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Energy and Density Distribution of Cosmic-Ray Neutrons*

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The theory of atmospheric neutron measurements is reviewed in the light of recent data on neutron scattering and capture cross sections in air and boron. It is concluded that the mean energy of atmospheric neutrons is 0.26 ev rather than the previous value of 0.06 calculated by Bethe, Korff, and Placzek. The spatial distribution is then recalculated and found to agree closely with earlier calculations by Flügge. Recent measurements of neutron intensity as a function of altitude are discussed in the light of the foregoing and are found to offer confirmation of the new calculations.

I. ENERGY DISTRIBUTION OF ATMOSPHERIC NEUTRONS

VARIOUS methods of neutron production in the atmosphere have been discussed by Korff[1] in the light of recent experiments, and it is concluded that by far the greatest number of atmospheric neutrons must be associated with nuclear disruptions (stars). Based on this assumption Bethe, Korff, and Placzek[2] have calculated the energy distribution, and S. Flügge[3] has calculated the expected density distribution of slow neutrons in the atmosphere. These distributions have been recalculated numerically by the author on the basis of more recent data on neutron cross sections in oxygen and nitrogen.

Using the Bohr model of the compound nucleus, Bethe, Korff, and Placzek[2] show that the mean diffusion length in the atmosphere for neutrons to be slowed down to 4 Mev by inelastic scattering is of the order of 36 g/cm^2. Below this energy, the lowest excitation level in oxygen or nitrogen, all further scattering is assumed to be elastic. When all scattering is elastic the fraction of neutrons which are slowed down to energy E without being captured is given by:[3]

$$N/N_0=\exp\left[-(M+1)\frac{\sigma_c(1\text{ ev})E^{-\frac{1}{2}}}{\bar{\sigma}_s}\right], \tag{1}$$

where σ_c is the neutron capture cross section in air as a function of E and $\bar{\sigma}_s$ is the mean scattering cross section over the energy region concerned. This fraction differs appreciably from unity only at low energies and in this region the Columbia group gives the total cross section of nitrogen in the form:[4]

$$\sigma_t=(9.96+0.34E^{-\frac{1}{2}})\text{ barn}, \tag{2}$$

where E is in ev. The constant term represents the scattering cross section, while the term in $E^{-\frac{1}{2}}$ is the capture cross section. On this basis the value of σ_c at 1 ev for nitrogen is 0.34 barn. Considering the capture cross section of oxygen to be negligible by comparison, we compute the effective cross section for air at 1 ev to be 0.28 barn.

In deriving Eq. (1) a constant value of σ_s was assumed throughout the region of integration. This is true for oxygen but not quite true in the case of nitrogen, although in the region where 99 percent of the value of the integral is developed (<40 ev), it holds very well.[5] By integrating the cross-section curves of Goldsmith, Ibser, and Feld, and again forming a weighted average for oxygen and nitrogen, a value of 8.6 barns for the mean scattering cross section of air over the entire region is obtained.

Using these values, and with $M=14.6$, the fraction of neutrons which get down to energy E without being captured is given by:

$$N/N_0=\exp(-0.509E^{-\frac{1}{2}}). \tag{3}$$

In particular, the exponent becomes unity for

$$\bar{E}_c=0.26\text{ ev}. \tag{4}$$

In other words. neutrons will on the average be slowed down to this energy before being captured.

The value originally computed in the same manner by Bethe, Korff, and Placzek[2] on the basis of earlier cross-section data was 0.06 ev. The calculation of absolute neutron intensities in the atmosphere shows the significance of this difference. If the average neutron energy had the lower value, then the use of a cadmium shield in the determination of atmospheric neutron intensities would introduce little error, since a large percentage of the neutrons in the atmosphere would have energies below the cadmium cut-off (about 0.4 ev). On the other hand, if we accept the larger value, then a relatively larger number of neutrons will have energies above the cut-off and a substantial error will be introduced. From experimental observation the latter conclusion appears to be correct.

II. DENSITY DISTRIBUTION

Let us now consider the mean distance travelled before capture by a neutron slowing down in the atmos-

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** Major, USAF, now at Los Alamos Scientific Laboratory, New Mexico.
[1] S. A. Korff, Rev. Mod. Phys. **20**, 327 (1948).
[2] Bethe, Korff, and Placzek, Phys. Rev. **57**, 573 (1940).
[3] S. Flügge, Chapter 14, *Cosmic Radiation* (Dover Publications, New York, 1946), W. Heisenberg, editor.
[4] E. Melkonian, Phys. Rev. **76**, 1750 (1949).
[5] Goldsmith, Ibser, and Feld, Rev. Mod. Phys. **19**, 259 (1947).

phere, and the mean absorption depth. When the scattering is done by elastic collisions, i.e., at energies below 4 Mev, Bethe, Korff, and Placzek[2] give the mean square diffusion length as:

$$\langle r^2\rangle_{Av}=(M_{eff}+1)\int_{0.26\text{ ev}}^{4\text{ Mev}} l^2(E)dE/E, \tag{5}$$

where $l(E)$ is the total mean free path in g/cm^2 as a function of the neutron energy, and $M_{eff}=14.6$.

This integral can now be evaluated more accurately since $\sigma(E)$ over the entire range of energies has been measured quite accurately.[4,5] Performing the integration and assuming a mean diffusion length of 36 g/cm^2 above 4 Mev we find the total mean diffusion length to be:

$$[\langle r^2\rangle_{Av}]^{\frac{1}{2}}=154\text{ g/cm}^2, \tag{6}$$

and the mean absorption depth

$$L=[\tfrac{1}{3}\langle r^2\rangle_{Av}]^{\frac{1}{2}}=89\text{ g/cm}^2. \tag{7}$$

Because of the uncertainty at higher energies these figures are probably uncertain by plus or minus ten percent.

According to Bethe, Korff, and Placzek,[2] if all neutrons are produced at the very top of the atmosphere the slow neutron density will vary as follows:

$$N(v,x)=x\exp(-x^2/2L^2), \tag{8}$$

and will thus have a maximum at $x=L$. Since the maximum of neutron production presumably takes place at some depth between 0 and L, we should expect the measured maximum of slow neutron density to fall at a somewhat greater depth.

If we assume that neutrons are produced primarily in nuclear disruptions then Flügge[3] gives the distribution of neutrons after diffusion to the mean energy in the form of a Fourier series. Because of insufficient cross-section data, several assumptions had to be made originally in the numerical calculation of terms. In the light of new data[4,5] the author re-calculated terms. It was found that although the assumptions made did not in general turn out to be correct, several factors balanced out so that the new solution agreed with one of Flügge's quite closely. The general expression is:

$$\rho_0(p)=2J_0\sum_{n=1}^{\infty}\frac{n\pi}{\mu^2+n^2\pi^2}0.995^{n^2}\left[\frac{a(1-0.0166n^2)!}{\alpha^{2-0.0166n^2}}+\frac{b(1-0.0166n^2)!}{\beta^{2-0.0166n^2}}\right]\sin n\pi p. \tag{9}$$

Here $\rho_0(p)$ is the neutron density as a function of pressure in atmospheres; μ is found from experiment to be approximately 7 atmos.$^{-1}$; $a=0.178$; $b=6.16$ (both in arbitrary units); $\alpha=0.08$ Mev^{-1}; and $\beta=0.35$ Mev^{-1}. J_0 is an arbitrary normalization factor.

The first seven terms (with $2J_0=1$) are:

$$\begin{aligned}\rho_0(p)=3.96\sin\pi p&+4.72\sin 2\pi p+3.81\sin 3\pi p\\&+2.65\sin 4\pi p+1.85\sin 5\pi p\\&+1.01\sin 6\pi p+0.74\sin 7\pi p+\cdots.\end{aligned} \tag{10}$$

A plot of this function expressed in terms of millibarns will be found in Fig. 1.

III. SLOW NEUTRON DETECTION

We turn now to the detectors used in measuring the neutron intensity in these experiments. Let us first consider an idealized counter in the form of a thin unshielded $1/v$ detector. We have previously shown that the number of neutrons outside the $1/v$ energy region in air is negligibly small under equilibrium conditions. Under these conditions, then, we need only consider the region in which this law holds in air.

It then follows that there is a mass of air which will, at all energies, capture the same number of neutrons as the detecting element. Put another way, n captures per sec. in the entire mass of the detector implies q captures per gram per sec. in air. If the detector is a gas this relationship between the two capture rates is given to be:[2]

$$q=n(\sigma_a/\sigma_d)(780/V_d p_{0d})\ g^{-1}\text{ sec.}^{-1}, \tag{11}$$

where V_d is the volume of the detector, p_{0d} is the filling

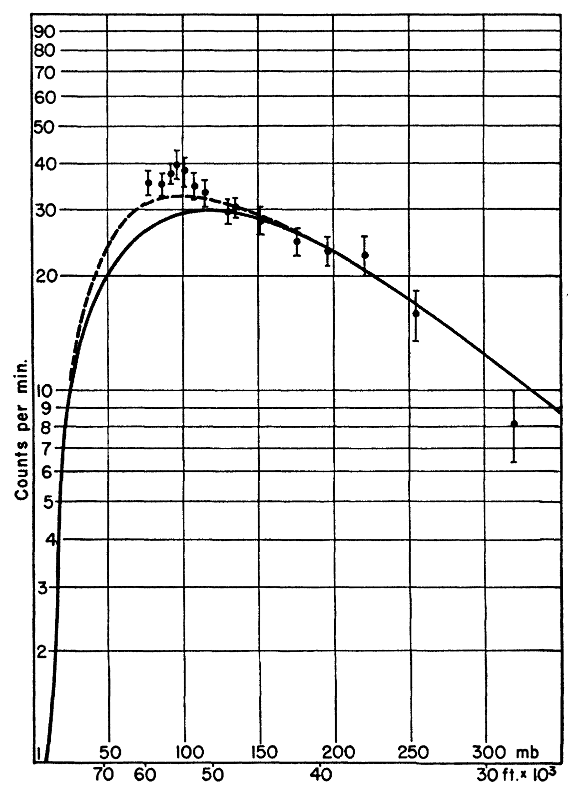

FIG. 1. Comparison of the data with theory. The theoretical curve is taken from reference 3 ($J_0=1.00$). The dashed curve is the same after correction for absorption in Pyrex.

TABLE I. Experimental neutron counting rates.

Pressure (mb)	N(Yuan) (c/min.)	N(Davis) (c/min.)	Absorption (percent)
300	13.6	11.6	15
250	19.3	16.5	15
150	34.7	31.0	11
100	39.0	36.0	8

pressure in atmospheres corrected to 0°C, and σ_a and σ_d are the capture cross sections of air and detector respectively. The ratio σ_a/σ_d is a constant since both are proportional to $1/v$.

Since the mean distance traveled by neutrons from the point of production is only about 150 g/cm^2, it follows that in a large region of the atmosphere the production rate will be in equilibrium with the absorption rate. In this region, which in practice lies between about 220 mb and 800 mb in the atmosphere (1 mb, $\cong 10^{-3}$ atmos.), the production rate is given directly by q.

On the basis of present day knowledge and proper instrumentation it is possible for us to consider an arrangement of BF_3 proportional counters as an excellent approximation to this theoretical detector. We now know the extension of the $1/v$ law for boron into higher energy regions[5] and find very slight variation in the important region. High energy neutrons will produce recoils in appreciable numbers, and other large cosmic ray events will be recorded from time to time, but this background may be subtracted in various ways.

In the present experiments two counters are used, identical in construction and filled to the same pressure, but one containing isotopically enriched BF_3 (96 percent B^{10}) and the other containing depleted BF_3 (10 percent B^{10}).[6] Neutrons are counted by means of the $B^{10}(n\alpha)$ reaction so that only the B^{10} component of the gas is active in capturing neutrons. If both counters are exposed to the same neutron flux, containing the same background of recoils and other cosmic-ray events, then the total counts recorded on the enriched counter will be:

$$A = 0.96N + b \text{ count/min.,} \tag{12}$$

while the depleted counter will record:

$$B = 0.10N + b \text{ count/min.,} \tag{13}$$

where N is the theoretical counting rate for a counter containing 100 percent $B^{10}F_3$ and b is the non-neutron background. Solution of these two equations gives:

$$N = (A - B)/0.86, \tag{14}$$

$$b = B - 0.10N. \tag{15}$$

This form of instrumentation effectively eliminates the high energy recoils and other background since B^{11} would presumably be as effective as B^{10} in this type of event. It has a major advantage over the cadmium shield method in that all neutrons are counted rather than merely those in the slowest region.

Before the BF_3 counter can be regarded as an ideal $1/v$ detector corrections must be made for the envelope with which it is surrounded. If a glass envelope is used, it is important to ascertain its boron content, since the most common laboratory glasses, for example Pyrex, are usually boro-silicates. To a first approximation the percentage absorption by a thin envelope as a function of energy is:[7]

$$H = \sigma(E) A \rho P t / M_{Av} \text{ (percent),} \tag{16}$$

where $\sigma(E)$ is the boron capture cross section in cm^2 as a function of E; A is Avogadro's number; ρ is the density of the glass used; P is the percentage by weight of boron in the glass; t is the thickness of the envelope; and M_{Av} is the mean atomic weight of the glass.

For example, in the counters used in the present experiments (Pyrex): $\rho = 2.23$; $P = 4.4$ percent; $t = 0.2$ cm; and $M_{Av} = 19.2$, so that:

$$H = 6.1 \times 10^{-2} \times \sigma(E) \text{ (percent),} \tag{17}$$

where $\sigma(E)$ is in barns.

If we assume the mean energy of neutrons in the atmosphere to be 0.26 ev as calculated, then the mean absorption

$$H = 14 \text{ percent} \tag{18}$$

for the present experiments.

After appropriate corrections have been made it is then possible to consider the BF_3 counter as a nearly ideal $1/v$ detector and to use it to measure the rate of absorption of neutrons in the atmosphere.

IV. EXPERIMENTAL

The general method of instrumentation has already been described. Counters were filled with multiply-distilled BF_3 gas and were carefully matched as to plateau characteristics. All the counters used had approximately 150-volt neutron plateaus in the proportional region centered at 2050 volts. Contamination backgrounds were found to be less than $\frac{1}{4}$ c/min. in all cases.

To prevent spurious counts due to corona discharge at high altitudes the entire high voltage system including the counters was pressurized and units were tested at a simulated altitude of 100,000 feet for an extended time in the high altitude chamber at Mitchel Air Force Base, New York. In addition, extensive temperature and pressure tests on all elements of circuitry were made and revealed negligible effects within the range of temperatures normally encountered.

Pulses from the two counters were fed alternately through the same amplifying system and data were telemetered to two ground stations.

[6] Obtained from the Oak Ridge National Laboratory of the AEC.

[7] S. A. Korff, *Electron and Nuclear Counters* (D. Van Nostrand Company, Inc., New York, 1946).

Apparatus was flown by means of a General Mills polyethylene balloon 30 feet in diameter and ballast controls were added to cause the balloon to float nearly level at two successive altitudes. In this way it was possible to investigate the regions of the atmosphere which were specifically of interest. Details of this technique and a discussion of the electronic circuits will be published elsewhere. All flights were made in the vicinity of Rome, New York, geomagnetic latitude 55°8′ N.

V. COMPARISON WITH THEORY

In agreement with Yuan[8,9] a maximum in the neutron density as a function of altitude was found to exist. Under the conditions outlined by Bethe, Korff, and Placzek,[2] and Flügge[3] the maximum was found at about 100 g/cm^2 absorption depth (7.3 cm Hg). Yuan, counting only those neutrons with energies below the cadmium cut-off, found a maximum at 8.5 cm Hg or 113.5 g/cm^2. Since the two experiments did not measure the same thing it is to be expected that results will differ somewhat. Interpretation of Yuan's data in the light of the calculations described earlier in this paper shows his results to be in agreement generally with those of the author.

The primary source of disagreement between the two experiments was, of course, in the method of instrumentation used. Since at the higher elevations the percentage of more energetic neutrons increases, the counting rate as measured by the cadmium shield method will have its maximum at a lower altitude.

Another factor which must be considered in comparing Yuan's data with that of the author's is the effect of the boron in the Pyrex envelopes used by the latter. Yuan employed soft glass counters with negligibly small boron content.

By comparing the present results with those of Yuan it is possible to check the calculated value of the mean energy of neutrons in the atmosphere. Table I was prepared using data from three flights of Yuan[10] and one of the author's. Correction was made for the difference in counter volume and filling pressure between the counters used in the various experiments. Noncapture background was assumed to be the same for all apparatus, after efficiency corrections were made. In this table N is the neutron counting rate for 100 percent B^{10} content in the BF_3. Data from Yuan's experiments were taken only from the unshielded counters.

In view of the uncertainties in the relative counter calibrations this was taken as good agreement with the calculated absorption of 14 percent. The absorption of 29 percent which would have resulted if the mean energy were 0.06 ev does not seem to be indicated.

It will be noted that absorption is less at the higher altitudes. This is to be expected in view of the increasing proportion of higher energy neutrons at the higher elevations, and resultant higher mean neutron energy in Eq. (17).

An attempt was made to fit the observed data to the theoretical curve drawn from Eq. (10). This plot may be seen in Fig. 1. The solid curve represents the calculations with $J_0=1.00$. The curve was fitted by inspection. The dashed curve represents a correction for the changing absorption in Pyrex as observed experimentally. It will be noted that the effect of this changing absorption is to move the maximum to a higher altitude, which seems to agree with experiment. Although the statistical fluctuation is rather large it is felt that no basic disagreement between theory and experiment is shown in this curve.

For the counters used in these experiments the quantities in Eq. (11) are: $V_d=234$ cm^3, $\sigma_d=585E^{-\frac{1}{2}}$ barn (based on 100 percent B^{10}); $\sigma_a=0.28E^{-\frac{1}{2}}$ barn; $p_{0d}=0.232$ atmos.; $n=N/60$ sec.$^{-1}$ when N is given in counts per minute, so that the production rate of neutrons in the atmosphere,

$$q=1.15\times10^{-4}\times N\ \text{g}^{-1}\ \text{sec.}^{-1}. \tag{19}$$

The production rate was calculated at two altitudes, namely those corresponding to 250 mb and 300 mb. The results were as follows:

$$\begin{aligned}&250\ \text{mb},\quad q=2.2\times10^{-3}\ \text{g}^{-1}\ \text{sec.}^{-1},\\&300\ \text{mb},\quad q=1.5\times10^{-3}\ \text{g}^{-1}\ \text{sec.}^{-1}.\end{aligned} \tag{20}$$

It is of interest to compare the neutron production rate in the atmosphere with that for protons. T. Coor[11] gives the proton production rate at 8.9 cm Hg (119 mb) in the copper-air mixture of an ionization chamber as 0.015 g^{-1} sec.$^{-1}$. In order to compare production rates it is necessary to extrapolate values obtained in the equilibrium region up to higher altitudes. If we assume in the lower region a pressure dependence of the form:

$$N=N_0\exp(-\mu p), \tag{21}$$

then from data we obtain:

$$N=109\exp(-0.007p). \tag{22}$$

Using this equation the extrapolated counting rate at 8.9 cm Hg (119 mb) is 47.5 c/min. Applying Eq. (19), the neutron production rate in air at this altitude is:

$$q=0.006\ \text{g}^{-1}\ \text{sec.}^{-1}. \tag{23}$$

In order to compare this figure with that for the proton production rate it is necessary to correct for the copper in Coor's apparatus. It can be shown that for a spherical ionization chamber of copper as used by Coor, the mean atomic weight of the gas and chamber is effectively that of the copper, to at least one part in 100.

If we assume that the neutron production rate, R, at a given altitude may be represented empirically by

[8] L. C. L. Yuan, Phys. Rev. **74**, 504 (1948).
[9] L. C. L. Yuan, Phys. Rev. **76**, 165 (1949).
[10] L. C. L. Yuan, private communication.
[11] T. Coor, private communication.

the relation

$$R = CZ^{\alpha}, \qquad (24)$$

where Z is the atomic weight and C is a constant, then from the data of Montgomery and Tobey[12] for carbon and aluminum we find α to be approximately 0.5. Using 14.6 for the mean atomic weight of air and 64 for copper we calculate the neutron production rate in copper at 119 mb to be

$$q_c = 0.013 \text{ g}^{-1} \text{ sec.}^{-1}. \qquad (25)$$

[12] C. G. Montgomery and A. R. Tobey, Phys. Rev. **76**, 1478 (1949).

When compared with the proton production rate at this altitude these data support the view that protons and neutrons are formed in approximately equal numbers in stars.

In conclusion the author wishes to extend grateful thanks to Professor S. A. Korff under whose direction this work was performed, and to the members of the New York University Cosmic-Ray Group: L. G. Collyer, H. A. C. Neuburg, M. Pavalow, W. P. Staker, and M. Swetnick.

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The Penetration and Diffusion of Co^{60} Gamma-Rays in Water Using Spherical Geometry*

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Ionization and Geiger-Müller counter measurements of Co^{60} gamma-rays were made in effectively spherical geometry up to a distance of 252.8 cm (16 mean free paths of the primary radiation) corresponding to an exponential attenuation of the primary radiation by a factor of 8.8×10^{6}. The "build-up factor" was observed to climb up to 33.8 and approached closely the theoretically predicted trend of asymptotic variation $r^{1.4}$. Preliminary information on the spectral distribution of the radiation is included.

I. INTRODUCTION

MUCH experimental work has been done to measure absorption coefficients under narrow beam conditions in which no scattered radiation can reach the detector. In other absorption experiments singly or multiply scattered radiation can reach the detector but the geometry dictated by practical considerations hinders a detailed theoretical analysis of the results.[1] Some data are available from arrangements with simple geometry but only at small distances from the source.[2,3]

Therefore, in connection with the progress on the theory of the penetration and diffusion of x-rays,[4] it seemed advisable to conduct a new experiment in a large mass of water and in simple geometry. The Naval Gun Factory in Washington, D. C., kindly made available its large water tank which is 25 ft. in diameter and 60 ft. deep. Measurements were made using two Co^{60} sources with an apparent strength of 0.33 and 4.75

TABLE I. Data from the 4.75 curie source.

Distance (cm)	46	61	76	91	106	120.5	135.5	150.5	180.5
Mr/min.	125.4	36.6	12.8	4.09	1.39	0.500	17.80×10^{-2}	6.4×10^{-2}	8.3×10^{-3}
Standard error	1.3	0.3	0.2	0.06	0.01	0.009	0.02×10^{-2}	0.2×10^{-2}	—
Build-up factor	4.51	5.98	8.34	9.83	11.65	13.5	15.5	17.6	22.0
Distance (cm)	135.5	150.5	165.5	180.5	195.5	210.5	225.5	240.5	252.8
Counts/sec.[a]	1682	647.4	221.4	82.4	29.92	11.2	4.36	1.64	0.72
Standard error	1	0.8	2.5	0.5	0.08	0.1	0.10	0.01	—
Build-up factor[b]	14.7	17.9	19.2	21.7	23.6	26.2	29.6	32.5	33.8

[a] Corrected for coincidence counts and a background of 0.55 counts/sec.
[b] Counter data are scaled to match ionization data in the overlappingregion.

* Work supported by the Applied Mathematics Branch of the ONR.
[1] Kennedy, Wyckoff, and Snyder, J. Research Nat. Bur. Stand. **44**, 157 (1950), RP2066.
[2] W. R. Faust, Phys. Rev. **77**, 227 (1950).
[3] Levin, Weil, and Goodman, M.I.T. Tech. Rep. 22 (June 15, 1949).
[4] Phys. Rev. **76**, 538, 739, 1843, 1885 (1949) and **77**, 425 (1950).