(K_2)]^2\\&+6A_3^2[a_t'^2Y_2^2(K_1)-a_t'b_t'Y_2(K_1)Y_2(K_2)+b_t'^2Y_2^2(K_2)]\\&+2A_4^2k^4\sin^2\theta[\alpha'Y_3(K_1)-\beta'Y_3(K_2)]^2,\end{aligned}\qquad(17)$$

$$a_s'=b_s'=a_s+b_s,\qquad K_1=2k\cos\theta/2,$$
$$a_t'=-b_t'=a_t-b_t,\qquad K_2=2k\sin\theta/2,$$
$$\alpha'=-\beta'=\alpha-\beta,$$

[9] See reference 3, Fig. 10, the curves marked $V_t=0$.

and

$$Y_1(K)=K^{-1}\int_0^\infty r\phi_1(r)\sin Kr dr,$$

$$Y_2(K)=K^{-1}\int_0^\infty r\phi_1(r)\times[\sin Kr+3(Kr)^{-1}\cos Kr-3(Kr)^{-2}\sin Kr]dr,$$

$$Y_3(K)=K^{-3}\int_0^\infty r\phi_2(r)[\sin Kr-Kr\cos Kr]dr.$$

For that part of (17) which is due to central and tensor forces we refer to the work of Ashkin and Wu, and of Rohrlich and Eisenstein.[10] The spin-orbit term is obtained using (8) and

$$\{(\mathbf{S}\cdot\mathbf{K})^2\}_{\mathrm{Av}}=K^2/2.$$

In estimating the 350-Mev cross section, one must consider the importance of relativistic corrections to a formula like (17). A consistent relativistic treatment falls, of course, outside the scope of the present work, where only phenomenological couplings are considered without reference to the dynamics of the interaction which generates them. Hence, the best we can do is to estimate the relativistic effects insofar as kinematical aspects alone are concerned. As will be shown in Appendix II, one obtains in this way corrections of around 20 percent to the non-relativistic value of σ. Clearly, therefore, it is hardly worth while to attempt to fit the 350-Mev data in too great detail as, moreover, corrections of that order due to the dynamical features neglected here may well be expected. Moreover, it follows that for still higher energies estimates of the present kind will be even less trustworthy. Thus the energy range covered till now in scattering experiments seems to extend about as far as one can analyze without a more refined theory of nuclear forces.

Using the first line of (17) and (18) we first estimate the contribution due to the singlet potential as given by the even-theory. For this purpose we take a Yukawa potential with range of $1.18\cdot 10^{-13}$ cm and depth 45.8 Mev. In the present units this corresponds to $3A_2-A_1=1.58$, while for 350 Mev $k\doteq 2.44$. From this one obtains $\sim$0.2 mb /sterad at 90°, in contrast with the experimental value[11] of $\sim$4 mb. Thus, on the present assumptions, the latter has to be considered as entirely an effect of spin orbit coupling.

Remembering that we also wish to account for 2 mb at 90° and 30 Mev as being due to *LS* coupling the ratio of $\sigma_{LS}(90°)$ for the energies 350 and 30 Mev is about 2. If we use the Born approximation at both energies this ratio is independent of the strength of the *LS* coupling. We now show that it depends very sensitively on its range and shape.

(c) Shape and Magnitude of the *LS* Interaction

We first consider the examples of a square well, a Yukawa potential and an $r^{-2}\exp(-r/r_0)$ potential, respectively, with $r_0=1.18\cdot 10^{-13}$ cm in each case. For the above ratio we obtain in this way: square well 26; Yukawa 0.066; $r^{-2}\exp(-r/r_0)$ 1. This behavior can easily be understood qualitatively. The square well has no tail; hence it gives too little scattering at 30 Mev so that the ratio comes out too large. The Yukawa potential has too much of a tail on the one hand (yielding too big a 30-Mev scattering), and is too weakly singular on the other hand, so that the 350-Mev cross section is too small. Hence, the ratio is much too small. For the $r^{-2}\exp(-r/r_0)$ case, the balance between the degree of singularity and the amount of tail is already much better. It is, in fact, possible here to get the right ratio of about 2 by decreasing the range by factor of approximately 2. The various *LS* parameters then turn out to have values sizably different from those of the central and tensor forces.

This is not the case if we take the radial dependence of the *LS* coupling to be

$$\phi_2(x)=\frac{1}{\lambda x}\frac{d}{d(\lambda x)}\frac{e^{-\lambda x}}{\lambda x},\tag{18}$$

where λ is the ratio of the range $1.18\cdot 10^{-13}$ cm of the central forces to that of the spin-orbit interaction. The choice (18) is in some way the most natural one to make from a field theoretical point of view. Also, we shall show presently that there is more experimental evidence which makes (18) preferable over other choices. For $\lambda\approx 1.1$ (corresponding to a range of $\sim 1.1\cdot 10^{-13}$ cm) one actually obtains the desired ratio of 2. Then in order to fit the $\sigma(90°)$ at 350 Mev to its value of $\sim$4 mb one obtains for the strength of the interaction

$$A_4\alpha'\sim 0.4$$

or $\sim$12 Mev in customary units. We note in passing that it is impossible in this way to find out anything about α' separately; i.e., about the precise exchange dependence of the spin-orbit interaction.

It should be emphasized that the *LS* contribution to the 30-Mev *p*–*p* scattering decreases very rapidly with increasing λ. The corresponding quantity at 350 Mev varies little with λ, so the ratio considered above is very sensitive to changes in the range occurring in (18). Indeed, one finds a ratio 1 for $\lambda=1$ and 4 for $\lambda=1.2$. Hence there are several reasons which make it premature to give here any but preliminary estimates for the parameters. First, there are the experimental uncertainties. Second, an inclusion of a relatively small amount of a central force in triplet states cannot be ruled out. This would lead to a contribution $\sim\cos^2\theta$ to

[10] J. Ashkin and T. Y. Wu, Phys. Rev. **73**, 973 (1948); F. Rohrlich and J. Eisenstein, Phys. Rev. **75**, 705, 1411 (1949).
[11] The values quoted in reference 3 are higher. The present value was communicated to us by Professor Chew.

the ^3P scattering cross section, and hence to an increase in the amount of 90° scattering one might like to account for as a spin-orbit effect. Finally, the Born approximation used here in estimating the effects of the LS coupling is rather crude (see below).

There is now a further feature of the p–p scattering which calls for our attention, namely the flatness of the differential cross section at 350 Mev. Here the radial dependence makes itself manifest in an even more striking way, as is exemplified by the following table which gives the ratio $\sigma(30°)/\sigma(90°)$ as a function of the shape:

(a) exponential	930;	(c) $r^{-2}\exp(-r/r_0)$	4.7;
(b) Yukawa	40;	(d) ϕ_2 from Eq. (18)	1.6;
(e) square well	0.42.		

The range has been taken $1.18\cdot10^{-13}$ cm with the exception of case (d), where again the range $1.1\cdot10^{-13}$ cm was used. Also, this ratio is independent of the potential strength (in the Born approximation). Its strong shape dependence shows again that we need a potential which is not only strongly singular, but which also has a small tail. This tail may, however, not be negligibly small, as we see from case (e) where the cross section decreases by a factor about 2 in going from 90° to 30°. Case (d) seems to be the most suitable one. Here, too, we have found a marked sensitivity to the value of λ, small increases of which tend to flatten $\sigma(\theta)$ even more.

Actually, it should be possible, when more experimental evidence will be available, to distinguish between the various cases in quite another manner. In case (d), $\sigma(90°)$ is, in the Born approximation,

$$\sigma(90°)\approx 8(A_4\alpha')^2k^4r_0^2/(\lambda^2+2k^2)^2, \tag{19}$$

from which one infers that $\sigma(90°)$ for p–p scattering should vary but little over an energy range from $\sim$150 Mev upward until relativistic effects make themselves strongly felt. This behavior contrasts with that which is to be expected for the cases (c) and (b), in which the corresponding energy dependence will be like E_0^{-1} and E_0^{-2}, respectively.

In all estimates made here we have used the Born approximation. It need hardly be said that this is a crude procedure, especially for the more singular potentials.[12] In particular, one must view with some trepidation the Born treatment of the potential (18) with its ^{3} singularity. On the other hand, one need not take this singularity too seriously in order to obtain the qualitative results stated above. In fact, one may consider (18) to be valid only down to the nucleon Compton wave-length, and then to be cut off at smaller distances. Even for the 350-Mev data this will not cause a drastic change, since the corresponding wave-length is still about 2.5 times larger than $\hbar/Mc$. Thus it seems to us to be fair to say that the p–p scattering data can be understood by considering an LS interaction with a radial dependence which has a strongly singular behavior and a small tail.

So far we have not made any commitments as to the sign of the spin-orbit coupling. This will now be determined from the n–p interaction.

IV. *LS* EFFECTS IN THE *N-P* SYSTEM

(a) Bound States

It has been stated that low energy phenomena are essentially unaltered by the LS term. Here one point requires closer inspection; *viz.* the influence on the $^3D_1$ part of the deuteron ground state. If the spin-orbit coupling were attractive in the $^3D_1$ state, the strong singularity would greatly counteract the centrifugal repulsion. This would result in a large $^3D_1$ admixture in the deuteron, in contradiction with the information obtained from the magnetic moment and electric quadrupole moment measurements. However, if the LS coupling is repulsive in this state, it will add to the already large centrifugal repulsion and hence have little effect on the deuteron ground state. Therefore, it would seem to be most reasonable to assume the spin-orbit term repulsive in the $^3D_1$ state. With this choice it follows from Eq. (14) that for given orbital angular momentum, the LS-interaction is attractive for the state $j=l+1$ and repulsive for $j=l$ and $j=l-1$.

The existence of the LS-coupling will also modify the ^3P states. In particular, it is important to see whether the singular behavior might result in some of these states being bound[13] From the above sign determination it follows that the $^3P_2$ potential is attractive, while the $^3P_1$ and $^3P_0$ are repulsive and of absolute magnitude once and twice as great, respectively. Thus, the problem is whether a bound $^3P_2$ state exists.

To investigate this we have considered as a simple model a square well which is so deep (namely corresponding to a strength $V_0\sim$130 Mev in the $^3P_2$ state, for $r_0=1\times10^{-13}$ cm) that it duplicates the results of our potential at 350 Mev. The model is relevant, since we have to examine whether a potential deep enough at small distances to account for the 350-Mev p–p data does or does not yield bound states. (The slight tail required at 30 Mev is not particularly significant in this connection.)

For a square well the condition for bound P-states is:

$$MV_0r_0^2/\hbar^2>\pi^2.$$

[12] As the LS interaction only is present for states with $L\geq1$, there is of course no question of divergence of the integrals. For example, in evaluating the P phases in the Born approximation for the dependence (19) one has

$$\int_0^\infty\left(\frac{1}{x}\frac{d}{dx}\frac{e^{-x}}{x}\right)J_{3/2}^2(kx)xdx = -\frac{1}{2\pi}\left[\frac{1}{2k}+k-\frac{1}{2k}\left(1+\frac{1}{4k^2}\right)\ln(1+4k^2)\right]$$

which varies as k^3 for small k.

[13] This was kindly pointed out to us by Professor Oppenheimer.

Hence, with the above values for the parameters a bound $^3P_2$ state cannot exist. It should be noted that in the $^3P_0$ state the absolute magnitude of the depth is $2V_0$. Therefore, had we chosen the other sign for the LS-interaction, the question of bound states would have been uncomfortably acute. Thus, the sign of the LS-coupling is such as to bind tighter states of highest total angular momentum (for a given value of l).

Here it is interesting to note a possible connection with the spin orbit interaction postulated by M. Mayer in her analysis of heavy nuclei.[6] The interpretation by this author also requires the sign of the LS-term to be such that states of highest j lie lowest. Furthermore, we have estimated how the present two-body spin-orbit coupling manifests itself in the interaction between a nucleon and a closed core in a heavy nucleus shell model.[14] Using a simple model according to which the nucleon moves in a homogeneous distribution of nuclear matter, it is found that intra-multiplet differences of the order of an Mev may be expected. This agrees in order of magnitude with M. Mayer's requirements.

With the sign as determined above, the ratio (for large l) of states in which the term is attractive to those in which it is repulsive is 1:2, and so tends to favor saturation. This is badly needed, since the even-theory involves only attractive forces. However, it should be borne in mind that this favorable effect may be counterbalanced by the strongly attractive force introduced in some states by the LS-coupling.

(b) *n-p* Scattering

The principal question here is how the addition of the LS interaction to the potential of the even theory will affect Christian and Hart's interpretation of the $n-p$ data.

While the exchange dependence of the LS interaction now of course becomes relevant, there is one statement which still can be made independent thereof: the relation between $\sigma_{LS}(90°)$ for the $p-p$ and the $n-p$ systems as given by Eq. (11). Assuming the form (18) for the radial dependence and keeping in mind the approximate constancy of $\sigma(90°)$ for energies greater than 150 Mev mentioned above, $\sigma_{LS}(90°)$ is $\sim$1 mb for the $n-p$ scattering at 260 Mev, at which energy experimental results are now available.[2] The data are at present still beset with considerable experimental uncertainties; the 90° scattering seems to lie between 1 and 2 mb/sterad. A precise determination of $\sigma(90°)$ for the $n-p$ case at high energies is of importance for the examination of the relative influences of tensor and spin-orbit forces. However, it would seem to be premature at the present stage to draw any quantitative conclusions on this point.[15]

It remains to consider the full angular dependence of the various types of forces. For this purpose the Born approximation (17) may again serve as a guide. Equation (17) is made to apply to the $n-p$ problem by dropping the dashes on the quantities a', b', α', β'.

The exchange dependence of the LS interaction is described by the ratio of α to β. As an instructive example one may consider that choice of α and β which leads to a $\sigma_{LS}{}^{n-p}$ symmetric around 90°. This can be done by taking $\alpha:\beta=1:-1$. It is immediately seen that then $\sigma_{LS}{}^{n-p}(\theta)$ is one-fourth of $\sigma_{LS}{}^{p-p}(\theta)$ for all angles. Bearing in mind the flatness of the high energy $p-p$ cross section, it follows that for angles not too close to 0° or 180°, the present determination[16] of α and β will give an approximately constant contribution of $\sim$1 mb/sterad. From this simple consideration it is already clear that, in view of the scanty evidence on high energy $n-p$ scattering and the remaining arbitrariness in the choice of the potential parameters, it is hardly possible to draw any conclusions about spin-orbit coupling from the $n-p$ analysis.

It remains to consider the $n-p$ scattering at the intermediate energies of 40 and 90 Mev. Here two points must be examined: the pure P-wave scattering and the SP-interference effect. As to the former, it is relatively small. Indeed, we must remember that $\sigma_{LS}{}^{p-p}(90°)$ is about 2 mb at 30 Mev and hence according to (11) and (15) $\sigma_{LS}{}^{n-p}(\theta)$ reaches a maximum at 90° which is $\sim$0.5 mb at 40 Mev. This would be but a small contribution to the observed value of about 17 mb. Finally, it follows from the expressions given in Appendix I that the SP contribution to the $n-p$ cross section, arising from interference in the triplet states is given by

$$\frac{3r_0^2}{2k^2}\sin\delta(^3S)\cdot\cos\theta\{(5/3)\sin\delta(^3P_2)\cos[\delta(^3P_2)-\delta(^3S)] +\sin\delta(^3P_1)\cos[\delta(^3P_1)-\delta(^3S)] +\tfrac{1}{3}\sin\delta(^3P_0)\cos[\delta(^3P_0)-\delta(^3S)]\}. \quad (20)$$

Now in the energy range considered the ^3S phase is large, while according to (16) the ^3P phases are still rather small. Hence, (20) can be approximately written as

$$(3r_0^2/2k^2)\sin\delta(^3S)\cdot\cos\theta\cdot\tfrac{1}{3}[5\delta(^3P_2)+{}^3\delta(^3P_1)+\delta(^3P_0)]$$

which is zero, however, according to (14a). This result is a consequence of the Born treatment of the P phase, due to LS interaction only (the S phase being computed rigorously). It can therefore also be considered to follow

[14] We are indebted to Dr. A. Bohr for a discussion on this subject.

[15] Qualitatively, an appreciable spin-orbit contribution will tend to shift the ratio of central to tensor forces in favor of the former. Such a change can be made without necessarily disturbing the interpretation of the low energy data. An increase in the relative amount of central force will tend to a larger differential scattering cross section around 0° and 180°.

[16] It should be noted that for this choice of α and β the spin-orbit interaction vanishes in states of even angular momentum. Thus, in particular, the deuteron ground state is then entirely unaffected. Cf. in this connection the comments of Blanchard, Avery, and Sachs (Phys. Rev. **78**, 292 (1950)) on the magnetic moment of the deuteron.

essentially from the first of the relations (8). Thus, provided the P phase shifts are mainly due to a spin-orbit coupling only, the symmetry disturbing SP interference will be negligible. The arguments presented by Christian and Hart in favor of the even-theory would therefore seem to be in no way incompatible with the inclusion of a spin-orbit coupling in the n–p interaction.

V. CONCLUDING REMARKS

The introduction of a spin-orbit coupling of type (1) and with a strongly singular radial dependence thus offers a possibility of interpreting the available p–p scattering data. Its inclusion in the n–p interaction proposed by Christian and Hart does not change drastically the theoretical n–p cross section, hence allowing at least qualitatively for a charge independent nuclear interaction. It need hardly be stressed that from this point of view the possibility of introducing an LS coupling is not tied to the even-theory. More generally, it can be said that at present there still seems to be enough flexibility to fit a spin-orbit term into an otherwise satisfactory description of the n–p interaction.

While the present interpretation might seem to be not unreasonable, it can lay no claim to uniqueness. Indeed, the use of a strongly singular tensor force[3] or of a hard core[4] so far seem equally acceptable from a phenomenological point of view. Nor does the argument of charge independence single out the LS coupling, as it may well be possible to choose the exchange dependence in such a way that the alternative interpretations also can be fitted in principle into a charge independent description. It is to be hoped that further experimental evidence may lead to a discrimination among the various possibilities. As discussed in Section II(c), a test is the dependence of $\sigma^{pp}(90°)$ on energy. Furthermore, it is noteworthy that the hard core model predicts a minimum at about 180 Mev for the total σ^{pp} as a function of energy, see reference 4, Fig. 6. The present model yields a monotonic cross section. Thus, it would seem to be possible to distinguish between the hard core model and the LS coupling with an r^{-3} behavior. A differentiation between the latter and a tensor force with a similarity strong singularity needs more detailed considerations, however, since their qualitative features are very much the same.

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APPENDIX I

We give here the main steps in the derivation of the cross section for the scattering due to a mixture of spin orbit and central forces. Proceeding in the standard way the triplet scattering amplitude is found to be

$$f_m(\theta)=(2/k)\sum_{l=0}^{\infty}(\pi(2l+1))^{\frac{1}{2}}\sum_{j=l-1}^{l+1}e^{\delta_{lj}}\sin\delta_{lj}a_l^{jm}\Pi_l^{jm},$$

where k is the wave number and m denotes the magnetic substate. δ_{lj} is the phase shift for the state lj. Π_l^{jm} are the normalized simultaneous triplet eigenfunctions of J^2, L^2 and J_z. (Of course it is not necessary for this problem to construct these explicitly.) a_l^{jm} is defined by

$$Y_l^0\,{}^3\chi_m=\Sigma_j\, a_l^{jm}\Pi_l^{jm},$$

where ${}^3\chi_m$ are the triplet spin eigenfunctions and Y_l^0 is the normalized spherical harmonic of order l. With the help of the orthogonality properties of the Π's, one readily finds for the total triplet cross section

$$\sigma=(\pi/k^2)\sum_{lij}(2l+1)\sin^2\delta_{lj}\sum_m|a_l^{jm}|^2.$$

Using $Q=\mathbf{L}\cdot\mathbf{S}$ as a projection operator, one derives

$$a_l^{jm}\pi_l^{jm}=[\alpha_{\nu l}Q^2+\beta_{\nu l}Q+\gamma_{\nu l}]Y_l^0\,{}^3\chi_m,\quad j=l+\nu,$$

where

$$\mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2).$$

Here

$$\alpha_{\nu l}=\frac{2\nu^2}{(2l+1)(2l+\nu+1)}-\frac{1-\nu^2}{l(l+1)},$$

$$\beta_{\nu l}=\frac{\nu^2(2l\nu+\nu+3)}{(2l+1)(2l+\nu+1)}-\frac{1-\nu^2}{l(l+1)},$$

$$\gamma_{\nu l}=\frac{\nu^2(2l\nu+\nu+1)}{(2l+1)(2l+\nu+1)}+1-\nu^2,$$

with the understanding $\alpha_{\nu 0}=\beta_{\nu 0}=0$.

Using

$$Tr(\mathbf{L}\cdot\mathbf{S})=0,\quad Tr(\mathbf{L}\cdot\mathbf{S})^2=2\mathbf{L}^2,\quad Tr(\mathbf{L}\cdot\mathbf{S})^3=-\mathbf{L}^2,$$
$$Tr(\mathbf{L}\cdot\mathbf{S})^4=2(\mathbf{L}^2)^2,$$

where Tr denotes the trace with respect to the spin variables, one obtains for the differential cross section

$$\sigma(\theta)=\frac{\pi}{k^2}\sum_{ll}\sum_{\nu\nu'}[(2l+1)(2l'+1)]^{\frac{1}{2}}\cdot\cos(\delta_{l\nu}-\delta_{l'\nu'})\times\sin\delta_{l\nu}\sin\delta_{l'\nu'}F_{\nu\nu'}{}^{ll'}(\theta),$$

in which

$$F_{\nu\nu'}{}^{ll'}=P_{l0}P_{l'0}[3\gamma\gamma'+2\alpha'\gamma l'(l'+1)+2\alpha\gamma' l(l+1)+\tfrac{3}{2}\alpha\alpha' l(l+1)l'(l'+1)]$$
$$+P_{l1}P_{l'1}[(\alpha-\beta)(\alpha'-\beta')+\beta\beta'][l(l+1)l'(l'+1)]^{\frac{1}{2}}$$
$$+\tfrac{1}{2}P_{l2}P_{l'2}\alpha\alpha'[(l-1)l(l+1)(l+2)(l'-1)l'(l'+1)(l'+2)]^{\frac{1}{2}}.$$

Here the P_{lm} denote the orthonormal associated Legendre functions. We have used the notation $\alpha=\alpha_{\nu l}$, $\alpha'=\alpha_{\nu' l'}$, etc. Finally, one obtains for the total triplet cross section

$$\sigma=(\pi/k^2)\Sigma_{lj}(2j+1)\sin^2\delta_{lj}.$$

APPENDIX II

We give here without derivation the formulas needed to take into account relativistic corrections to the scattering cross section. As emphasized in Section II(b), only kinematic features will be dealt with. For a more detailed discussion of the following relations we refer especially to Møller's work on the relativistic electron-electron scattering.[17]

Let v be the velocity in the laboratory system of the incoming nucleon and $\gamma=(1-v^2/c^2)^{-\frac{1}{2}}$. For the angle θ_{lab} corresponding to a scattering angle θ^* in the system where the momenta are equal and opposite, we have

$$tg\theta=[2/(\gamma+1)]^{\frac{1}{2}}tg\theta^*/2.$$

For 350-Mev nucleons, $\gamma=1.37$. It follows that the relativistic distortion of the angular distribution as compared with the non-relativistic limit ($\gamma=1$) is about 5°, which can be considered

[17] C. Møller, Ann. d. Physik 14, 531 (1932).

negligible with the present experimental uncertainties. The relation between the elements of solid angle is:

$$4\cos\theta_{\rm lab}d\Omega_{\rm lab}=\frac{8(\gamma+1)}{[(\gamma+3)+(\gamma-1)\cos\theta^*]^2}d\Omega^*.$$

Again, within experimental accuracy it is sufficient to use the relation obtained by putting $\gamma=1$.

In the moving reference system each particle has a velocity u where

$$u=\gamma v/(\gamma+1).$$

The relative flux is then $2u$. The density of final states is

$$(p^*E_f^*/32\pi^3\hbar^3)d\Omega^*.$$

Here p^* is the momentum of one of the particles and E_f^* is the total final energy:

$$p^*=M\gamma v/[2(\gamma+1)]^{\frac{1}{2}},\quad E_f^*=2M[\tfrac{1}{2}(\gamma+1)]^{\frac{1}{2}}.$$

Inserting these quantities into the Born approximation formulae one finds the net effect is to multiply the non-relativistic expression for the cross section by $\frac{1}{2}(\gamma+1)$ and to replace the non-relativistic wave vector $\mathbf{k}$ by a wave vector $\mathbf{k}^*$ corresponding to the momentum $\mathbf{p}^*$. Even at 350 Mev the difference between $\mathbf{k}$ and $\mathbf{k}^*$ is very small. Hence, the total "kinematic" relativistic effect is to multiply the cross section by $\frac{1}{2}(\gamma+1)$. For 350 Mev this factor is 1.18, while at 260 Mev it is 1.14.

Since comparable dynamic relativistic effects are to be expected,[18] there is little point to try for anything better than about 20 percent accuracy.

Notes added in proof.—(1) It has been kindly pointed out to us by Dr. Christian that while the *S–P* interference vanishes the *P–D* does not. While small, this contributes an asymmetry to the *n–p* scattering which is of the same order as that observed. However, the sign of the interference term turns out to be the opposite of that observed. This suggests the above sign determination of the *LS* term is wrong. With the "odd" exchange dependence this will not affect the deuteron. However, the connection with the work of M. Mayer is then lost.

(2) Professor Serber has emphasized to us that irrespective of the *p–p* effects the *LS* term does not *lower* the *n–p* total cross section which calculation always gives as too large compared with experiment.

(3) Preliminary exact calculations indicate that a considerably smaller range of the spin-orbit term is needed to achieve a really flat 350-Mev cross section. This would mean the constants given in this paper may need large alteration and the possibility of simultaneously fitting the 30-Mev data may be lost. These exact calculations will be reported elsewhere.

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The Cross Section for Photo-Disintegration of the Deuteron at Low Energies

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Precision measurements have been made of the photo-disintegration of the deuteron with the gamma-rays of Ga^{72} and Na^{24}. Relative cross sections have been measured at 2.51, 2.62, and 2.76 Mev. Calibration of the sources gave absolute cross sections at 2.62 and 2.76 Mev.

I. INTRODUCTION

The variation with energy of the photoelectric and photo-magnetic cross sections of the deuteron is essential data for the determination of the parameters of the nuclear forces. So far measurements of the total cross sections $\sigma_T=\sigma_e+\sigma_m$ have been made by several groups [1–8] using γ-rays of quantum energy 2.62, 2.76, 6.3, and 17 Mev.

The measurement of these cross sections falls naturally in two parts: the determination of the number of γ-quanta emitted per unit time by the source and the number of photo-disintegrations produced by this source in a system containing heavy hydrogen.

The disintegrations are now usually counted by observing the photo-protons either in an ionization chamber counter or in a photographic plate. These methods have the advantage of being independent of a neutron standard needed for older methods relying on the counting of photo-neutrons. Recently, deuterium filled ionization chamber counters with electronic collection have been developed sufficiently to allow a precision of 1 percent in comparing the number of photo-protons produced by γ-rays of different energy.

Several methods can be used for determining the number of γ-quanta emitted from the source. In the case of the 2.62 Mev γ-rays emitted by RdTh, one can use the results of Ricoux[9] or Winand[10] giving the number of disintegrations per second of a RdTh source which gives the same ionization as 1 mg of radium in an ionization chamber of the Curie type. Unfortunately,

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