The ^2S and ^4S effective two-body neutron-deuteron interactions were represented by square-well central potentials having depths V_2 and V_4, respectively, and the same range r_0. It was then assumed that[2]

$$r_0=(5.0\pm2.0)\times10^{-12}\text{ cm}. \quad (1)$$

The analysis proceeded with the deduction of a theorem to the effect that the binding energy of the ^2S or ^4S ground state of H^3, calculated on the basis of such an effective two-body model, is not greater than the actual binding energy of a neutron to a deuteron in forming that state. The existence of H^3 in a ^2S ground state of known binding energy and with no bound excited states consequently places a restriction on the maximum values of V_2 and V_4 for a given value of r_0.

The value of V_x ($x=2$ or 4) can now be calculated *uniquely*, for a given value of r_0, from the corresponding scattering length of the deuteron a_x ($x=2$ or 4) by means of the relationship:

$$\tan(K_xr_0)/K_xr_0=1-a_x/r_0, \quad (2)$$

where

$$K_x^2=4MV_x/3\hbar^2,$$

provided the above-mentioned restrictions on V_2 and V_4 are considered. At the time this work was undertaken, the experimental results on the scattering of slow neutrons from deuterons provided only two fairly definite conclusions regarding the values of a_2 and a_4;[3-5] namely: a_2, $a_4>0$; and Eq. (5) below. From the fact that $a_4>0$, it was then concluded from (2) that V_4 cannot be greater than zero, and consequently,

$$0<a_4<r_0. \quad (3)$$

From the fact that $a_2>0$, it was deduced from (2) that, for any given value of r_0 in the range of (1),

$$a_2\geqq(a_2)_{\min}\geqq0.4\times10^{-12}\text{ cm}. \quad (4)$$

Corresponding to four different values of r_0 in the range of (1), sixteen sets of values for V_2 and V_4 were then calculated from Eq. (2) by using values of (a_2, a_4) consistent with (3), (4), and the experimental value of

$$\sigma_D=(4\pi/3)(a_2^2+2a_4^2) =3.44\text{ barns}, \quad (5)$$

which was assumed to be correct. It proved convenient to combine (5) with (2), and consider a_2 and r_0 as the independent variables for the functions V_2 and V_4. On the assumption that the specifically nuclear *n-n* and *p-p* interactions are the same, these sets of potentials were then used to calculate the angular distributions obtained in *p-d* scattering at 250 and 275 kev,[6] and the 90° (center-of-mass system) cross sections obtained for *p-d* scattering from 1.5 to 3.0 Mev.[7] It was found (as was to be expected) that the fit of the calculated values to the experimental data was relatively insensitive to the value of r_0 within the range of (1) (or even to the assumption of two different values of r_0, in that range, for the ^2S and ^4S effective interactions). It was found, however, that a good fit to the data could be obtained only for

$$a_2\approx0.8\times10^{-12}\text{ cm},\quad a_4\approx0.3\times10^{-12}\text{ cm}, \quad (6)$$

a_4 being obtained from a_2 via (5). These values, moreover, provided this fit over the entire energy range of the *p-d* data considered.

This work was carried out before the accurate results obtained at Chalk River for the scattering of slow neutrons from D_2 became available.[8] These latter results, together with (3), (4), and (5), give

$$a_2=(0.826\pm0.012)\times10^{-12}\text{ cm},\quad a_4=(0.26\pm0.02)\times10^{-12}\text{ cm}, \quad (7)$$

which is in excellent agreement with (6) above, deduced from the analysis of *p-d* scattering. This agreement implies that an effective two-body model of the nucleon-deuteron interaction is indeed a practical means of correlating the low energy nucleon-deuteron scattering data, and, that the hypothesis of the equality of the *n-n* and *p-p* interactions is corroborated under these conditions.

A detailed report on the above work will be submitted for publication shortly.

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Asymptotic Solutions of Ordinary Linear Differential Equations of the Second Order

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In a recent paper[1] Kuhn has applied the WKB method to the calculation of the cohesive energies of monovalent metals. His analysis is based on a new procedure proposed by the writer.[2] In consideration of the possibility that this method may be capable of wide applicability, it will be justifiable to present it here in a more explicit form.

We consider the asymptotic solutions of the second-order linear differential equation

$$(d^2\Phi/dx^2)+a^2P(x)\Phi=0 \quad (\text{for } a\rightarrow\infty), \quad (1)$$

where

$$P(x)=a_1x+a_2x^2+\cdots \quad (a_1\neq0).$$

We make the transformations

$$z=\int_0^x [P(y)]^{1/2}dy,$$
$$Q=P^{-1/4}d^2P^{1/4}/dz^2=-P^{-3/4}d^2P^{-1/4}/dx^2 =-[(5/36)z^{-2}+\lambda z^{-2/3}+\lambda_0+\lambda_1z^{2/3}+\cdots], \quad (2)$$
$$Q_1(z)=\int^z Q(y)dy.$$

It can be shown that the two independent solutions Φ_1, Φ_2, can be expressed in the form

$$\Phi_{1,2}=P^{-1/4}\Psi_{1,2} \quad (3)$$

with

$$\Psi_{1,2}=\exp[\pm iaz\pm(i2a)^{-1}Q_1(z)+(4a^2)^{-1}Q(z)]+O(a^{-3}) \quad (4)$$

for $z\neq0$, and

$$\Psi_{1,2}=(\tfrac{1}{2}\pi a)^{1/2}i^{\pm5/6}\exp\left\{\mp(i2a)^{-1}\times\int_0^\infty[Q+(5/36z^2)]dz\right\}\cdot z^{1/6}\zeta H_{1/3}^{(1,2)}(\eta)+O(a^{-3}) \quad (5)$$

for $z=O(a^{-1})$.

Here

$$\zeta=[\xi-\tfrac{1}{5}\lambda_1\kappa^{-2}\xi^2]^{1/2},\quad \eta=\kappa[\xi+\tfrac{1}{5}\lambda_1\kappa^{-2}\xi^2]^{3/2},\quad \xi=z^{2/3}+\lambda\kappa^{-2},\quad \kappa^2=a^2+\lambda_0, \quad (6)$$

and the symbol H stands for the Hankel functions.

As an example, the asymptotic formula for the Hankel functions for the "transition region" can be written in the form

$$H_a^{(1,2)}(a\sec\theta)=i^{\pm1/3}\cot^{1/2}\theta\cdot z^{1/6}\zeta H_{1/3}^{(1,2)}(\eta)+O(a^{-\eta/2}), \quad (7)$$

where

$$z=\tan\theta-\theta,\quad \lambda=-3^{1/3}/105,\quad \lambda_0=2/75,\quad \lambda_1=-(69/13475)3^{2/3}.$$

This is to be compared with Langer's formula,[3] which can be obtained from the above by setting $\lambda=\lambda_0=\lambda_1=0$, giving

$$H_a^{(1,2)}(a\sec\theta)=i^{\pm1/3}\cot^{1/2}\theta\cdot z^{1/2}\cdot H_{1/3}^{(1,2)}(az)+O(a^{-5/3})$$

and Watson's formula[4]

$$H_a^{(1,2)}(a\sec\theta)=i^{\pm1/3}3^{-1/2}\tan\theta\cdot H_{1/3}^{(1,2)}(\tfrac{1}{3}a\tan^3\theta)+O(a^{-1}).$$

One of the merits of the new formula is that it permits the easy computation of the zeros of Φ (for example, of $J_a(x)$), from those of the Bessel functions of order 1/3.

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