Studies of the Propagation Velocity of a Ferromagnetic Domain Boundary*

H. J. Williams, W. Shockley, and C. Kittel
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received August 4, 1950)

Experimental results are given on the velocity of propagation of a single domain boundary in a crystal of silicon iron with a simple domain structure. In weak applied magnetic fields (~0.003 oersted) the velocity is given by a relation of the form $v=G(H-H_0)$, where G is a constant ~4 cm/sec./oersted in this crystal, and $H_0 \cong 0.003$ oersted is the *starting field*. Calculation of the eddy current losses accompanying the motion of a plane boundary gives a theoretical expression for G in good agreement with experimental values; the predicted linear dependence on the resistivity was approximately verified by measurements at 78°, 194°, and 293°K. In stronger fields (>5 oersteds) there is evidence that the wall closes on itself, and the experimental velocity of collapse of the wall as deduced from flux changes agrees with the theoretical result based on a model of eddy current losses accompanying a collapsing cylindrical boundary. The results have a bearing on the well-known *eddy current anomaly*, namely, the fact that the total loss in a ferromagnetic material undergoing a.c. magnetization is often two or three times larger than the eddy-current and hysteresis losses calculated in the usual way assuming a spatially uniform and isotropic classical permeability.

I. INTRODUCTION

THIS paper discusses the results and interpretation of measurements of the propagation velocity of a ferromagnetic domain boundary in the single crystal of silicon iron with a simple domain structure employed previously by Williams and Shockley.[1] The experiment is similar in principle to the Sixtus-Tonks experiment,[2] with the important difference that in the present experiment the eddy current configuration is amenable to exact mathematical calculation, thereby enabling a quantitative comparison with observation. Experiments and analysis similar to those described in III and V below have been carried out by K. H. Stewart[2a] and were reported at the Grenoble Conference on Ferromagnetism and Antiferromagnetism as were the principal results of this article. However, it appears from Stewart's hysteresis loops unlikely that his specimen had as simple a domain structure as that encountered in our experiments.

II. DESCRIPTION OF SPECIMEN

The specimen was cut from a single crystal of 4 weight percent silicon iron so as to form a hollow rectangle (Fig. 1) having all surfaces parallel to [100] planes. The outside dimensions of the rectangle are 1.34×1.71 cm and the cross section of a leg is 0.114 ×0.152 cm.

Figure 1 shows the domain structure of the crystal. The broken lines represent domain boundaries or Bloch walls. There is a boundary at each corner and one extending completely along the length of the specimen, forming a total of eight domains. If the four inner domains carry flux around the crystal in a clockwise manner the four outer domains carry flux in a counter-clockwise manner. The net flux depends on the position of the boundary which can be moved by an applied field. The field is obtained by winding a number of turns of wire around the specimen and then passing a current through the wire. The change in flux is measured by a secondary winding connected to a fluxmeter.

III. LOW FIELD MEASUREMENTS

With this specimen there is a direct correlation between the position of the boundary and the net magnetization in the specimen, so that the velocity of the boundary movement for low fields can be measured by timing the deflections on a Cioffi recording fluxmeter with a stop watch. When the magnetization changes from saturation in one direction to saturation in the other direction the boundary moves the width of a leg in the hollow rectangle. At low fields the boundary is thought to be plane, as shown in Fig. 2a, the surface tension of the boundary overcoming the tendency of the eddy current drag to make it curve. The effective driving field was taken to be the difference between the applied field and the minimum value of the field which would make the boundary move. First a hysteresis loop was traced using the smallest value of the field that would move the boundary. This value of the field was not constant but varied somewhat as the boundary moved across the crystal perhaps due to different imperfections that the boundary encountered in various places. Then another loop was traced keeping the excess field constant by manipulating the current control in the magnetizing circuit. This was done by observing the loop as it was being traced and keeping the recording pen a constant distance away from the side of the previous loop. A series of such loops was traced with constantly increasing field and the times for the deflections measured. Then the velocity was plotted as a function of the field. The maximum excess field was 0.003 oersted. A linear relation was obtained; the slope of the line gave the velocity per oersted. Sets of measurements were made at 78°, 194°, and 293°K.

* A preliminary report was presented at the New York Meeting, Phys. Rev. 78, 341 (1950).
[1] H. J. Williams and W. Shockley, Phys. Rev. 75, 178 (1949).
[2] K. J. Sixtus and L. Tonks, Phys. Rev. 37, 930 (1931); 39, 357 (1932); 42, 419 (1932); 43, 70, 931 (1933).
[2a] K. H. Stewart, Proc. Phys. Soc. 63A, 761 (1950).

These results are compared with the theoretical values in Fig. 3.

The criterion for low fields is that the effect of surface tension for appreciable curvature of the wall be large compared to the driving field. A surface tension γ of 2 ergs/cm^2 and a field H of 0.003 oersted correspond to a radius of $r=\gamma/2IH=2/2\times1500\times0.003=0.22$ cm or more than twice the edge of the rectangle cross section. For such a condition, no matter what forces tended to hold the wall back, the applied field could not deform it appreciably from a plane. On the other hand, for the high field conditions described below, the H values are 1000 times larger and consequently surface tension forces are negligible for the curvatures that are expected to occur, such as those in Fig. 2(b), for example.

IV. HIGH FIELD MEASUREMENTS

Some preliminary work was done in which a condenser was discharged through the primary winding and the deflections observed on a galvanometer scale to determine the extent of the boundary movement. During these tests it was observed that if a reversing field was applied to the specimen in an initially saturated condition the net change in flux increased uniformly with condenser voltage up to about the point at which half the flux was reversed. For larger discharges it was observed that the ballistic kick was followed by a slow motion which proceeded with gradually increasing velocity until the specimen became spontaneously magnetized to saturation in the reverse direction. This was interpreted as meaning that under high fields the eddy currents tend to retard the motion of the boundary more in the middle of the crystal than near the surface so that the boundary curves and finally forms a cylinder which collapses due to its surface tension as is shown in Fig. 2b. This idea was checked by making a series of measurements at comparatively high fields.

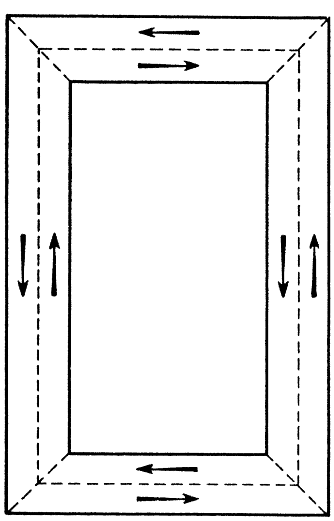

FIG. 1. Simple domain structure in Si-Fe single crystal in form of hollow rectangle.

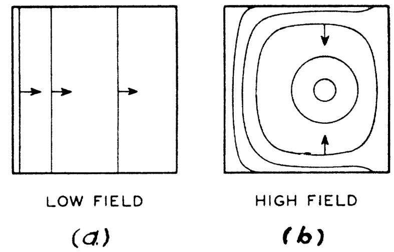

FIG. 2. Boundary motion in low and high fields.

For high field measurements secondary voltage pulses were photographed on a type 304H Dumont cathode-ray oscillograph using a 5C PA tube. A variable delay circuit was used so that the field was applied after the sweep had been triggered. The secondary voltage pulses were photographed with a Leica camera having an f/3.5 lens and using a super xx panchromatic film. Figure 4 shows a typical voltage pulse.

Voltage pulses were obtained for fields ranging from 5 to 80 oersteds. For each pulse values of V/H were plotted as a function of the corresponding values of $T\times H$ as shown in Fig. 5, which also shows the theoretical curve for a collapsing cylindrical boundary. The experimental results are in good agreement with theory.

We go on to derive the theoretical expressions for the low and high field situation.

V. EXACT SOLUTION OF EDDY CURRENT LOSSES FOR PLANE WALL IN RECTANGULAR BAR

This calculation is intended to apply at low fields. The width of the bar in the x-direction is $2L$, and in the y-direction is d. The origin of coordinates is at the center of the cross section, and the wall is in the plane $x=0$. We neglect H in comparison with B_s, as is readily justified. We employ Gaussian units.

We require solutions of the following equations for the current density $\mathbf{i}$:

$$\nabla^2\mathbf{i}=0; \tag{1}$$

$$\mathrm{curl}\,\mathbf{i}=0; \tag{2}$$

$$\mathrm{div}\,\mathbf{i}=0; \tag{3}$$

except within the wall, where

$$\mathrm{curl}\,\mathbf{i}=-(1/\tau c)(d\mathbf{B}/dt), \tag{2a}$$

τ being the electrical resistivity. The boundary conditions are

$$\mathbf{i}_n=0 \tag{4}$$

on all outer surfaces, and at the wall position

$$\pm i_y=B_s v/\tau c, \tag{5}$$

where v is the wall velocity. We suppose that the wall is moving sufficiently uniformly so that dissipation of energy by purely local eddy currents caused by structural irregularities may be neglected. This appears to

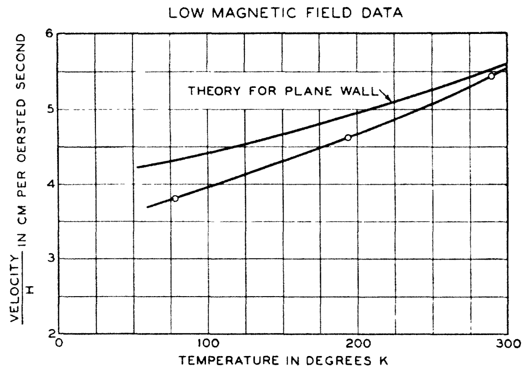

FIG. 3. Comparison of experimental and theoretical values of the wall velocity in low fields. The theoretical calculation, based on eddy current losses for a plane wall in uniform motion, does not contain any disposable constants. The deviations between the two curves are within the estimated accuracy of the measurements.

be a valid supposition in our experiments, but in the case of cold worked materials the local losses may be appreciable, and we have in these circumstances a probable explanation of the effects of cold-working reported by Dijkstra and Snoek[3] in their work on the Sixtus-Tonks experiment.

The solution may be verified to be of the form:

$$i_x = -\sum_{\substack{\text{odd}\\ n}} D_n \sin(n\pi y/d)\sinh[(L-x)n\pi/d]; \tag{6}$$

$$i_y = \sum_{\substack{\text{odd}\\ n}} D_n \cos(n\pi y/d)\cosh[(L-x)n\pi/d], \tag{7}$$

where

$$D_n = \pm 4(B_s v/\tau c)/[n\pi \cosh(Ln\pi/d)], \tag{8}$$

the plus sign obtaining for $n=1, 5, 9, \cdots$, and the minus sign for $n=3, 7, 11, \cdots$.

We denote by P the power loss per unit length in the z-direction, and find

$$P = 4\tau \int_0^L \int_0^{d/2} (i_x^2 + i_y^2)\,dxdy, \tag{9}$$

which comes out after carrying out the indicated integrations, to be

$$P = (16d^2B_s^2v^2/\pi^3\tau c^2) \sum_{\substack{\text{odd}\\ n}} n^{-3}\tanh(n\pi L/d). \tag{10}$$

The series is rapidly converging, and for a square rod $d=2L$ is equal to 0.97, while for the actual dimensions of the crystal employed the sum is 1.00.

If we set the eddy current losses equal to the rate $2HI_svd$ at which work is done by the applied field on the specimen, per unit length, we find, taking the sum as equal to unity, the result

$$v = (\pi^2\tau c^2/32B_s d)H, \tag{11}$$

[3] L. J. Dijkstra and J. L. Snoek, Philips Res. Rep. 4, 334 (1949).

which is in excellent (± 20 percent or less) agreement with the experimental results shown in Fig. 3. The observed velocities are slightly lower, and this is in the expected direction as we have not considered purely relaxation effects on the wall motion. In the experiments $v/H \approx 4$ cm/sec./oersted.

Landau and Lifshitz[4] in their classic paper on domains show that relaxation effects alone will give the following relation:

$$v = (\gamma^2 I_s \Delta/\Lambda)H, \tag{12}$$

where Λ is the relaxation frequency, γ is the magnetomechanical ratio and $\Delta = (A/K)^{\frac{1}{2}}$ is the usual wall thickness parameter; in Si-Fe we have $\Delta \approx 3\times 10^{-6}$ cm; $\gamma \approx 2\times 10^7$ radians/sec./oersted; $I_s \approx 1600$; $\Lambda \approx 3\times 10^9$ sec.$^{-1}$ as estimated from microwave resonance results; so that $v/H = 600$ cm/sec./oersted. This is much higher than the actual velocity, which is apparently limited almost entirely by eddy currents. But in very thin sheets or in high resistivity material (such as ferrites) we may expect to find that relaxation processes[5] are important in determining wall velocities. Further, in the case of experiments of the Sixtus-Tonks type the wall makes a small glancing angle with the propagation direction, so that the effective value for Δ to be used in Eq. (12) may be greater by a factor of the order of 100 than the normal wall thickness. This is the explanation which we offer to account for the high values ($\sim$50000 cm/sec./oersted) for v/H reported, for example, by Dijkstra and Snoek[3] in a Sixtus-Tonks experiment.

VI. STUDY OF COLLAPSING CYLINDRICAL DOMAIN WALL IN CYLINDRICAL SPECIMEN

This calculation is intended to apply at high fields; we approximate the rectangular cross section of a crystal leg by a circle of equal area. We consider a long cylinder of magnetic material of radius R, and suppose that within and concentric with this there is a tube-like domain boundary of radius ρ separating two domains running in the direction of the cylinder axis, but antiparallel to each other. We suppose that the wall is collapsing under the action of an external field H alone, and neglect the surface tension of the wall,

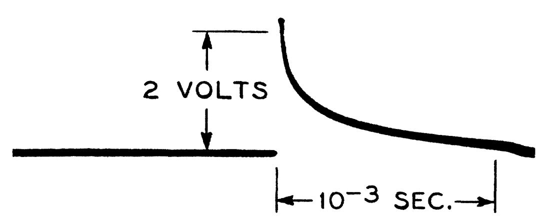

FIG. 4. Photograph of voltage pulse resulting from collapse of Bloch wall in strong fields ($\sim$10 oersteds).

[4] L. Landau and E. Lifshitz, Physik. Zeits. Sowjetunion 8, 153–169 (1935).

[5] C. Kittel, Proceedings of Grenoble Conference, 1950 (to be published); Phys. Rev. 79, 214 (1950).

which is quite legitimate. We study the voltage induced in an external winding which is concentric with the cylinder.

The flux associated with the collapsing wall induces at radius r a field E_θ:

$$2\pi r E_\theta = -\frac{1}{c}\frac{d\varphi}{dt} = -\frac{1}{c}8\pi M_s \cdot \frac{d}{dt}\pi\rho^2 = -\frac{16\pi^2}{c}M_s\rho v_r, \quad (13)$$

where $v_r = d\rho/dt$ is the radial velocity of the wall. We have

$$E_\theta = -(8\pi/c)M_s v_r \rho/r. \quad (14)$$

Dropping the minus sign, the eddy current density is

$$j_\theta = 8\pi M_s v_r \rho/\tau c r, \quad (15)$$

where τ is the electrical resistivity. The total current per unit length of cylinder is

$$i_\theta = \int_\rho^R j_\theta dr = (8\pi/\tau c)M_s v_r \rho \ln(R/\rho). \quad (16)$$

We have again neglected secondary eddy current effects, as is quite justified in our velocity range.

The longitudinal field inside the wall is

$$H = 4\pi i_\theta/c,$$
$$H = (32\pi^2/\tau c^2)M_s v_r \rho \ln(R/\rho). \quad (17)$$

The velocity of the wall is determined by the energy balance $A = B$,

where

A = energy dissipation by eddy currents,
B = energy release by magnetization change.

$$A = \int_\rho^R jE2\pi r dr = \int_\rho^R (64\pi^2/\tau c^2)M_s^2 v_r^2 \rho^2 \cdot 2\pi dr/r$$
$$= (128\pi^3/\tau c^2)M_s^2 v_r^2 \rho^2 \ln(R/\rho); \quad (18)$$
$$B = 4\pi H M_s \rho v_r. \quad (19)$$

Equation (19) gives for the velocity

$$v_r = H\tau c^2/[32\pi^2 M_s \rho \ln(R/\rho)] \quad (20)$$

and, for the voltage,

$$V = H\tau c N/2 \ln(R/\rho), \quad (21)$$

where N is the number of turns on the secondary winding. The comparison with experiment is shown in Fig. 5.

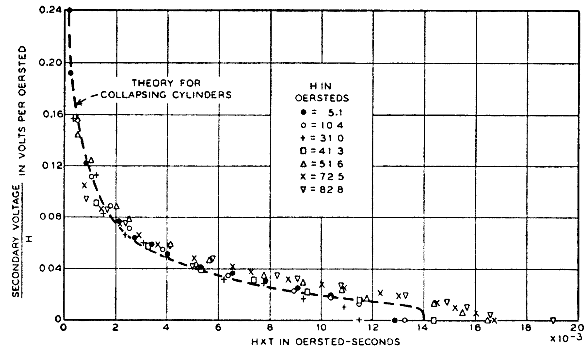

FIG. 5. Comparison of experimental and theoretical values of the voltage pulse arising from the wall collapse at high fields. The theoretical calculation is based on eddy current losses for a circular cylinder and does not contain any disposable constants.

VII. THE EDDY CURRENT ANOMALY

The calculation of eddy current losses in ferromagnetic materials is a standard engineering calculation in connection with transformer cores and rotating electrical machinery. It is well known that the calculation does not give correct results in ferromagnetic materials, even when hysteresis effects are considered: the observed losses are always greater than the calculated losses, often by a factor of two or three. The discrepancy is known as the eddy current anomaly.[6]

At the basis of the standard calculation lies the assumption that the permeability is homogeneous and isotropic; that is, it is assumed that every element of volume of the specimen is characterized by a scalar permeability μ_0. Actually we know that the important magnetization changes in ferromagnetic materials are associated with domain rotation and domain boundary displacement. We are concerned in this paper only with domain boundary displacement. Here the effective local permeability is very inhomogeneous. In the case of a field applied parallel to a 180 degree boundary the local permeability is unity away from the boundary, but assumes extremely high values within the boundary. It is obvious that the values inside the boundary must be very high if the average of the permeability over the entire volume is to account for the high average permeabilities observed in transformer materials.

It is reasonable to expect that such an inhomogeneous distribution of permeability will lead in general to higher eddy current losses than one would calculate on the basis of an equivalent uniform average permeability. A given flux change will cause a definite e.m.f. $\int E \cdot dl$ around a fixed path, independent of the origin of the flux change. But the power dissipation is proportional to E^2, and the average value of E^2 will be higher for an inhomogeneous distribution of flux change than for a uniform distribution, even though the total flux change may be equal in both cases. Therefore, we expect on this qualitative reasoning to find that the uniform permeability assumption leads to calculated losses lower than observed losses.

We give below exact calculations for the comparison of losses on the classical and domain models for the case of a square rod in an a.c. field parallel to the axis of the rod. For the domain structure we assume a single

[6] F. Brailsford, J. Inst. Elec. Eng. **95**, II, 38 (1948); O. I. Butler and C. Y. Mang, J. Inst. Elec. Eng. **95**, II, 25 (1948); R. Feldtkeller, Frequenz **3**, 229 (1949); V. E. Legg, Bell Sys. Tech. J. **15**, 39 (1936); L. W. McKeehan and R. M. Bozorth, Phys. Rev. **46**, 527 (1934).

plane domain wall dividing the square into two equal rectangles. A suitable basis for comparison of the losses is found in the comparison of Q's, that is, the ratio of energy stored to energy dissipated per radian. The domain model has a Q lower than the classical model by the factor $\frac{1}{3}$. This is of the same order of magnitude as the observed discrepancy, but of course the calculated results will depend on the particular geometry employed.

In the case of a circular cylinder with a concentric domain boundary the ratio $Q(\text{domain})/Q(\text{classical})$ can be varied between 0 and ∞, according to the position of the domain boundary. This situation is rather special, however, and in general one would expect the domain Q to be lower, on the basis of the qualitative argument given above relating to the average values of E^2 and E.

There are theoretical grounds for believing that the eddy current anomaly may be larger in thin sheets of material than in thicker sheets, if we suppose the number of domain boundaries to be constant. The thinner the sheet, the more concentrated are the lines of E about each domain boundary, thus leading to a high average value of E^2 and high eddy current loss.

VIII. CLASSICAL THEORY OF EDDY CURRENT LOSSES IN SQUARE ROD

We consider the eddy current losses in a square rod of side a, permeability μ, and conductivity σ. The rod is situated in a uniform field of amplitude H_0 parallel to the axis and varying with angular frequency ω. We treat first the low frequency limit at which the skin depth δ is $\gg a$. The field then penetrates the rod without significant change in amplitude, so that we have

$$(\text{curl}\ \mathbf{E})_z = -j\omega\mu H_0/c, \tag{22}$$

or

$$(\text{curl}\ \mathbf{i})_z = -j\omega\mu\sigma H_0/c. \tag{23}$$

This equation is satisfied by

$$i_x = Fy;\quad i_y = -Fx, \tag{24}$$

if

$$F = j\omega\mu\sigma H_0/2c. \tag{25}$$

The time-average power loss per unit length of rod is

$$\begin{aligned} P &= \frac{1}{2\sigma}\int\int (i_x^2 + i_y^2)dxdy \\ &= 4(F^2/2\sigma)\int_0^{a/2}\int_0^{a/2}(x^2+y^2)dxdy \qquad (26) \\ &= \sigma\omega^2\mu^2H_0^2a^4/48c^2. \qquad (27) \end{aligned}$$

The maximum energy stored is $\mu H_0^2a^2/8\pi$, so that the Q is

$$Q = 12(\delta/a)^2. \tag{28}$$

At the high frequency limit $a \gg \delta$, and we employ appropriate approximations to find for the average power

$$P = \omega(\mu H_0^2/16\pi)(4a\delta), \tag{29}$$

while the maximum energy stored is $\mu H_0^2 4a\delta/16\pi$, giving

$$Q = 1. \tag{30}$$

The effective permeability may be written as

$$\mu(1 + jQ^{-1}), \tag{31}$$

which is consistent with the familiar equation $Q = \omega L/R$.

IX. DOMAIN THEORY OF EDDY CURRENT LOSSES IN SQUARE ROD

We treat a model in which there is a single plane wall running parallel to the axis of a square rod, and parallel to two of the sides. We suppose that the equilibrium position of the wall is at the midpoint of the rod; the losses will depend somewhat on the choice of equilibrium position. If there is a restoring force $-qx$ per unit area of wall the permeability at sufficiently low frequencies will be real and given by

$$\mu_0 = B_s^2/\pi qa, \tag{32}$$

where a is the side of the square, and we suppose that $\mu \gg 1$.

We suppose that in the absence of restoring force there is a relation of the form

$$v = GH \tag{33}$$

connecting the wall velocity and the applied field. It is shown in Section V above that

$$G^{-1} = (32B_s a/\tau c^2\pi^2)\sum_{\substack{\text{odd}\\ n}} n^{-3}\tanh(n\pi/2), \tag{34}$$

where the sum is approximately equal to 0.97.

The complete force equation now is

$$2HI_s = qx + j2\omega I_s x/G, \tag{35}$$

which gives

$$\mu = \mu_0[1 + j(B_s\omega/2\pi Gq)]^{-1}. \tag{36}$$

We have

$$B_s\omega/2\pi Gq = (8/\pi^3)(a/\delta)^2(0.97), \tag{37}$$

where the effective skin depth δ is defined in the classical way, using μ_0 as the permeability.

At low frequencies $\delta \gg a$, and

$$\begin{aligned} Q &= (\pi^3/7.76)(\delta/a)^2 \\ &\approx 4(\delta/a)^2, \end{aligned} \tag{38}$$

which is only one-third of the value of the classical Q given by Eq. (28).

At high frequencies $\delta \ll a$, and

$$\mu \approx -4j(\delta/a)^2\mu_0 \tag{39}$$

so that the material behaves as a resistive element. The calculation is valid only so long as the skin depth for permeability unity is greater than the side of the specimen, or when $\mu_0\delta^2 > a^2$. It is seen that also at high frequencies the domain model gives lower Q's than the classical mode.

We wish to thank Mr. H. R. Moore for electronics assistance and Mr. J. G. Walker for general technical assistance.