The Meson as a Composite Particle*

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Calculations are carried out along the lines of the work of Fermi and Yang in which the π-meson is considered as a composite particle formed from a proton and an anti-neutron. On the assumption of a vector interaction it is found that the ${}^1S_0$ state must be excluded because its energy goes to zero as the interaction goes to zero, while the ${}^3P_0$ state appears to give an acceptable solution. On the assumption of a tensor interaction it is found that ${}^1S_0$ and ${}^3P_0$ solutions both exist, but for opposite signs of the interaction. The tensor interaction must therefore be excluded since it would lead to the formation of a composite particle by a proton and a neutron. Using the vector interaction one finds that the ground state is a ${}^3P_1$, but that there are other states with $j=0$, 1 and 2 lying near it, the proximity depending on the interaction range assumed.

I. INTRODUCTION

SOME time ago the suggestion was made that particles of spin 0 and 1 are composite, consisting of two particles each of spin $\frac{1}{2}$, closely bound to each other.[1] The method of calculation used was essentially equivalent to that of Kemmer in his attempt at a relativistic treatment of the deuteron.[2] Recently this idea was applied by Fermi and Yang[3] to the case of a π-meson, regarded as a composite particle formed by a nucleon and an anti-nucleon. The present paper is devoted to a further consideration of this problem.

II. GENERAL TREATMENT

For two particles, individually satisfying the Dirac equation and interacting with each other directly, without any intermediary field, the wave equation can be written in the usual notation

$$\{-i\hbar c\boldsymbol{\alpha}_1\cdot\nabla_1+m_1c^2\beta_1-i\hbar c\boldsymbol{\alpha}_2\cdot\nabla_2+m_2c^2\beta_2+H_{12}\}\psi=W\psi. \quad (1)$$

In order to have relativistic invariance,[2] the interaction term H_{12} must be of the form

$$H_{12}=-\delta(\mathbf{x}_1-\mathbf{x}_2)\textstyle\sum_i b_i\omega_i, \quad (2)$$

where the b_i are constants and

$$\begin{array}{ll}\omega_1=\beta_1\beta_2 & \text{(scalar interaction)},\\ \omega_2=\frac{1}{2}(1-\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2) & \text{(vector)},\\ \omega_3=\frac{1}{2}\beta_1\beta_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2) & \text{(tensor)},\\ \omega_4=\frac{1}{2}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-\Gamma_1\Gamma_2) & \text{(pseudovector)},\\ \omega_5=\beta_1\beta_2\Gamma_1\Gamma_2 & \text{(pseudoscalar)}.\end{array} \quad (3)$$

Here $\boldsymbol{\sigma}$ is the spin vector ($\sigma_z=-i\alpha_x\alpha_y$, etc.) and $\Gamma=-i\alpha_x\alpha_y\alpha_z$.

The wave function ψ in (1) will have 16 components, which can be written ψ_{ij} ($i, j=1, 2, 3, 4$), the subscripts representing spinor indices associated with the two particles. However, in place of these spinor components, one can introduce linear combinations of them which will transform like components of tensors of various ranks. Thus, if one takes the Dirac matrices in the usual form,[4] one can write

$$\begin{aligned}I&=\tfrac{1}{2}(\psi_{12}-\psi_{21}+\psi_{34}-\psi_{43}),\\ A_1&=\tfrac{1}{2}(-\psi_{13}+\psi_{24}+\psi_{31}-\psi_{42}),\\ A_2&=(i/2)(-\psi_{13}-\psi_{24}+\psi_{31}+\psi_{42}),\\ A_3&=\tfrac{1}{2}(\psi_{14}+\psi_{23}-\psi_{32}-\psi_{41}),\\ A_4&=\tfrac{1}{2}(-\psi_{12}+\psi_{21}+\psi_{34}-\psi_{43}),\\ B_{23}=G_1&=\tfrac{1}{2}(-\psi_{11}+\psi_{22}-\psi_{33}+\psi_{44}),\\ B_{31}=G_2&=(i/2)(-\psi_{11}-\psi_{22}-\psi_{33}-\psi_{44}),\\ B_{12}=G_3&=\tfrac{1}{2}(\psi_{12}+\psi_{21}+\psi_{34}+\psi_{43}),\\ J&=\tfrac{1}{2}(\psi_{14}-\psi_{23}+\psi_{32}-\psi_{41}),\\ U_1&=\tfrac{1}{2}(-\psi_{11}+\psi_{22}+\psi_{33}-\psi_{44}),\\ U_2&=(i/2)(-\psi_{11}-\psi_{22}+\psi_{33}+\psi_{44}),\\ U_3&=\tfrac{1}{2}(\psi_{12}+\psi_{21}-\psi_{34}-\psi_{43}),\\ U_4&=\tfrac{1}{2}(-\psi_{14}+\psi_{23}+\psi_{32}-\psi_{41}),\\ -iB_{41}=F_1&=\tfrac{1}{2}(-\psi_{13}+\psi_{24}-\psi_{31}+\psi_{42}),\\ -iB_{42}=F_2&=(i/2)(-\psi_{13}-\psi_{24}-\psi_{31}-\psi_{42}),\\ -iB_{43}=F_3&=\tfrac{1}{2}(\psi_{14}+\psi_{23}+\psi_{32}+\psi_{41}),\end{aligned} \quad (4)$$

where I is a scalar, A_μ a four-vector, $B_{\mu\nu}$ an antisymmetric tensor (expressed in terms of two three-vectors $\mathbf{F}$ and $\mathbf{G}$), U_μ a pseudovector, and J a pseudoscalar.

Equation (1) can be rewritten as a set of equations for the above tensor components. It is convenient to introduce, in place of the coordinates $\mathbf{x}_1$ and $\mathbf{x}_2$, new variables $\mathbf{x}=\mathbf{x}_1-\mathbf{x}_2$ and $\mathbf{X}=\frac{1}{2}(\mathbf{x}_1+\mathbf{x}_2)$. However, it is easily seen that the total linear momentum of the system, i.e., the momentum conjugate to $\mathbf{X}$, will be a constant of the motion and by a Lorentz transformation can be made to vanish. In that case, derivatives with respect to components of $\mathbf{X}$ drop out of the equations and the resulting relations can be written as follows:

$$\begin{aligned}(W+p\Omega)I+2i\hbar c\nabla\cdot\mathbf{F}+2Mc^2A_4&=0,\\ (W-s\Omega)\mathbf{A}+2\hbar c\nabla\times\mathbf{U}-2\mu c^2\mathbf{F}&=0,\\ (W+s\Omega)A_4+2Mc^2I&=0,\\ (W-v\Omega)\mathbf{G}+2i\hbar c\nabla J-2Mc^2\mathbf{U}&=0,\\ (W+v\Omega)\mathbf{F}+2i\hbar c\nabla I-2\mu c^2\mathbf{A}&=0,\\ (W+u\Omega)U_4+2\mu c^2J&=0,\\ (W-\mu\Omega)\mathbf{U}+2\hbar c\nabla\times\mathbf{A}-2Mc^2\mathbf{G}&=0,\\ (W+q\Omega)J+2i\hbar c\nabla\cdot\mathbf{G}+2\mu c^2U_4&=0,\end{aligned} \quad (5)$$

* Part of this work was done while one of the authors (H.M.M.) held an AEC Predoctoral Fellowship.

[1] N. Rosen, Phys. Rev. **74**, 128(A) (1948). H. M. Moseley, Phys. Rev. **76**, 197(A) (1949). However it should be pointed out that a similar idea, although involving a different point of view, was proposed earlier by L. de Broglie, "Une Nouvelle Théorie de la Lumière" (Hermann, Paris, 1940) and "Théorie Générale des Particules à Spin" (Gauthier-Villars, Paris, 1943).

[2] N. Kemmer, Helv. Phys. Acta **10**, 48 (1937).

[3] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).

[4] W. Pauli, *Handbuch der Physik* **24** (2nd ed.) Part 1, p. 219.

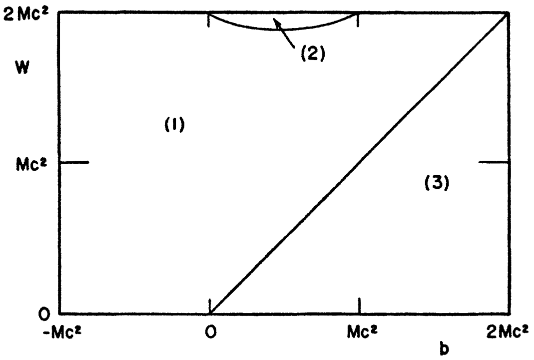

FIG. 1. The (W,b) plane, separated into regions according to the sign of T^2.

where the operator ∇ refers to the relative coordinates $\mathbf{x}$, $M=\frac{1}{2}(m_1+m_2)$, $\mu=\frac{1}{2}(m_1-m_2)$, $\Omega=\delta(\mathbf{x})$ and

$$\begin{aligned} p&=b_1+2b_2-3b_3-2b_4+b_5,\\ q&=-b_1+2b_2+3b_3-2b_4-b_5,\\ s&=b_1-b_2-b_4-b_5,\\ u&=-b_1-b_2-b_4+b_5,\\ v&=-b_1-b_3-b_5. \end{aligned} \tag{6}$$

Now Kemmer found in his deuteron calculation (taking $m_1=m_2$) that if the interaction involves $\delta(\mathbf{x})$, as is required for relativistic invariance, then there are no bound states of finite energy. Fermi and Yang, considering the problem from the standpoint of quantized fields, pointed out that the states of the system involving two particles are mixed with states in which additional particle pairs are formed. On the basis of this fact they were led to take for the effective interaction between a nucleon and an anti-nucleon a range of the order of $\hbar/Mc$, where M is the mass of a nucleon. On the other hand, if one accepts the idea of the existence of a universal length of the order of the classical electron radius as a limit to the accuracy of measurement of position,[5] one is led to a range of this order of magnitude (since it is large compared to $\hbar/Mc$). Hence in place of $\delta(\mathbf{x})$ we shall take hereafter

$$\Omega=\begin{cases}1, & r\leqslant\sigma,\\ 0, & r>\sigma,\end{cases} \tag{7}$$

where r is the inter-particle distance and σ a constant, the range of the interaction. This "square-well" potential is taken for simplicity; it is clear that its use can lead only to very approximate results.

III. THE MESON

As already mentioned, Fermi and Yang[3] assumed that the π-meson was composed of a nucleon and an anti-nucleon of equal rest-mass M. In this case then $\mu=0$ and Eqs. (5) can be transformed by a change of variables into the equations given by Kemmer.[2] Denoting the components of any one of the three-vectors $\mathbf{A}$, $\mathbf{U}$, $\mathbf{F}$, $\mathbf{G}$ by H_1, H_2, H_3 one takes instead

$$\mathcal{H}_1=-2^{-\frac{1}{2}}(H_1-iH_2),\quad \mathcal{H}_2=H_3,\quad \mathcal{H}_3=2^{-\frac{1}{2}}(H_1+iH_2). \tag{8}$$

With a suitable change of notation one then gets Kemmer's equations and one can use his method of solution.[6]

At this point is is desirable to make a more specific assumption concerning the nature of the interaction between the particles. Fermi and Yang pointed out that of the five types of interactions in (3), the vector and tensor interactions change sign if the anti-nucleon is replaced by a nucleon, whereas the other interaction terms remain unchanged. Hence, on the assumption that a proton and an anti-neutron form a composite particle, while a proton and neutron do not, they restricted the interaction to the vector and tensor types. While a linear combination of the two is possible, it is natural to take only one of them for simplicity. Fermi and Yang chose the vector interaction. However, the tensor interaction has *a priori* an equal claim. In the present paper both types of interactions will be considered.

Following Fermi and Yang, we shall restrict ourselves to the case $\mu=0$. These authors also restricted their discussion to the case of zero total angular momentum ($j=0$). To permit a ready comparison with their work, the next section deals with the case $j=0$, while the case of a general integral value of j is discussed in the following section.

IV. CASE $j=0$

There are two types of solutions for states with $j=0$. One will be labelled $^1S_0$. In the relativistic case it is a mixture of $^1S_0$ and $^3P_0$, but goes over into $^1S_0$ in the non-relativistic approximation. The other will be labelled $^3P_0$. It is also a mixture of $^3P_0$ and $^1S_0$ in the general case, but goes over in $^3P_0$ in the non-relativistic approximation. These two types of solutions differ in their behaviors under a reflection of coordinates.[2]

For the $^1S_0$ solutions the non-vanishing components are taken in the form:

$$I=f_1(r),\quad A_4=f_2(r),\quad \mathbf{F}=f_3(r)\mathbf{r}/r. \tag{9}$$

One obtains for $r>\sigma$

$$f_1=Br^{-1}\exp(-kr),\quad f_2=-2Mc^2f_1/W,\quad f_3=(-2i\hbar c/W)df_1/dr, \tag{10}$$

and for $r<\sigma$

$$f_1=Ar^{-1}\sin Tr,\quad f_2=-2Mc^2f_1/(W+s),\quad f_3=-2i\hbar c(df_1/dr)/(W+v), \tag{11}$$

[5] L. de Broglie, Comptes Rendus **200**, 361 (1935); A. March, Naturwiss. **26**, 649 (1938); N. Rosen, Phys. Rev. **72**, 298 (1947).

[6] The more general equations for $\mu\neq0$ and their solutions are discussed by H. M. Moseley, dissertation, University of North Carolina (in preparation).

where

$$k^2=(4M^2c^4-W^2)/4\hbar^2c^2,$$
$$T^2=(W+v)[(W+p)(W+s)-4M^2c^4]/4\hbar^2c^2(W+s), \quad (12)$$

and A, B are constants.

The boundary conditions at $r=\sigma$ determine the energy through the equation

$$T\sigma\,\mathrm{ctn}(T\sigma)=1-(1+k\sigma)(W+v)/W. \quad (13)$$

For the $^3P_0$ case, the non-vanishing components are

$$J=f_4(r),\quad \mathbf{G}=f_5(r)\mathbf{r}/r,\quad \mathbf{U}=f_6(r)\mathbf{r}/r. \quad (14)$$

One finds that, for $r>\sigma$

$$f_4=Br^{-1}\exp(-kr),$$
$$f_5=-2i\hbar cW(df_4/dr)/(W^2-4M^2c^4), \quad (15)$$
$$f_6=2Mc^2f_5/W,$$

while for $r<\sigma$

$$f_4=Ar^{-1}\sin Kr,$$
$$f_5=-2i\hbar c(W-u)(df_4/dr)/[(W-u)(W-v)-4M^2c^4], \quad (16)$$
$$f_6=2Mc^2f_5/(W-u).$$

Here

$$K^2=(W+q)[(W-u)(W-v)-4M^2c^4]/4\hbar^2c^2(W-u), \quad (17)$$

and the energy is determined by the boundary conditions through the equation

$$K\sigma\,\mathrm{ctn}(K\sigma)=1+(1+k\sigma)W[(W-u)(W-v)-4M^2c^4]/(4M^2c^4-W^2)(W-u). \quad (18)$$

From the preceding relations one can determine the energy for each of the two types of interactions.

(a) *Vector Interaction.* For the $^1S_0$ state, if we write b in place of the interaction constant b_2, then the relations (12) and (13) on the basis of (6) can be written

$$(T\sigma)^2=W\sigma^2[(W+2b)(W-b)-4M^2c^4]/4\hbar^2c^2(W-b), \quad (19)$$

$$T\sigma\,\mathrm{ctn}(T\sigma)=-k\sigma. \quad (20)$$

Let us now consider $(T\sigma)^2$ as a function of W and b, according to Eq. (19). Note that for $W=b$, $(T\sigma)^2=\infty$, and for $(W+2b)(W-b)=4M^2c^4$, $(T\sigma)^2=0$. The curves represented by these two equations divide up the (W, b) plot into three regions, as shown in Fig. 1. In the region marked (1) in this figure, $(T\sigma)^2<0$ so that Eq. (20) has no solution in this region, since $x\,\mathrm{ctnh}x\geq 1$. In the regions marked (2) and (3), $(T\sigma)^2>0$, so that solutions are possible. But if b is taken to lie in region (2), then $W>4\sqrt{2}Mc^2/3$, so that the energy is too large for the system to represent a π-meson. Hence only the region (3) remains.

Figure 2 shows a plot of the ground-state solution $W_1(b)$ and the solution for the first excited state $W_2(b)$ for values in the region (3) and with the range $\sigma=\hbar/Mc$. The lower curve passes through the point corresponding to the solution given by Fermi and Yang, i.e.,

$$W=Mc^2/6.46,\quad b=53.0Mc^2.$$

However, if we accept the criterion that only those solutions have physical significance for which the energy goes to $2Mc^2$ as the interaction is cut out adiabatically (in this case, as $b\to 0$), we see that no acceptable solution exists in this case. This conclusion is independent of what value one takes for the range σ, as can be seen from the shape of the region (3) in Fig. 1.

Let us now consider the $^3P_0$ state. The relations which determine the energy are

$$(K\sigma)^2=(W+2b)[W(W+b)-4M^2c^4]\sigma^2/4\hbar^2c^2(W+b), \quad (21)$$

$$K\sigma\,\mathrm{ctn}(K\sigma)=1+W[W(W+b)-4M^2c^4](1+k\sigma)/(W+b)(4M^2c^4-W^2). \quad (22)$$

By proceeding as above one finds that all solutions for which $b<0$ are of the type for which $W\to 0$ as $b\to 0$. On the other hand for $b>0$ acceptable solutions exist. Assuming that the ground state represents a π-meson, so that $W=Mc^2/6.46$ one obtains

$$\text{for } \sigma=\hbar/Mc,\quad b=286\,Mc^2, \quad (A)$$

$$\text{for } \sigma=2.8\cdot 10^{-13}\,\text{cm}=a_0,\quad b=2.71\,Mc^2. \quad (B)$$

(b) *Tensor Interaction.* For the $^1S_0$ state, the conditions determining the energy are

$$(T\sigma)^2=(W-b_3)[W(W-3b_3)-4M^2c^4]\sigma^2/4\hbar^2c^2W, \quad (23)$$

$$T\sigma\,\mathrm{ctn}(T\sigma)=1-(W-b_3)(1+k\sigma)/W. \quad (24)$$

It is found that acceptable solutions exist only for $b_3<0$. For example, to get a ground-state energy corresponding to the mass of a π-meson one finds

$$\text{for } \sigma=\hbar/Mc,\quad b_3=-9.86Mc^2, \quad (C)$$
$$\text{for } \sigma=a_0,\quad b_3=-8.57Mc^2. \quad (D)$$

Finally, for the $^3P_0$ state the equations determining the energy are

$$(K\sigma)^2=(W+3b_3)[W(W+b_3)-4M^2c^4]\sigma^2/4\hbar^2c^2W, \quad (25)$$

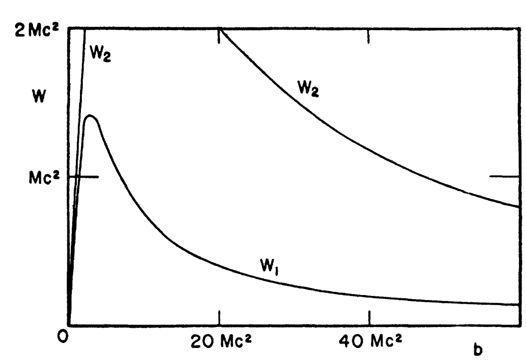

FIG. 2. The energies of the two lowest states of the $^1S_0$ type as functions of the interaction parameter b.

TABLE I. Energies of lowest states of various types.

State	W/Mc^2 $\sigma=\hbar/Mc$	$\sigma=2.8\times10^{-13}$ cm
$^3P_1$	0.1548	0.1548
$^3D_2$	0.166	0.15487
$^3P_0$	1.350	0.162
$^3S_1+{}^3D_1$	0.165	0.15486
$^3P_2+{}^3F_2$	0.180	0.15494

$$K\sigma \operatorname{ctn}(K\sigma)=1+[W(W+b_3)-4M^2c^4](1+k\sigma)/(4M^2c^4-W^2). \quad (26)$$

In this case acceptable solutions exist only for $b_3>0$, and to get the π-meson energy one finds

$$\text{for } \sigma=\hbar/Mc, \quad b_3=26.7Mc^2, \quad \text{(E)}$$
$$\text{for } \sigma=a_0, \quad b_3=25.7Mc^2. \quad \text{(F)}$$

The preceding results indicate that it is necessary to discard the tensor interaction. Since it has been found that bound states exist for both signs of b_3 it follows that if a proton and an anti-neutron form a composite particle, then a proton and a neutron will do likewise, in general. For example, if one accepts the solution (C) above for the $^1S_0$ state as describing a π-meson, then with a change in the sign of b_3 (corresponding to a change from an anti-neutron to a neutron) one can obtain a solution for the $^3P_0$ state. This is found to have an energy of $0.509Mc^2$. Similarly, if one accepts the solution (E) for the $^3P_0$ state, then changing the sign of b_3 leads to a solution for the $^1S_0$ state corresponding to a smaller mass than that of the π-meson. To avoid such neutron-proton systems one must discard the tensor type of interaction.

We see then that Fermi and Yang were justified in adopting the vector rather than the tensor interaction. However, it appears that one should take the $^3P_0$ solution rather than the $^1S_0$, as long as one restricts himself to states with $j=0$.

The question arises as to whether, for a given value of the interaction parameter, there may not be states with j different from zero lying below the ones considered previously. This is investigated in the next section.

V. CASE OF HIGHER j

For an arbitrary integral value of j, there are in general three distinct types of solutions of Eqs. (5). We shall denote them here by A, B, C corresponding to Kemmer's[2] type IIb, Ib, Ia, respectively. To characterize these solutions we shall make use of Kemmer's[2] "vectors" $\mathfrak{X}_j{}^m$, $\mathfrak{Y}_j{}^m$, $\mathfrak{Z}_j{}^m$, which he defined in terms of normalized spherical harmonic functions $P_j{}^m$.

(a) *Solutions of type A*. The solutions of type A have for the non-vanishing components

$$I=\alpha P_j{}^m, \quad A_4=\beta P_j{}^m, \quad \mathfrak{F}=\gamma\mathfrak{X}_{j-1}{}^m+\Delta\mathfrak{Z}_{j+1}{}^m, \quad (27)$$

where α, β, γ, Δ are functions of r. With respect to a reflection of coordinates through the origin the solution has a parity of $(-1)^j$. The $^1S_0$ solution found above is a special case of this type. It is found that, as in the $^1S_0$ case, the energy in general tends to zero rather than $2Mc^2$ as $b\to 0$. Hence this type of solution will be discarded.

(b) *Solutions of type B*. The solutions of this type have for the non-vanishing components

$$\mathbf{A}=\epsilon\mathfrak{X}_{j-1}{}^m+\eta\mathfrak{Z}_{j+1}{}^m, \quad \mathbf{G}=\lambda\mathfrak{Y}_j{}^m, \quad \mathbf{U}=\zeta\mathfrak{Y}_j{}^m, \quad (28)$$

where ϵ, η, λ, and ζ are functions of r. Substituting into (5) enables one to express ϵ, η and λ in terms of ζ which satisfies the equations

$$\begin{aligned} &\frac{d^2\zeta}{dr^2}+\frac{2}{r}\frac{d\zeta}{dr}+\left[k^2-\frac{j(j+1)}{r^2}\right]\zeta=0, \quad r>\sigma,\\ &\frac{d^2\zeta}{dr^2}+\frac{2}{r}\frac{d\zeta}{dr}+\left[N^2-\frac{j(j+1)}{r^2}\right]\zeta=0, \quad r<\sigma, \end{aligned} \quad (29)$$

where

$$N^2=(W+b)[W(W+b)-4M^2c^4]/4\hbar^2c^2W. \quad (30)$$

One can show that at $r=\sigma$, where the interaction has a discontinuity, ζ and $(W+b\Omega)^{-1}(d\zeta/dr+\zeta/r)$ must be continuous. The solutions of this type have a parity $(-)^j$ with respect to a reflection through the origin.

No solution of type B exists for $j=0$. For $j=1$, one gets solutions which will be denoted by $^3P_1$, but which in the relativistic case include also some $^3S_1$ and $^3D_1$. For these solutions one takes

$$\begin{aligned} \zeta&=Ar^{-2}(Nr\cos Nr-\sin Nr), \quad r<\sigma,\\ \zeta&=Br^{-2}(1+kr)\exp(-kr), \quad r>\sigma. \end{aligned} \quad (31)$$

The boundary conditions at $r=\sigma$ give the relation which determines the energy,

$$N\sigma\operatorname{ctn}(N\sigma)=1+(N\sigma)^2(1+k\sigma)/[k^2\sigma^2+(b/W)(1+k\sigma+k^2\sigma^2)]. \quad (32)$$

A calculation shows that for the values of b determined in the previous section the lowest $^3P_1$ level lies below the $^3P_0$ level. Hence it is necessary to modify the interaction constant b so as to make the energy of the lowest $^3P_1$ state correspond to the mass of the π-meson ($0.1548Mc^2$). One finds that

$$\text{for } \sigma=\hbar/Mc, \quad b=28.3Mc^2,$$
$$\text{for } \sigma=a_0, \quad b=25.7Mc^2.$$

It follows then that the energy of the $^3P_0$ state in the first case is $1.350Mc^2$ and in the second case $0.162Mc^2$. There will also be a number of excited $^3P_1$ states, the first one in the first case ($\sigma=\hbar/Mc$) having an energy of $0.196Mc^2$.

In the case of $j=2$, one gets solutions labelled $^3D_2$. For the values of b chosen above one gets as the energy of the lowest state of this type for

$$\sigma=\hbar/Mc, \quad W=0.166Mc^2,$$
$$\sigma=a_0, \quad W=0.1549\ Mc^2.$$

(c) *Solutions of type C.* The solutions of this type have as non-vanishing components

$$J=\alpha P_j{}^m,\quad \mathbf{A}=\zeta\mathfrak{Y}_j{}^m,$$
$$\mathbf{G}=\gamma\mathfrak{X}_{j-1}{}^m+\Delta\mathfrak{Z}_{j+1},\quad \mathbf{U}=\epsilon\mathfrak{X}_{j-1}{}^m+\eta\mathfrak{Z}_{j+1}{}^m, \tag{33}$$

where α, ζ, γ, Δ, ϵ, and η are functions of r. This type of solution has a parity $(-1)^{j+1}$ under a reflection through the origin.

In the case $j=0$ the equations are fairly simple and one obtains the $^3P_0$ solution discussed in the preceding section. For larger values of j the equations, and also the boundary conditions giving the energy, are somewhat complicated and will therefore not be given here.[7]

For $j=1$ the solution is a mixture of $^3S_1$ and $^3D_1$ (with small amounts of $^3P_1$ and $^1P_1$, the proportions of these increasing as one goes to higher excited states). The lowest state of this type has an energy

$$\text{for } \sigma=\hbar/Mc,\quad W=0.165Mc^2,$$
$$\text{for } \sigma=a_0,\quad W=0.15486Mc^2,$$

where the value has been written with excessive precision to show its relation to the energy of the ground state. For $j=2$ the solution is largely a mixture of $^3P_2$ and $^3F_2$. The lowest state of this type has energy values $0.180Mc^2$ and $0.15494Mc^2$ for the above ranges, respectively. The preceding results are summarized in Table I.

In conclusion it should be emphasized that the numerical values for the energy levels are not significant because of the approximation introduced by the use of the square-well interaction. However, it is interesting that there are a number of different states lying close to the ground state with angular momenta 0, 1, and 2. These would be interpreted as particles with nearly equal masses and spins 0, 1, and 2. Further measurements and analysis of data on π-mesons should show whether variations in mass and spin actually occur.

[7] These will be found, in different notation, in reference 2. However, in Eq. (21b) of that paper there appears to be an error of sign in the third term of each of the two factors on the left-hand side.

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The Emission of Long-Range Charged Particles in the Slow Neutron Fission of Heavy Nuclei

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Light charged particles with ranges greater than 6 cm of air produced in the slow neutron fission of U^{233}, U^{235}, and Pu^{239} have been studied in detail by coincidence counting methods. In each case particles with a continuous range distribution extending to about 50 cm of air were observed, the distribution showing a broad maximum in the neighborhood of 20 cm. Direct measurement of the energies of the light particles from U^{235} showed a fairly symmetrical distribution about 15 Mev with a maximum energy of about 26 Mev. Comparison of the energy and range distributions shows that all the long-range particles are α-particles. The frequency of emission of these α-particles was found to be

1 in 405±30 fissions for U^{233},

1 in 505±50 fissions for U^{235}, 1 in 445±35 fissions for Pu^{239}.

No protons were observed, although the apparatus would have detected any with ranges lying between 10 and 100 cm of air. The energy distribution of fission fragments coincident with long-range α-particles was also measured. The usual two peaks were observed indicating asymmetric division of mass, but each peak was shifted to a lower energy than is observed in binary fission. Quantitative comparison of the energies involved showed that, on the average, the total kinetic energy carried away in fission accompanied by α-emission is about equal to that liberated in binary fission. Possible explanations for α-emission in fission are discussed.

I. INTRODUCTION

THE occasional emission of light charged particles in the fission of uranium by slow neutrons seems to have been noticed first by Alvarez.[1] Subsequent investigations in several laboratories using coincidence counting techniques and uranium-loaded photographic emulsions have confirmed and extended the original observations. Farwell, Segrè, and Wiegand,[2] using a coincidence arrangement, reported that light particles with ranges up to 23 cm of air are produced in about 0.4 percent of fission events in U^{235} and in 0.2 percent of fissions in Pu^{239} by slow neutrons. By comparison of the ionization of the light particles with that of α-particles from polonium it was shown that they were probably α-particles, a conclusion in accord with the majority of measurements of grain density in photographic emulsions.[3–7] On the other hand, the work of Tsien and his associates[8] suggests that not all the light

[1] L. W. Alvarez, mentioned in reference 2. Earliest publications in the open literature are by Green and Livesey, Proc. Int. Conf., Cambridge, July, 1946, and Tsien, Chastel, Ho, and Vigneron, Comptes Rendus **223**, 986 (1946).

[2] Farwell, Segrè, and Wiegand, Phys. Rev. **71**, 327 (1947).

[3] P. Demers, Phys. Rev. **70**, 974 (1946).

[4] Wollan, Moak, and Sawyer, Phys. Rev. **72**, 447 (1947).

[5] L. L. Green and D. L. Livesey, Phil. Trans. A **241**, 323 (1948).

[6] L. Marshall, Phys. Rev. **75**, 1339 (1949).

[7] E. W. Titterton (unpublished). We are grateful to Dr. Titterton for communicating to us some of his results prior to publication.

[8] Tsien, Ho, Chastel, and Vigneron, J. de phys. et rad. **8**, 165, 200 (1947).