TABLE I. List of isomers. For comparison with shell structure the odd mass number isotopes (odd A) are separated into odd proton and odd neutron.

$l=4$ Odd A Odd P	Odd N	Even or unknown A		$l=5$ Odd A Odd P	Odd N		Even or unknown A
Rh^{103}	Se^{77}	Sc^{46}	Sb^{124}	Y^{91}	Zn^{69}	Te^{131}	Sc^{44}
Ag^{107}	Se^{81}	Co^{58}	Sb^{124}	Nb^{91}	Sr^{87}	Xe^{131}	In^{114}
Ag^{109}	Kr^{79}	Co^{60}	Cs^{134}	Nb^{95}	Sn^{117}	Xe^{135}	Pb^{204}
	Kr^{81}						
Au^{197}	Kr^{83}	Br^{80}	Ta^{182}	Tc^{97}	Sn^{119}	Ba^{133}	Pt
	Sr^{85}	Nb^{94}	Ir^{192}	In^{113}	Te^{121}	Ba^{135}	
	Cd^{111}	Tc^{94}	Er	In^{115}	Te^{123}	Ba^{137}	
	Xe^{127}	Rh^{104}	Yb		Te^{125}	Pt^{197}	
	Dy^{165}	In^{112}	Yb		Te^{127}	Hg^{199}	
	Hf^{179}	Sb^{122}	W		Te^{129}		

energy levels of odd A nuclei. A comparison of these predictions with the lifetime-energy l value assignments demonstrates some major inconsistencies. The presently accepted isomers are arranged in Table I according to l value.

The odd nucleon shells of interest are those from 29 to 50, 51 to 82, and 83 to 126. In agreement with theory, none of these isomers are found below nucleon number 39. From 39 to 50, shell structure predicts both the $p_{1/2}$ and $g_{9/2}$ levels. A mixture of magnetic 2^4 and electric 2^5 pole radiation ($l=5$) is therefore possible. The nearby $p_{3/2}$ level would give electric 2^3 pole radiation ($l=3$). Thus, while the 8$l=5$ isomers in this region are consistent with shell theory, the 9$l=4$ isomers are not. The $f_{7/2}$ subshell, which could combine with $p_{1/2}$ to give $l=4$, is in the shell below 28 and shell binding energies are much greater than the excitation energy of these $l=4$ isomers (<200 kev).

From 51 to 82 nucleons the available subshells are: $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $s_{1/2}$, and $h_{11/2}$. At 63 neutrons, the $g_{7/2}s_{1/2}$ combination could give an $l=4$ transition. However, the ground-state spin of Cd^{111} is $\frac{1}{2}$ and the isomeric transition occurs between two excited states. In the upper part of the shell, $h_{11/2}$ can combine with either $d_{3/2}$ or $s_{1/2}$ to give $l=5$; no $l=4$ is predicted. Therefore, Xe^{127} and Au^{197} are inexplicable. The 13 $l=5$ isomers in this region do fit shell theory. However, in 4 cases (Sn^{117}, Te^{121}, Te^{123}, and Te^{125}), the isomeric transition is followed by a gamma-ray. In each case the cross-over transition is absent in contradiction to predictions based on shell theory.

In the final shell, there is consistency for both $l=4$ and $l=5$. At 99 and 107 neutrons, $l=4$ is consistent with the $h_{9/2}p_{3/2}$ or $h_{9/2}p_{1/2}$ combinations. At 119 neutrons, both $l=5$ transitions can be explained by the $i_{13/2}p_{3/2}$ or $i_{13/2}f_{5/2}$ combinations.

To summarize, the simple spin-orbit coupling shell model would predict no $l=4$ transitions below nucleon number 50 and none in the upper parts of higher shells. Of the 14 odd A nuclei only the 2 with nucleon number above 82 are consistent with this prediction. On the other hand, the 23 odd A isomers in the $l=5$ group would fit nicely into this shell model if the absence of cross-over transitions could be explained. Internal conversion measurements can provide valuable additional tests of both isomeric classification and shell structure models.

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On the Determination of the *P-P* Interaction from Scattering Experiments

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AS is well known[1,2] low energy $P-P$ scattering can be accounted for by several different potential functions. On the other hand, it has recently been proved by Jauho[3] that the nuclear $P-P$ phase shift under certain physically reasonable conditions determines the potential function uniquely. Thus it seems to be of some interest to make a direct attempt at a calculation of the $P-P$ interaction from available experiments.

Starting from the radial equation

$$y''+\left(1-\frac{2\alpha k}{r}\right)y=-V(r)y \qquad (1)$$

we can easily prove the relation

$$k\sin\eta(k)=\int_0^\infty uVy\,dr \qquad (2)$$

[$y(0)=0$, $y(\infty)\sim\sin(\xi+\eta)$; u is the regular Coulomb wave function, determined by $u(0)=0$, $u(\infty)\sim\sin\xi$, where $\xi=kr-\alpha\ln 2kr+\arg\Gamma(i\alpha+1)$]. Replacing y by u we can solve for V

$$V=-\frac{2}{\pi}\kappa^2\cdot\int_0^\infty\frac{\varphi(\alpha,\beta)\sin\eta}{\alpha^3}d\alpha.$$

Here

$$2\alpha=k/\kappa;\quad \kappa r=\beta;\quad kr=\rho;$$
$$\varphi(\alpha,\beta)=\{(d/d\rho)[u(\alpha,\rho)v(\alpha,\rho)]\}_{\rho=\beta/2\alpha};$$

v is the irregular Coulomb wave function, determined by $v(\infty)\sim\cos\xi$.

Since we know practically nothing of $\sin\eta$ for small values of α (i.e., for high energies), it is natural to use a cut-off convention. Choosing the lower limit in the integral (3) proportional to β, which seems to be reasonable, we find that V can be represented very nicely by Yukawa functions, and we obtain the following values of the range r_0 and the strength J using the cut-off radius a:

a	r_0(cm)	J(Mev)
$\beta/2$	$1.21\cdot10^{-13}$	18.2
β	$1.00\cdot10^{-13}$	22.0

A fuller account of the investigation will be published elsewhere (*Arkiv. f. Fysik*) together with a table of irregular Coulomb wave functions.

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Theory of Natural Alpha-Radioactivity

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RECENT measurements of cross sections of (α,n), (α,2n), and (α,3n) reactions[1-3] give excitation functions which, when compared with Weisskopf's calculation of alpha-capture cross sections,[4] require that the parameter r_0 in the expression for the nuclear radius

$$r=r_0A^{\frac{1}{3}}\times10^{-13}\text{ cm} \qquad (1)$$

be taken as 1.5 in the calculation of the Coulomb barrier penetration factor.[5] Using this in the expression for the decay constant, λ,

$$\lambda=\Gamma_\alpha/\hbar=\Gamma_{0\alpha}P(r_0)/\hbar \qquad (2)$$

(where Γ_α is the width for alpha-emission, $\Gamma_{0\alpha}$ is the alpha-width without barrier, and $P(r_0)$ is the Coulomb barrier penetration factor which is a very sensitive function of the assumed value of r_0), requires, in order to obtain the observed values for λ,

$$\Gamma_{0\alpha}\sim10^5\text{ ev.} \qquad (3)$$

Condition (3) is in considerable disagreement with the value given by Bethe in his many-body theory of alpha-decay[6]

$$\Gamma_{0\alpha}\sim1\text{ ev.} \qquad (4)$$

This discrepancy may perhaps be explained as follows. Bethe assumed that neutron widths and alpha-particle widths without barrier are roughly equal, and calculated the neutron widths from

$$\Gamma_n/E^{\frac{1}{2}} \sim 4\times 10^{-4}(\text{ev})^{\frac{1}{2}}. \quad (5)$$

Which was empirically determined from the available data on neutron capture cross sections. Relation (5) is furthermore in agreement with the expected relation that, all other things being equal,

$$\Gamma \propto E^{\frac{1}{2}} \quad (6)$$

which can be derived from considerations of the momentum space degeneracy of the outgoing particles.

In line with the suggestions of Weisskopf, Feshbach, and Peaslee,[7] Wigner[8] has shown that widths are also proportional to level spacing, D, whence (6) becomes

$$\Gamma \propto E^{\frac{1}{2}}D. \quad (7)$$

Relation (5) must then be considered valid only in cases of neutron capture. Since level spacings in neutron capture are about 10 ev, in general, (5) must be replaced by

$$\Gamma/E^{\frac{1}{2}}D \sim 4\times 10^{-5}(\text{ev})^{-\frac{1}{2}}. \quad (8)$$

Applying (8) to the case of alpha-decay,

$$E^{\frac{1}{2}} \sim 2.5\times 10^{3}(\text{ev})^{\frac{1}{2}}, \quad D\sim 10^{5}\text{ ev}$$

whence

$$\Gamma \sim 10^{4}\text{ ev}. \quad (9)$$

The value of D was obtained from the measured energy differences between various energy groups in natural alpha-decay.[9] Comparison of (9) with (3) indicates that the ratio of neutron width to alpha-width without barrier is about 0.1; and comparison of (3) with the width from the one-body theory (0.8 Mev) indicates that the "probability of formation of an alpha-particle" in a nucleus (i.e., the correction for the many-body theory over the one-body theory) is about one-eighth. Either of these last two values may be in error by a factor of 10 or more.

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Ionization by Recoil Particles from Alpha-Decay

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SINCE the energy loss of a slow heavy particle is predominantly to recoiling gas atoms, ionization by secondary heavy particles contributes a large fraction of the total ionization resulting from a slow heavy particle that is stopped in a gas.

Let η be the ionization efficiency $\omega^e I/E$ of a primary particle of energy E which gives rise to the number of ion pairs I, where ω^e is the energy loss per ion pair of a particle the energy of which is very high. The ionization efficiency satisfies an equation of the form

$$\frac{d}{dE}(E\eta)=\mu+\lambda\int_0^{E_m'} dE'k(E,E')\eta'(E'), \quad \eta(0)=0, \quad (1)$$

in which $\eta'=\omega^e I'/E'$ is the ionization efficiency of a gas atom of energy E' in its own gas. The functions μ and λ are given by

$$\mu=\omega^e\sigma^e/(b^e+b^\nu), \quad \lambda=b^\nu/(b^e+b^\nu),$$

where σ^e is the cross section for the production of an ion pair in a collision between the primary particle and an atom of the gas including ionization by ejected electrons, and b^e and b^ν are the stopping cross sections per atom for the loss of energy by the primary particle to excitation and ionization and to atomic recoil, respectively. The kernel $k(E, E')$ is

$$\sigma(E,E')E'\Big/\int_0^{E_m'} dE'\sigma(E,E')E',$$

where $\sigma(E, E')$ is the cross section per unit energy range for the production of a recoil atom of energy E'; the maximum energy transferred to an atom is $E_m'=4MM'E/(M+M')^2$. The ionization efficiency η' of a gas atom satisfies the differential-integral equation obtained from (1) by regarding the initial particle as identical in nature with the recoil atom; the corresponding functions are designated μ', λ', etc.

We consider a heavy particle ($Z=82$, $M=208$ proton masses) in argon. For velocities less than about $0.4v_0$, where $v_0=e^2/\hbar$, the primary scattering is very nearly spherically symmetrical in the center of gravity system and therefore $k(E,E')\approx 2E'/E_m'^2$. Deviations from spherically symmetrical secondary scattering are unimportant until velocities of the primary particle considerably greater than $0.12v_0$ are attained ($E\gg 78$ kev). Up to these velocities b^ν and $b^{\nu\prime}$ are practically constant,[1] except for negligible linear decrease at very small velocities.

There is evidence[2] to indicate that $\sigma^{e\prime}$ increases very roughly as the square root of the energy in the kev range. Assuming $\mu'\approx a'v'$ and $\lambda'\approx 1$, we obtain $\eta'\approx 10a'v'/7$. Since primary ionization alone gives $(2/3)a'v'$, it is seen that on this basis 53 percent of the ionization due to a gas atom in its own gas is secondary heavy particle ionization.

Assuming, likewise, $\mu\approx av$ and $\lambda\approx 1$, (1) gives

$$\eta\approx[\tfrac{2}{3}a+(16a'\gamma/21)]v, \quad v/v_0\ll 1, \quad (2)$$

where $\gamma=2M/(M+M')=1.68$.

The ionization in argon by single recoil particles from Po, ThC, and ThC′ has been measured by Madsen.[3] His data are well represented by (2) with the proportionality factor

$$\tfrac{2}{3}a+16a'\gamma/21=\omega^e/(15.4\text{ ev})v_0.$$

In order to estimate $\sigma^{e\prime}$, we put a equal to a' and obtain $a'\approx\omega^e/(30\text{ev})v_0$. With[1] $b^{\nu\prime}\approx 30\pi a_0^2\epsilon_0$, this gives $\sigma^{e\prime}\approx 30\pi a_0^2(v/v_0)$ which at 1 kev is about 10^{-16} cm^2, in satisfactory agreement with the measurements of Berry,[2] who found $0.7\cdot 10^{-16}$ cm^2 at 1 kev, and of Rostagni,[4] who found $0.8\cdot 10^{-16}$ cm^2 at 600 ev. It is found from (2) that about 66 percent of the ionization by a recoil particle from natural alpha-decay is heavy-particle secondary ionization.

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The Meson Spectrum and Meteorological Variations in Cosmic-Ray Intensity

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THE production spectrum of mesons has been derived by Sands[1] from calculations based on the sea-level spectrum and the known behavior of mesons produced in the atmosphere. His results indicate approximately an inverse power law spectrum of the form $dE/E^{2.5}$ over a wide range of energies. Calculations of a similar nature have been made in Ottawa in an attempt to present a physical picture of the meteorological variations in cosmic-ray intensity at sea level. These calculations when compared with measured values of the "barometer effect" give a surprising amount of information about the meson spectrum in the higher momentum range (above about 1.8 Bev/c).