correction to the scattering of the nucleon, by an external field, and the scattering of meson by meson. We can easily see that the integral M for the last process converges, since its divergent part is identical with that of the scattering of light by light which has been shown to vanish by cancellation. The conventional perturbation method can now be used again, the only precaution being that the graph should not contain any meson self-energy part of order g^2.

Similar calculations applied to scalar meson with vector coupling shows that the corresponding expression (10) vanishes, since M_s is given only by the first term of (4) owing to the absence of the γ_5 factor; therefore, the divergence cannot be removed in this case by renormalizaion of meson mass and charge. It would be of interest to apply the present theory to investigate the nuclear potential and the nucleon magnetic moments. Further details will be published later.

The author is indebted to Professor T. Y. Wu and Dr. J. Pirenne for the helpful discussions.

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949). This paper contains many terms and concepts used in the present note.
[2] R. P. Feynman, Phys. Rev. **76**, 769 (1949). This paper also gives the explanation of most of the notation used in the present note.

Preparation of Co^{58m} by a (γ, n) Reaction*

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RECENTLY Strauch has reported the preparation and identification of a radioactivity, Co^{58m}, with a half-life of 8.8 hr.[1] The identification was based on evidence of its formation from manganese by a (α, n) reaction. It was also produced by bombardment of cobalt and nickel by deuterons and fast neutrons, and by irradiation of copper with deuterons in the 184-in. cyclotron. He showed that the isomer decayed by the emission of a highly converted gamma-ray with an energy of 25 kev to the well-known 72-day Co^{58} activity which undergoes positron emission and K-capture.[2] Counting of the principal radiation, which consists primarily of 17-kev and 24-kev conversion electrons, requires a sample of high specific activity and use of a windowless counter.

We have prepared Co^{58m} by a (γ, n) process in irradiations in the Iowa State College synchrotron. The necessary high specific activity was obtained by a Szilard-Chalmers-type decomposition of a Co^{III} complex induced by the recoil from the nuclear reaction. In this process the radioactive atoms were converted to the Co^{II} oxidation state, which could then be separated chemically from the target material. To accomplish this separation, 5 g of crystals of the complex salt, $K_3Co(C_2O_4)_3 \cdot 3H_2O$, were placed in a test tube with a 1-cm diameter. The sample tube was carefully aligned with the 65-Mev beam of the synchrotron and irradiated for one hour. After irradiation the target salt was dissolved in water, and the Co^{II} was separated from the $[Co(C_2O_4)_3]^{---}$ anion by adsorption on IRC-50 cation exchange resin. The radioactive Co^{II} was eluted from the resin with HCl and samples were prepared for counting by electroplating cobalt metal on platinum disks. The recovery of cobalt appeared to be better than 95 percent. From the weight of the deposits it was estimated that about three percent of the complex decomposed during irradiation, so that the enrichment amounted to a factor of about 30.

The decay was followed with a windowless, gas-flow G-M counter. A half-life of 9.2±0.2 hr. was observed for the short-lived component. This radiation was completely stopped by a 2 mg/cm² aluminum absorber. Intensities have not yet been satisfactory for the identification of the Co^{58} daughter activity.

A portion of the target salt was mounted for counting under standard conditions and the decay of the C^{11} was followed. The half-life checked the previously reported value,[2] 20.5±0.5 min. By use of the C^{11} as an "internal standard," a tentative ratio for the cross section for the formation of Co^{58m} to the cross section for C^{11} was estimated to be 2.5. Rather large uncertainties in this value were due to the large correction for the self-absorption in the cobalt sample. It is intended to obtain a more accurate value for this quantity, and if possible to evaluate the cross-section ratio for Co^{58}.

We wish to acknowledge our appreciation to Dr. L. J. Laslett and Dr. D. J. Zaffarano for their assistance in providing the synchrotron irradiations.

* Contribution No. 130 from the Institute for Atomic Research and Department of Chemistry, Iowa State College, Ames, Iowa. This work was performed in the Ames Laboratory of the AEC.
[1] K. Strauch, Phys. Rev. **79**, 487 (1950).
[2] G. J. Seaborg and I. Perlman, Rev. Mod. Phys. **20**, 585 (1948).

The Possible Existence of a Constant Third-Order Difference among the Nuclear Magic Numbers

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October 13, 1950

IT has been called to the writer's attention that the results published in a previous Letter to the Editor under this same title[1] are identical with those given in an earlier note by E. Bagge.[2] Although the writer's considerations were made independently of those of Bagge, it is desired to acknowledge the priority of Bagge's work.

[1] F. A. Valente, Phys. Rev. **78**, 77 (1950).
[2] E. Bagge, Naturwiss. **35**, 375 (1948).

Evidence Concerning Equality of n-n and p-p Forces

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COULOMB energies of light nuclei offer strong evidence of equality of proton-proton (p-p) and neutron-neutron (n-n) forces. Comparison of neutron-deuteron (n-d) with proton-deuteron (p-d) scattering, which results, respectively, from the work of Wantuch[1] and Rosen[2] leads to similar conclusions. While the observations of Rosen have been made by bombarding ordinary hydrogen with 10.5-Mev deuterons, they are equivalent to the observation of the scattering of 5.25-Mev protons by stationary deuterons. This energy is comparable with neutron energies of 4.5 and 5.5 Mev in Wantuch's work. The interference effect between the Rutherford and the specifically nuclear waves is too high to make the comparison completely quantitative. Upper limits of the effect of the cross-product term between these two waves may be calculated as in Table I.

Here θ is the scattering angle in the center-of-mass system, and the last row in the table gives the ratio of the absolute value of the interference term $2|\psi\psi^c|$ to the square of the absolute value of the wave function ψ. The Coulomb wave is denoted by ψ^c. Rosen's values were used to obtain $|\psi|$.

In Table I no explicit account is taken either of the two independent spin orientations of the colliding particles or of the phase differences between ψ and ψ^c. It is not likely from a statistical viewpoint that the phase relations for both spin orientations are such as to give a maximum possible interference effect and it is not surprising, therefore, that the two sets of data show very similar values, as may be seen from Table II. The agreement is best, on the whole, around the minimum which is close to $\theta=90°$. A 5 percent Coulomb interference effect would remove an appreciable part of the discrepancy at $\theta=160°$. It will be noted

TABLE I. Values of maximum fractional interference with Coulomb wave.

$\theta=$	60°	90°	120°	150°	180°
$2\lvert\psi^c/\psi\rvert=$	0.23	0.16	0.12	0.055	0.041

TABLE II. Values of collision cross section per unit solid angle in barns.

$\theta=$	70°	80°	100°	110°	120°	140°	160°	180°	E (Mev)
(p-d)	0.115	0.095	0.060	(0.052)	0.055	0.105	0.230		5.25
(n-d)	0.125	0.088	0.050		0.055	0.105	0.200	0.235	5.5
(n-d)		0.095	0.050		0.055	0.130	0.210	0.275	4.5

that for $\theta=130°$ the difference between the values of σ at 5.5 and 4.5 Mev is sufficiently large to make reasonable an estimate of errors of the order of 10 percent in the n-d measurements. The outstanding differences at $\theta=160°$ may not be real, therefore.

On account of effects in the interior of the nucleus one may expect the phase shifts to be affected when a neutron is changed into a proton. An exact correspondence cannot be expected, therefore, even apart from the interference with Coulomb scattering. The author is indebted to Dr. Louis Rosen for helpful discussion of his data.

* Assisted by the joint program of the ONR and AEC.
† Released by declassification authorities following request of September 25, 1950.

[1] E. Wantuch, Phys. Rev. **79**, 729 (1950).
[2] J. C. Allred and L. Rosen, Phys. Rev. **79**, 227 (1950).

A Note on the Classical Spin-Wave Theory of Heller and Kramers*

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October 30, 1950

IN 1934 Heller and Kramers[1] obtained the Bloch energy levels[2] for a ferromagnet by starting with a classical theory of spin-waves and then quantizing this theory. Certain obscurities remained in the theory, however, as recognized by Heller and Kramers, and it is the purpose of this note to clarify these obscure points. We shall discuss particularly the physical and mathematical origin of the apparent zero-point energy, which HK had to omit, and the canonical nature of the variables used. Reference will be made to the quantal spin-wave theory as given by Holstein and Primakoff.[3]

The Hamiltonian for the simplest case, a linear chain of N atoms with nearest-neighbor interactions, is

$$\mathcal{H}=-2\beta H\sum_{l=1}^{N} S_{l,z}-2J\sum_{l=1}^{N}\mathbf{S}_l\cdot\mathbf{S}_{l+1}, \quad (1)$$

where $\mathbf{S}_l$ is the spin vector (operator in the quantal case) of the lth atom in units of $\hbar$, β the Bohr magneton, J the exchange integral, and H the z-directed magnetic field.

Considering $\mathbf{S}_l$ as a classical vector, HK observe that near saturation $S_{l,x}$ and $S_{l,y}$ are small and, therefore,

$$S_{l,z}=[\check{S}^2-(S_{l,x}{}^2+S_{l,y}{}^2)]^{\frac{1}{2}}\cong S[1-(S_{l,x}{}^2+S_{l,y}{}^2)/2S^2], \quad (2)$$

where $\check{S}$ is the magnitude of the spin vector. In a quantal treatment $\check{S}$ is to be replaced by $[S(S+1)]^{\frac{1}{2}}$, where S is now the maximum z component of the spin.

Using Eq. (2) causes the Hamiltonian to become

$$\mathcal{H}\cong\mathcal{H}_{\mathrm{HK}}(\check{S})=-2N\check{S}[\beta H+J\check{S}]+2J\sum_{l=1}^{N}$$
$$\times\{(1+\alpha)(S_{l,x}{}^2+S_{l,y}{}^2)-(S_{l,x}S_{l+1,x}+S_{l,y}S_{l+1,y})\}, \quad (3)$$

where $\alpha=\beta H/2J\check{S}$.

We may compare Eq. (3) with the quantal Hamiltonian obtained[4] by HP after introduction of approximations appropriate near saturation. In the same notation:

$$\mathcal{H}_{\mathrm{HP}}=\mathcal{H}_{\mathrm{HK}}(S)+2J(1+\alpha)i\sum_{l=1}^{N}(S_{l,x}S_{l,y}-S_{l,y}S_{l,x}). \quad (4)$$

We notice that the quantal Hamiltonian has an additional sum of commutators, which vanish classically, and that it is S rather than $\check{S}$ which appears. We shall discuss this below.

An orthogonal transformation is now made by HK to make H a sum of squares. The new variables P_λ, Q_λ after renormalization are determined by

$$S_{l,x}=(S)^{\frac{1}{2}}\sum_{\lambda=0}^{N-1}a_{l\lambda}P_\lambda;\quad S_{l,y}=(S)^{\frac{1}{2}}\sum_{\lambda=0}^{N-1}a_{l\lambda}Q_\lambda, \quad (5a)$$

where

$$a_{l\lambda}=(2/N)^{\frac{1}{2}}\cos[(2\pi\lambda/N)+(\pi\lambda/2N)]\quad(\lambda\neq0)$$
$$a_{l0}=(1/N)^{\frac{1}{2}}. \quad (5b)$$

The variables P_λ, Q_λ are obviously wave-like in nature.

Quantization of the HK theory is now possible, since the classical Poisson bracket is

$$\{P_\lambda, Q_{\lambda'}\}_{\mathrm{P.B.}}=(\delta_{\lambda\lambda'}/S)\{S_{l,x}, S_{l,y}\}_{\mathrm{P.B.}}=\delta_{\lambda\lambda'}S_{l,z}/S\cong\delta_{\lambda\lambda'} \quad (6)$$

from Eq. (5) and with $S_{l,z}/S$ approximately equal to unity, corresponding to conditions near saturation. Hence, $P_\lambda{}^2+Q_\lambda{}^2$ has eigenvalues $2(n_\lambda+\frac{1}{2})$ with $n_\lambda=0, 1, 2, \cdots$.

The energy levels for $\mathcal{H}_{\mathrm{HK}}$ are:

$$E_{\mathrm{HK}}(n_\lambda,\check{S})=-2N\check{S}[\beta H+J\check{S}]$$
$$+4JS\sum_{\lambda=0}^{N-1}\left(1+\alpha-\cos\frac{2\pi\lambda}{N}\right)(n_\lambda+\tfrac{1}{2}), \quad (7)$$

a result differing from the quantal energy levels in the appearance of $\check{S}$ and the additional $\frac{1}{2}$ in the factor $(n_\lambda+\frac{1}{2})$ as will now be shown. For the additional terms in Eq. (4) can readily be calculated using the HP approximations[5] to be

$$S_{l,x}S_{l,y}-S_{l,y}S_{l,x}=iS. \quad (8)$$

(Equation (8) indicates that the same approximation is made in the quantal theory as in Eq. (6); i.e., $S_{l,z}$ is replaced by S.) It follows that, using the same orthogonal transformation and quantizing the result, we obtain

$$E_{\mathrm{HP}}(n_\lambda)=E_{\mathrm{HK}}(n_\lambda, S)-2JS\sum_\lambda(1+\alpha)$$
$$=E_{\mathrm{HK}}(n_\lambda, S)-2JS\sum_\lambda\left(1+\alpha-\cos\frac{2\pi\lambda}{N}\right), \quad (9)$$

which verifies the statement made above.

The physical origin of the extra $\frac{1}{2}$ is not hard to see. It arises from Eq. (2): in the HK theory the distinction between S and $[S(S+1)]^{\frac{1}{2}}$ is not maintained, so that, in effect, the expression $S-S_z$, the deviation of the z component of the spin from its maximum value, is replaced by $[S(S+1)]^{\frac{1}{2}}-S_z$. Now the former quantity has, rigorously, integral values, while the latter is approximately $S-S_z+\frac{1}{2}$ (expanding $[S(S+1)]^{\frac{1}{2}}$). The $\frac{1}{2}$ comes from the fact that, quantum-mechanically, the spin vector never lies along the z axis and $S_x{}^2+S_y{}^2$ is never rigorously zero. In the formal theory this is expressed by the appearance of the commutator of S_x and S_y as mentioned above.

* Supported in part by the ONR.

[1] G. Heller and H. A. Kramers, Proc. Amst. Acad. Sci. **37**, 378 (1934), referred to as HK.
[2] See A. Sommerfeld and H. Bethe, *Handbuch der Physik*, Vol. 24, Part 2, p. 601.
[3] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940), referred to as HP. See also D. Polder, Phil. Mag. **40**, 99 (1949).
[4] Reference 3, Eq. (7).
[5] $S_{l,x}+iS_{l,y}\cong(2S)^{\frac{1}{2}}a_l$, $S_{l,x}-iS_{l,y}\cong(2S)^{\frac{1}{2}}a_l{}^*$, $S_{l,z}=S-a_l{}^*a_l$, $a_la_{l'}{}^*-a_{l'}{}^*a_l=\delta_{ll'}$.
See reference 3, Eqs. (3) to (6) and following discussion.

The Scattering Lengths of the Deuteron and p-d Scattering*

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AN analysis of the S-wave scattering of nucleons by deuterons has been carried out by means of a spin-dependent, "effective" two-body model of the nucleon-deuteron interaction.[1] This analysis was begun by considering the deuteron as a single structureless particle so that the actual nucleon-deuteron interaction could be replaced by an "effective" two-body interaction.