TABLE I. Values of ρ from spectroscopic data.

Ion	State	ρ/cm^{-1}	Reference
Ti^{++}	$3d^2\,{}^3F$	0.24 ± 0.03	a
V^{++}	$3d^3\,{}^4F$	$0.4\ \pm0.1$	a
V^{+++}	$3d^2\,{}^3F$	0.24 ± 0.05	a
Cr^{++}	$3d^4\,{}^5D$	0.42 ± 0.05	b
Cr^{+++}	$3d^3\,{}^4F$	0.44 ± 0.05	c
Mn^{+++}	$3d^4\,{}^5D$	$0.8\ \pm0.2$	c
Fe^{++}	$3d^6\,{}^5D$	0.95 ± 0.1	d
Co^{++}	$3d^7\,{}^4F$	1.50 ± 0.05	e
Ni^{++}	$3d^8\,{}^3F$	5.31 ± 0.03	e

[a] Atomic Energy Levels, Circular 467, National Bureau of Standards (1949).
[b] F. L. Moore, Princeton thesis (1950).
[c] I. S. Bowen, Phys. Rev. **52**, 1153 (1937).
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Detailed calculation shows that the coefficient D arising from (a) is given by

$$D=\frac{6}{25}\left(\frac{e\hbar}{mc}\right)^2\frac{eH'}{\Delta E}(3w-2v)\int r^4\psi_d(r)\psi_s(r)dr, \qquad (1)$$

where

$$v=\int_0^\infty\frac{1}{r}\psi_d(r)\psi_s(r)\int_0^r r'^2\psi_d^2(r')dr'dr,$$

$$w=\int_0^\infty\frac{1}{r^3}\psi_d^2(r)\int_0^r r'^4\psi_d(r')\psi_s(r')dr'dr.$$

Here ψ_d and ψ_s are the radial wave functions of the $3d$ and $4s$ electrons, and ΔE is the energy of $3d^44s\,{}^6D$ above $3d^5\,{}^6S$. It is difficult to estimate the integrals, which depend strongly on the overlap of ψ_d and ψ_s, but reasonable assumptions give the right order of magnitude ($D\sim0.02$ cm^{-1}).

In the other ions of the iron group W_{SS} has diagonal elements within the ground term, representable by a sub-Hamiltonian quadratic in $\mathbf{S}$, and which from arguments of rotational covariance must have the form

$$W_{SS}=-\rho\{\tfrac{1}{2}L_iL_j+\tfrac{1}{2}L_jL_i-\tfrac{1}{3}L(L+1)\delta_{ij}\}S_iS_j$$
$$=-\rho\{(\mathbf{L}\cdot\mathbf{S})^2+\tfrac{1}{2}(\mathbf{L}\cdot\mathbf{S})-\tfrac{1}{3}LS(L+1)(S+1)\}. \qquad (2)$$

One finds, for the lowest term,

$$\rho=\frac{1}{7S(2L-1)}\left(\frac{e\hbar}{mc}\right)^2\left\{(4S-5)p+\frac{1}{7}(100-62S)q\right\}, \qquad (3)$$

where

$$p=\int_0^\infty\frac{1}{r}\psi_d^2(r)\int_0^r r'^2\psi_d^2(r')dr'dr,$$

$$q=\int_0^\infty\frac{1}{r^3}\psi_d^2(r)\int_0^r r'^4\psi_d^2(r')dr'dr.$$

If hydrogen-like wave functions with effective nuclear charge Z' are used to calculate p and q, the result is

$$p=12q/7=(11/25920)(Z'me^2/\hbar^2)^3.$$

Second-order effects from W_{LS} via other terms of the $3d^n$ configuration will also give contributions of the form (2), which will add to (3), and in the second half of the transition series they may be larger than the W_{SS} contribution.

In chromous and manganic salts ($3d^4\,{}^5D$) the splitting of the spin quintet which is left lowest by the crystalline field is probably mainly determined[4] by W_{SS}. In chromic and vanadous salts ($3d^3\,{}^4F$), and nickel salts ($3d^8\,{}^3F$), the term in ρ, and also a mechanism analogous to (a), may contribute appreciably to the splitting, and previous estimates of the magnitude of V_2 from the splitting should be treated with reserve.

In some cases ρ can be inferred from the departures from the interval rule in spectroscopic data. These are listed in Table I.

I wish to thank Mr. Martin Redlich for help in the calculations of the coefficients in (1) and (3).

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The New Isotope Pu^{242} and Additional Information on Other Plutonium Isotopes

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October 20, 1950

INVESTIGATION of the higher isotopes of plutonium produced by neutron irradiation has revealed the existence and properties of a new plutonium isotope, namely Pu^{242}, and has also yielded some additional information about the previously reported isotopes Pu^{240} and Pu^{241}. This letter will give only a brief report of these new results and a detailed description of the experiments will be deferred until a later date. The investigation of these isotopes was made possible by the intensive irradiation with neutrons of plutonium and americium samples originally consisting essentially completely of the isotopes Pu^{239} and Am^{241}, respectively. The production of Pu^{240} and Pu^{241} by the neutron irradiation of Pu^{239} has been reported previously.[1] Following the irradiation of the samples, which occurred over periods up to a few years, the plutonium, americium, and curium were chemically separated from each other and from fission products and impurities. The isotopic composition of each plutonium sample was then determined by use of a mass spectrograph which employed a thermal ionization source. Lines corresponding to plutonium isotopes of masses 241 and 242 produced by (n,γ) reactions were observed in addition to lines due to the other well-known plutonium isotopes (Pu^{238}, Pu^{239}, and Pu^{240}).

Pu^{240}.—Using very thin samples formed by volatilization from a hot filament, the width of the observed alpha-pulse analysis peak corresponding to the alpha-particles in a sample containing the two isotopes Pu^{239} and Pu^{240} was such that there is not more than about 20-kev difference in the alpha-particle energies of these two isotopes. Therefore, the alpha-particle energy[2] of Pu^{240} is 5.16 ± 0.02 Mev.

Pu^{241}.—A low abundance group of alpha-particles at the energy 4.91 ± 0.03 Mev was observed in neutron-bombarded plutonium using the 48-channel differential alpha-energy pulse analyzer. This group is present in the amount expected if it is due to the isotope Pu^{241}, i.e., best agreement with the alpha-decay systematics[3] is obtained if it is due to Pu^{241}, and it cannot be due to Pu^{242} in view of the results discussed below. To be a low energy alpha-group of the isotope, Pu^{240}, its abundance would apparently be greater by a factor[4] of 5 to 10. If this alpha-particle group is ascribed to Pu^{241}, its alpha-intensity and isotopic abundance lead to a partial half-life of Pu^{241} for alpha-particle decay of roughly 4×10^5 years in agreement with previous results.[5]

The half-life of Pu^{241} for beta-particle decay was estimated from the growth of the daughter Am^{241} using tracer Cm^{242} to determine chemical yield in the separations of the Am^{241} from the plutonium. The separations were made with measured amounts of plutonium in which the daughter had been allowed to grow over successively measured intervals of time. Using the mass spectrographically determined value for the isotopic abundance of Pu^{241} in the plutonium, and a value of 475 years for the half-life[6] of Am^{241}, the half-life of the Pu^{241} for beta-decay was calculated as 14 years, which is in rough agreement with the previous value[5] ($\sim$10 years).

A rough estimate of the cross section for the reaction $Pu^{241}(n,\gamma)Pu^{242}$ with pile neutrons was obtained using an estimated value for the neutron flux and the isotopic compositions determined in the mass spectrograph. The cross section so obtained was very roughly 250 barns, but it is subject to large error owing to uncertainty in the estimation of the neutron flux.

Pu^{242}.—The isotope Pu^{242} was observed in a mass spectrographic analysis of neutron-bombarded plutonium, but its abundance was too small to make it possible to identify the radioactive decay properties in such samples. However, O'Kelley, Crane, Barton, and Perlman[7] have observed that the 16-hour Am^{242m} undergoes appreciable branching decay through the electron capture process. Advantage was taken of this by separating a plutonium fraction from a sample of americium (Am^{241}) which had been bombarded

with neutrons for several months. Mass spectrographic analysis of the total plutonium produced during the irradiation showed that it consisted of about 50 percent Pu^{242} and 50 percent Pu^{238}, the former produced by the electron capture decay of Am^{242m} and the latter through the decay chain $Am^{242m}(\beta^-/16\text{ hr.}) > Cm^{242}(\alpha/162\text{ days}) > Pu^{238}$. Alpha-pulse analysis of this plutonium showed the presence of alpha-particles of 4.88 Mev in abundance corresponding to a half-life of roughly 5×10^5 years for Pu^{242}. This energy and half-life agrees well with those expected for Pu^{242} from the alpha-decay systematics, which also indicate that this isotope should be beta-stable. A smaller sample of daughter plutonium containing a higher proportion of Pu^{242} with respect to Pu^{238} was obtained by separating plutonium from a sample of 16-hour Am^{242m} which was initially free of plutonium and which was allowed to decay for about a day. The alpha-pulse analysis of this sample also showed the 4.88-Mev alpha-particle of Pu^{242}, in this case present in greater relative abundance as expected.

We wish to acknowledge the advice and assistance of Professor Glenn T. Seaborg, whose help contributed greatly to the success of this work.

The successful handling in a safe manner of the radioactivity involved was made possible through the operation and use of remote control equipment and excellent protective devices provided by Nelson Garden and the members of his Health Chemistry group. In this connection we especially wish to thank C. M. Gordon, W. G. Ruehle, and J. M. Davis for assistance during the experiments.

* This work was performed under the auspices of the AEC.
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The S-Matrix in Meson Theory

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IT has been shown by Dyson[1] that there are only a few types of "primitive divergent graphs" in quantum electrodynamics (different types being specified by different values of E_n and E_m defined below). This result leads to the separation of the divergence from the S-matrix obtained by conventional perturbation method. We shall now extend the investigation to the meson theory. We first consider the pseudoscalar meson under pseudovector coupling with the nucleons. Let us consider any primitive graph with n vertices, E_n external nucleon lines, and E_m external meson lines. The numbers of internal nucleon and meson lines are respectively $F_n=n-E_n/2$ and $F_m=(n-E_m)/2$. The contribution of this graph to the S-matrix is an integral M which contains F_n factors of the form $S_F(\mathbf{k}_i)=(\mathbf{k}_i+m)/(\mathbf{k}_i^2-m^2)$ and F_m factors of the form $D_F(\mathbf{k}_j)=1/(\mathbf{k}_j^2-\kappa^2)$ and $n-E_m$ factors $g\gamma_5\mathbf{k}_j$ integrated over $d^4k_1d^4k_2\cdots d^4k_F$, where $F=F_n+F_m-n+1$, g is the interaction constant, κ and m are the masses of the meson and nucleon. The bold type letter $\mathbf{k}$ stands for $\gamma_\mu k_\mu$. M will be convergent if

$$2E_m+\tfrac{3}{2}E_n-n-4\geqslant 1. \quad (1)$$

(1) will always be violated for any given values of E_n and E_m if n is big enough. This means that the number of types of primitive divergent graphs is infinite. It is the aim of the present note to show that we can still separate the divergence from the S-matrix.

The self-energy integral of the meson of the lowest order g^2 is

$$M_s=-\frac{g^2}{\pi i}\int Sp\left[\gamma_5\mathbf{k}\frac{\mathbf{p}-\frac{1}{2}\mathbf{k}+m}{(\mathbf{p}-\frac{1}{2}\mathbf{k})^2-m^2}\gamma_5\mathbf{k}\frac{\mathbf{p}+\frac{1}{2}\mathbf{k}+m}{(\mathbf{p}+\frac{1}{2}\mathbf{k})^2-m^2}\right]d^4p. \quad (2)$$

For free meson we have $\mathbf{k}^2=\kappa^2$. For virtual mesons k_μ are four independent variables of integration. We shall consider the general case in which $\mathbf{k}^2\neq\kappa^2$. Noticing that γ_5 anti-commutes with γ_μ and $\gamma_5\gamma_5=1$, we have

$$M_s=-\frac{g^2}{\pi i}k_\mu k_\nu\int Sp\left[\gamma_\mu\frac{\mathbf{p}-\frac{1}{2}\mathbf{k}-m}{(\mathbf{p}-\frac{1}{2}\mathbf{k})^2-m^2}\gamma_\nu\frac{\mathbf{p}+\frac{1}{2}\mathbf{k}+m}{(\mathbf{p}+\frac{1}{2}\mathbf{k})^2-m^2}\right]d^4p. \quad (3)$$

Since the spur of an odd number of factors of γ_μ vanishes, the term linear in m must vanish. Equation (3) can therefore be written as

$$M_s=k_\mu k_\nu K_{\mu\nu}+\frac{2g^2}{\pi i}k_\mu k_\nu\int Sp(\gamma_\mu\gamma_\nu)\times\frac{m^2d^4p}{[(\mathbf{p}-\frac{1}{2}\mathbf{k})^2-m^2][(\mathbf{p}-\frac{1}{2}\mathbf{k})^2-m^2]}, \quad (4)$$

where

$$K_{\mu\nu}=-\frac{g^2}{\pi i}\int Sp\left[\gamma_\mu\frac{\mathbf{p}-\frac{1}{2}\mathbf{k}+m}{(\mathbf{p}-\frac{1}{2}\mathbf{k})^2-m^2}\gamma_\nu\frac{\mathbf{p}+\frac{1}{2}\mathbf{k}+m}{(\mathbf{p}+\frac{1}{2}\mathbf{k})^2-m^2}\right]d^4p \quad (5)$$

is the vacuum polarization integral obtained in electrodynamics. The integral in (5) and the second term of (4) have been evaluated by Feynman[2] using his regularization method. We have

$$K_{\mu\nu}=-\frac{g^2}{\pi}(k_\mu k_\nu-\delta_{\mu\nu}\mathbf{k}^2)\left[-\tfrac{1}{3}\ln\frac{\lambda^2}{m^2}-\frac{4m^2+\mathbf{k}^2}{3\mathbf{k}^2}\left(1-\frac{\theta}{\tan\theta}\right)+\tfrac{1}{9}\right], \quad (6)$$

where $\mathbf{k}^2=4m^2\sin^2\theta$. It is seen that $k_\mu k_\nu K_{\mu\nu}$ vanishes identically. The second term of (4) gives

$$M_s=\frac{g^2}{\pi}\mathbf{k}^2\left[4m^2\left(1-\frac{\theta}{\tan\theta}\right)-\tfrac{1}{3}\mathbf{k}^2+2(\lambda^2+m^2)\ln\left(\frac{\lambda^2}{m^2}+1\right)\right]. \quad (7)$$

For free mesons, $\mathbf{k}^2=\kappa^2$, M_s is equal to the mass renormalization

$$\Delta\kappa=\frac{g^2}{\pi}\kappa^2\left[4m^2\left(1-\frac{\theta_0}{\tan\theta_0}\right)-\tfrac{1}{3}\kappa^2+2(\lambda^2+m^2)\ln\left(\frac{\lambda^2}{m^2}+1\right)\right], \quad (8)$$
$$\kappa^2=4m^2\sin^2\theta_0.$$

We have

$$M_s=\Delta\kappa+(\mathbf{k}^2-\kappa^2)\frac{g^2}{\pi}\left[4m^2-\tfrac{1}{3}\kappa^2+2(\lambda^2+m^2)\ln\left(\frac{\lambda^2}{m^2}+1\right)\right]+\frac{g^2}{\pi}\left[\tfrac{1}{3}\mathbf{k}^2(\kappa^2-\mathbf{k}^2)+4m^2\left(\kappa^2\frac{\theta_0}{\tan\theta_0}-\mathbf{k}^2\frac{\theta}{\tan\theta}\right)\right]. \quad (9)$$

The second term on the right-hand side of (9) is the renormalization of the mesic charge, g. The first and second terms in (9) should be omitted if we use the observed values of κ and g. The only observable effect in (9) is the third term

$$D_C(\mathbf{k})=\frac{g^2}{\pi}\left[\tfrac{1}{3}\mathbf{k}^2(\kappa^2-\mathbf{k}^2)+4m^2\left(\kappa^2\frac{\theta_0}{\tan\theta_0}-\mathbf{k}^2\frac{\theta}{\tan\theta}\right)\right], \quad (10)$$

which will contribute to the radiative effect of more complicated processes involving this virtual meson. Let us consider any internal meson line of a given graph. It can be seen easily that the total radiative effect from the self-energy parts of the meson is obtained by replacing the $D_F(\mathbf{k})$ function by

$$D_F'(\mathbf{k})=D_F(\mathbf{k})+D_F(\mathbf{k})D_C(\mathbf{k})D_F(\mathbf{k})+D_F(\mathbf{k})D_C(\mathbf{k})D_F(\mathbf{k})D_C(\mathbf{k})D_F(\mathbf{k})+\cdots=D_F(\mathbf{k})/[1-D_C(\mathbf{k})D_F(\mathbf{k})], \quad (11)$$

where the first term is the original D_F function with no radiative correction, the second term is the radiative correction when one self-energy part is inserted into the meson line, and the third term is the radiative correction when two self-energy parts are inserted, and so on. All of the self-energy parts thus inserted are of the lowest order in g, and no self-energy part of higher order has been inserted. At large values of $\mathbf{k}^2$, $D_F'(\mathbf{k})$ behaves as $1/(\mathbf{k}^2)^2$, while $D_F(\mathbf{k})$ behaves only like $1/\mathbf{k}^2$. After the replacement has been introduced into every internal meson line, the degree of the integrand of M is decreased by $2F_m=(n-F_m)$. Therefore, the condition of convergence (1) is now replaced by

$$E_m+\tfrac{3}{2}E_n-4\geqslant 1, \quad (12)$$

which is the same condition as is obtained in electrodynamics. Therefore, the divergence in our case is the same as in the case of electrodynamics after the renormalizations of charge and mass of the meson have been taken care of. As in electrodynamics, the only possible types of divergences are the self-energy of the nucleon, the self-energy of the meson (of order higher than g^2), the radiative