tween 8 and 11 Mev; any gamma-rays in this region could not have intensities as large as one-half percent of the 4.45-Mev intensity.

## V. CONCLUSIONS

The polonium-beryllium gamma-ray spectrum has been found to consist of a single gamma-ray of energy $4.45\pm0.09$ Mev, with the possibility of radiation at lower energy remaining because of the decrease in sensitivity of the spectrometer below this energy. Since no gamma-radiation is observed at 7 Mev, one of the important bits of evidence for the 7-Mev level in C$^{12}$ appears to be removed. However, since there is good evidence that there are no levels in C$^{12}$ below 4.45 Mev, the 2.7-Mev gamma-ray found by early observers cannot be accounted for except by postulating the 7-Mev level. Thus it seems possible that this low energy gamma-ray may also not exist, the evidence for it being of the same statistical accuracy as the evidence for the 7-Mev gamma-ray. Further doubt is cast on the existence of this level by recent experimental results[8] on the inelastic scattering of protons by carbon nuclei, which indicate levels in C$^{12}$ only at 4.8 and 10.1 Mev.

The author wishes to thank Professor T. W. Bonner for guidance and assistance in all phases of this work, and to express his indebtedness to the Los Alamos Scientific Laboratory for the loan of the polonium-beryllium source.

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[8] Levinthal, Martinelli, and Silverman, Phys. Rev. **78**, 199 (1950).

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# On the Deep Configurations of V I and Cr II

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The theoretical formulas for $d^3s^2$, $d^4s$, and $d^5$ are compared with the experimental data of V I and Cr II; the Cr II terms are assigned to configurations. The agreement remains practically unimproved by taking into account the interactions between configurations.

## I. INTRODUCTION

THIS study constitutes a direct continuation of Many's work[1] on 3- and 4-electron spectra in the iron group, applied to 5-electron spectra. The procedure adopted is essentially the same in our case, and the reader is referred to Many's article for abbreviations, designations, etc., In both spectra, we may still presume that we are confronted with *LS* coupling.

Following Many's example, we first compared the theoretical formulas with the experimental term values, not taking into account any configuration interactions. Thereafter we repeated our calculations allowing for interactions between configurations. In either case, we calculated the Slater-Condon *F*'s and *G*'s by means of least squares, evaluated the terms, and, finally, compared the agreements in both cases.

All theoretical formulas are taken from Racah.[2]

## II. V I

This spectrum is well known for its clean *LS* coupling. The experimental term values are taken from Meggers and Russell.[3] The two lowest configurations $d^3s^2$ and $d^4s$ are moderately well mixed up. Of their 32 terms 17 are found experimentally; besides a high lying $^6S$ term, a still higher $^4F$ assigned to $d^5$ is given in the new tables of Moore.[4] Both are excluded from our calculations and dealt with separately afterward.

At first, separate calculations for $d^4s$ and $d^3s^2$ were carried out, although there is but little theoretical justification for such a procedure, and then the calculations for $d^4s$ and $d^3s^2$ combined were made. Finally, the configuration interactions were taken into account. It is seen from Table I that, on the one hand, the small lowering of the mean deviation and the rise of the mean error in the case of $d^3s^2$, when taking separate parameters $B$ and $C$ in $d^4s$ and $d^3s^2$ (see column 1) do not warrant this procedure; on the other hand, the mean error rises also when taking into account the electrostatic interactions between $d^4s$ and $d^3s^2$ (see column 3). Therefore, the best result is obtained by calculating with the same $B$ and $C$ in both configurations and neglecting the interactions between configurations (see column 2). As regards $d^3s^2$ and $d^4s$, there is no possibility of interchanging the assignments of any two terms.

Concerning the two $d^5$ terms, there can be no doubt about the identity of the $^6S$. Computing $A(d^5)$ from this latter term and taking the values of $B$ and $C$ from our previous calculation, we can compute the theoretical

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[1] A. Many, Phys. Rev. **70**, 511 (1946).

[2] G. Racah, Phys. Rev. **62**, 438 (1942); **63**, 367 (1943).

[3] H. N. Russell and W. F. Meggers, J. Research Nat. Bur. Stand. **17**, 125 (1936).

[4] C. E. Moore, *Atomic Energy Levels* (National Bureau of Standards, 1949), Vol. I.

TABLE I. The deep configurations of V I. Column (1). Neglecting configuration interactions, separate parameters in the two configurations. Column (2). Neglecting configuration interactions. Same $B$ and $C$ in both configurations. Column (3). With configuration interactions.

| Term | | Obs. | (1) | | (2) | | (3) | |
|---|---|---|---|---|---|---|---|---|
| | | | Calc. | Diff. | Calc. | Diff. | Calc. | Diff. |
| $d^3s^2$ | $^4F$ | 319 | 377 | +58 | 1593 | +1274 | 1699 | +1380 |
| $d^4s$ | $^6D$ | 2297 | 2217 | −80 | 1651 | −646 | 1683 | −614 |
| $d^4(^5D)s$ | $^4D$ | 8596 | 8743 | +147 | 8186 | −410 | 8344 | −252 |
| $d^3s^2$ | $^4P$ | 9715 | 9857 | +142 | 10269 | +554 | 10157 | +442 |
| $d^3s^2$ | $^2G$ | 11008 | 10868 | −140 | 10727 | −281 | 10702 | −306 |
| $d^3s^2$ | $^2P$ | 13805 | 14027 | +222 | 13619 | −186 | 13497 | −308 |
| $d^3s^2$ | $^2D^-$ | 14535 | 15425 | +890 | 14743 | +208 | 14670 | +135 |
| $d^4s$ | $^4H$ | 14992 | 14282 | −710 | 14365 | −627 | 14315 | −677 |
| $d^3s^2$ | $^2H$ | 15192 | 14027 | −1165 | 13619 | −1573 | 13628 | −1564 |
| $d^4(^3P^-)s$ | $^4P$ | 15389 | 15506 | +117 | 15779 | +390 | 15937 | +548 |
| $d^4(^3F^-)s$ | $^4F$ | 15725 | 15902 | +177 | 16076 | +351 | 16042 | +317 |
| $d^4s$ | $^4G$ | 17166 | 17193 | +17 | 17257 | +91 | 17217 | +51 |
| $d^4(^3P^-)s$ | $^2P$ | 19061 | 19422 | +361 | 19700 | +639 | 19869 | +808 |
| $d^4(^3H)s$ | $^2H$ | 19090 | 18198 | −892 | 18286 | −804 | 18376 | −714 |
| $d^4(^3F^-)s$ | $^2F$ | 19056 | 19817 | +761 | 19997 | +941 | 19667 | +611 |
| $d^4(^3D)s$ | $^4D$ | 20789 | 21269 | +480 | 21301 | +517 | 21281 | +492 |
| $d^4(^3G)s$ | $^2G$ | 21622 | 21109 | −513 | 21178 | −444 | 21276 | −346 |
| $d^5$ | $^6S$ | 20202 | | | | | | |
| $d^5$ | $^4F$ | 37032 | | | | | | |
| $d^4s$ | $^2I$ | | | | | | 21297 | |
| $d^4(^1G^-)s$ | $^2G$ | | | | | | 22287 | |
| $d^3s^2$ | $^2F$ | | | | | | 22592 | |
| $d^4(^1S^-)s$ | $^2S$ | | | | | | 25231 | |
| $d^4(^3D)s$ | $^2D$ | | | | | | 25285 | |
| $d^4(^1D^-)s$ | $^2D$ | | | | | | 26586 | |
| $d^4(^1F)s$ | $^2F$ | | | | | | 30151 | |
| $d^4(^3F^+)s$ | $^4F$ | | | | | | 33288 | |
| $d^4(^3P^+)s$ | $^4P$ | | | | | | 33643 | |
| $d^3s^2$ | $^2D^+$ | | | | | | 34117 | |
| $d^4(^3F^+)s$ | $^2F$ | | | | | | 37282 | |
| $d^4(^3P^+)s$ | $^2P$ | | | | | | 37615 | |
| $d^4(^1G^+)s$ | $^2G$ | | | | | | 38675 | |
| $d^4(^1D^+)s$ | $^2D$ | | | | | | 50611 | |
| $d^4(^1S^+)s$ | $^2S$ | | | | | | 64343 | |
| | | | $d^3s^2$ | $d^4s$ | | | | |
| $A(d^3s^2)$ | | | 9757 | — | 10269 | | 10443 | |
| $A(d^4s)$ | | | — | 19664 | 19025 | | 19202 | |
| $A(d^5)$ | | | — | — | — | | (40537) | |
| $B=F_2-5F_4$ | | | 632 | 582 | 578 | | 581 | |
| $C=35F_4$ | | | 2654 | 2108 | 2273 | | 2244 | |
| $G_2$ | | | — | 1305 | 1307 | | 1332 | |
| $H_2=R^2(dd, ds)/35$ | | | — | — | — | | 106 | |
| Mean error | | | ±864 | ±607 | ±828 | | ±850 | |
| Mean deviation | | | ±611 | ±484 | ±696 | | ±684 | |

term value of the $^4F$ belonging to $d^5$; this is found to be 48692 cm$^{-1}$. Since the experimentally found $^4F$ lies at 37032 cm$^{-1}$, this term can hardly be assigned to $d^5$. On the other hand, it is pretty close to $d^4(^3F^+)s\,^4F$ to be found at 33288 cm$^{-1}$ (see Table I), and this new assignment corresponds, therefore, much better to the facts. Furthermore, the separations between the levels of $^4F$ are rather big for a $d^5$ term; the two intervals $J:2\frac{1}{2}-3\frac{1}{2}$ and $J:3\frac{1}{2}-4\frac{1}{2}$ obey the Landé interval rule pretty well.

TABLE II. Comparison between V II[a] and V I.

| V II Term | | Calc.-obs. | V I Term | | Calc.-obs. |
|---|---|---|---|---|---|
| $d^3s$ | $\frac{1}{2}(^{(4)3}F+^5F)$ | +1495 | $d^3s^2$ | $^4F$ | +1274 |
| $d^3s$ | $\frac{1}{2}(^{(4)3}P+^5P)$ | +776 | $d^3s^2$ | $^4P$ | +554 |
| $d^3s$ | $\frac{1}{2}(^3G+^1G)$ | +104 | $d^3s^2$ | $^2G$ | −281 |
| $d^3s$ | $\frac{1}{2}(^{(2)3}P+^1P)$ | +177 | $d^3s^2$ | $^2P$ | −186 |
| $d^3s$ | $\frac{1}{2}(^3D^-+^1D^-)$ | +421 | $d^3s^2$ | $^2D^-$ | +208 |
| $d^3s$ | $\frac{1}{2}(^3H+^1H)$ | −980 | $d^3s^2$ | $^2H$ | −1573 |
| $d^4$ | $^5D$ | −1342 | $d^4s$ | $\frac{1}{2}(^6D+^{(5)4}D)$ | −528 |
| $d^4$ | $^3H$ | −1512 | $d^4s$ | $\frac{1}{2}(^4H+^2H)$ | −716 |
| $d^4$ | $^3P^-$ | +231 | $d^4s$ | $\frac{1}{2}(^4P^-+^2P^-)$ | +515 |
| $d^4$ | $^3F^-$ | −135 | $d^4s$ | $\frac{1}{2}(^4F^-+^2F^-)$ | +646 |
| $d^4$ | $^3G$ | −517 | $d^4s$ | $\frac{1}{2}(^4G+^{(3)2}G)$ | −177 |
| $d^4$ | $^3D$ | +177 | $d^4s$ | $^{(3)4}D$ | +517 |

[a] See reference 1.

TABLE III. Experimental Landé factors in $d^4s$ of Cr II.

| Term | $(E_J-E_{J-1})/J$ $=\zeta(SL)$ | $\zeta(SL)/\alpha$ | $\zeta_{3d}$ |
|---|---|---|---|
| $(^5D)^6D$ | 43 | 43×5 | 215 |
| $(^5D)^4D$ | 66 | 666/3 | 222 |
| $(^3D)^4D$ | −18 | 18×18 | 324 |
| $(^3D)^2D$ | −35 | 35×9 | 315 |
| $(^3G)^4G$ | 25 | 25×10 | 250 |
| $(^3G)^2G$ | 12 | 12×5 | 60 |
| $(^3H)^4H$ | 14 | 14×15 | 210 |
| $(^3H)^2H$ | 30 | 30×7.5 | 225 |
| $(^3P^-)^4P$ | 230 | 230×1 | 230 |
| $(^3P^-)^2P$ | 465 | 465/2 | 232 |
| $(^3F^-)^4F$ | 14 | 14×15 | 210 |
| $(^3F^-)^2F$ | 11 | 11×7.5 | 83 |
| $(^3P^+)^4P$ | −24 | 24×1.5 | 36 |
| $(^3F^+)^4F$ | 80 | −80×90 | −7200 |

TABLE IV. The deep configurations of Cr II. Column (1). Neglecting configuration interactions. Column (2). With configuration interactions.

| Term | | Obs. | (1) | | (2) | |
|---|---|---|---|---|---|---|
| | | | Calc. | Diff. | Calc. | Diff. |
| $d^5$ | $a^6S$ | 00000 | −1616 | −1616 | −1524 | −1524 |
| $d^4(^5D)s$ | $a^6D$ | 12281 | 14258 | +1977 | 14164 | +1883 |
| $d^4(^5D)s$ | $a^4D$ | 19833 | 21553 | +1720 | 21230 | +1397 |
| $d^5$ | $a^4G$ | 20521 | 19473 | −1048 | 19443 | −1078 |
| $d^5$ | $a^4P$ | 21829 | 22870 | +1041 | 22779 | +950 |
| $d^5$ | $b^4D$ | 25046 | 24513 | −533 | 24973 | −73 |
| $d^5$ | $a^2I$ | 30155 | 28528 | −1627 | 28588 | −1567 |
| $d^4s$ | $a^4H$ | 30292 | 29708 | −584 | 29692 | −600 |
| $d^4(^3P^-)s$ | $b^4P$ | 30535 | 31410 | +875 | 31582 | +1047 |
| $d^4(^3F^-)s$ | $a^4F$ | 31172 | 31813 | +641 | 31540 | +368 |
| $d^{5-}$ | $a^2D$ | 31432 | 32725 | +1293 | 32346 | +914 |
| $d^{5-}$ | $a^2F$ | 32471 | 33364 | +893 | 33277 | +806 |
| $d^5$ | $b^4F$ | 32857 | 33669 | +812 | 33863 | +1006 |
| $d^4s$ | $b^4G$ | 33598 | 33308 | −290 | 33319 | −279 |
| $d^4(^3H)s$ | $a^2H$ | 34740 | 34086 | −654 | 34150 | −590 |
| $d^4(^3P^-)s$ | $a^2P$ | 35132 | 35787 | +655 | 35985 | +853 |
| $d^4(^3F^-)s$ | $b^2F$ | 35601 | 36190 | +589 | 35601 | −0 |
| $d^5$ | $b^2H$ | 35673 | 35524 | −149 | 35733 | +60 |
| $d^{5-}$ | $a^2G$ | 36206 | 36446 | +244 | 35375 | −831 |
| $d^4(^3D)s$ | $c^4D$ | 38324 | 38347 | +23 | 38316 | −8 |
| $d^4(^3G)s$ | $b^2G$ | 38549 | 37686 | −863 | 38243 | −306 |
| $d^4(^1G^-)s$ | $c^2G$ | 39773 | 39386 | −387 | 39929 | +156 |
| $d^{5+}$ | $c^2F$ | 39829 | 39326 | −503 | 39882 | +53 |
| $d^4s$ | $b^2I$ | 40225 | 38164 | −2061 | 38234 | −1991 |
| $d^4(^3D)s$ | $b^2D$ | 42944 | 42724 | −220 | 42949 | +5 |
| $d^4(^1D^-)s$ | $c^2D$ | 45718 | 44738 | −980 | 44587 | −1131 |
| $d^{5+}$ | $d^2G$ | 52325 | 53521 | +1196 | 53456 | +1131 |
| $d^4(^3F^+)s$ | $c^4F$ | 53656 | 53215 | −441 | 53002 | −654 |
| $d^4(^3P^+)s$ | $c^4P$ | 55062 | (53618) | (−1444) | (53398) | (−1664) |
| $d^4(^1S^-)s$ | $^2S$ | | | | 40898 | |
| $d^5$ | $^2S$ | | | | 45685 | |
| $d^5$ | $^2D$ | | | | 48663 | |
| $d^4(^1F)s$ | $^2F$ | | | | 49081 | |
| $d^4(^3P^+)s$ | $^2P$ | | | | 57446 | |
| $d^4(^3F^+)s$ | $^2F$ | | | | 57710 | |
| $d^4(^1G^+)s$ | $^2G$ | | | | 59585 | |
| $d^5$ | $^2P$ | | | | 65948 | |
| $d^{5+}$ | $^2D$ | | | | 70900 | |
| $d^4(^1D^+)s$ | $^2D$ | | | | 74354 | |
| $d^4(^1S^+)s$ | $^2S$ | | | | 91185 | |
| $A(d^4s)$ | | | | 35211 | | 35192 |
| $A(d^5)$ | | | | 23579 | | 23317 |
| $B=F_2-5F_4$ | | | | 720 | | 710 |
| $C=35F_4$ | | | | 2778 | | 2790 |
| $G_2$ | | | | 1459 | | 1531 |
| $H_2=R^2(dd, ds)/35$ | | | | — | | 150 |
| Mean error | | | | ±1116 | | ±1073 |
| Mean deviation | | | | ±1011 | | ±952 |

The mean error of ±850 cm$^{-1}$ on a configuration width of 22000 cm$^{-1}$ constitutes an accuracy of four percent.

For a further comparison we show in Table II the differences between theoretical and experimental term values in V I and their parents in V II as taken from Many.[1] It is important to note that, for related terms, the differences in question are mostly of the same order of magnitude and have the same signs.

### III. Cr II

The recent classification of this spectrum by Kiess appears to be still unpublished. Therefore, we had to compile the experimental term values from the classification of the Cr II spectrum as given by Moore.[5]

Since Moore did not assign the terms to specific configurations, the problem of assignments naturally arose. After several calculations best agreement was reached by ascribing all of the 29 terms given by Moore to $d^5$ and $d^4s$, $d^3s^2$ being apparently too high. $d^5$ and $d^4s$ combined should theoretically contain 40 different terms. Our most powerful means for testing these assignments was the check-up on the Landé splittings, which, according to theory, should vanish for all $d^5$ terms and all doublets of $d^4s$ which are based on singlets. In Table III we evaluated $\zeta_{3d}$ from the experimental splittings of each term assigned to $d^4s$.

It should be noted that the values of $\alpha$ shown against the last six terms in Table III, which are non-linear, are approximate only. As the average, resulting from this calculation, we obtained roughly $\zeta_{3d}=220$ cm$^{-1}$. Most of the poor agreements can be explained. With $(^3D)^4D$, it suffices to use instead of an average value of 18 cm$^{-1}$ the least one, namely, 12.5 cm$^{-1}$, in order to reach agreement with the above-quoted value of $\zeta_{3d}$. The unsatisfactory values in the cases of $(^3G)^2G$ and $(^3F^-)^2F$ can be explained qualitatively by spin interactions with neighboring terms.

On the other hand, we can make, starting from $\zeta_{3d}=220$ cm$^{-1}$, rough calculations for the presumably not weak spin interactions in the two cases of $d^5(^2D_{5/2}^- - ^2F_{5/2})$ and $d^5(^2G_{9/2}^- - ^2H_{9/2})$. The results obtained are 270 and 170 cm$^{-1}$, respectively, in pretty good agreement with the actual splittings found in these four $d^5$ terms; according to theory the latter, belonging to a half-filled shell, should show no splittings at all. The agreement is very bad in the case of $c^4F$, and it is interesting to note here that, taking the hypothetical case of $c^4F$ as the lowest term of $d^3s^2$, we should receive the much better result, $\zeta_{3d}=240$ cm$^{-1}$.

Finally, the usual calculations have been carried out, first disregarding electrostatic interactions and, thereafter, taking them into account. In both cases $c^4P$ has been neglected because of its uncertainty. The results are compiled in Table IV. The improvement, when allowing for electrostatic interactions, is but insignificant. The mean error of ±1076 cm$^{-1}$ on a configuration width of 53000 cm$^{-1}$ constitutes an accuracy of two percent.

I am deeply grateful to Professor G. Racah for suggesting this problem and for his constant assistance throughout my work.

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[5] C. E. Moore, *A Multiplet Table of Astrophysical Interest* (Princeton University Press, Princeton, New Jersey, 1945).

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# The Electromechanical Behavior of $BaTiO_3$ Single-Domain Crystals*

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The electromechanical response of $BaTiO_3$ single-domain crystals was investigated as a function of temperature from room temperature to above the Curie point. It was found that when we eliminate domain effects, either by using a perfect single-domain crystal or by aligning the domains by an external electric field, we are dealing with a true linear piezoeffect. The linear piezoeffect was measured by both a static and a dynamic method. The values of the $d_{31}$ piezoelectric coefficient were compared with those calculated from the dielectric constant and the spontaneous polarization. The behavior of the piezoelectric resonance frequency was observed also through the same temperature range. Although the crystal is cubic above the Curie point and has a center of symmetry, the piezoelectric response persisted under the application of an external d.c. field because the external field will induce a tetragonal crystal structure through the electrostrictive effect, so that the crystal becomes piezoelectric again above the Curie point. Optical and dielectric measurements were found to verify these results.

## I. INTRODUCTION

THE practical importance of $BaTiO_3$ as transducers makes it essential to know the electromechanical response of this substance. A number of measurements of the electromechanical coupling have been made on ceramics and also on multidomain crystals by Mason,[1] Jaffe,[2] and Rzhanov.[3] However,

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* Sponsored by the ONR, the Army Signal Corps, and the USAF.

[1] W. P. Mason, Phys. Rev. 74, 1134 (1948).

[2] H. Jaffe, Ind. Eng. Chem. 42, 264 (1950), and a private communication.

[3] A. V. Rzhanov, J. Exp. Theor. Phys. U.S.S.R. 19, 502 (1949).