

## Second Sound in He<sup>3</sup>–He<sup>4</sup> Mixtures

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Second sound has been investigated in He<sup>3</sup>–He<sup>4</sup> mixtures for He<sup>3</sup> concentrations between 0.09 percent and 0.80 percent in the temperature range between 1.24°K and 1.95°K. The velocity of second sound was found to increase markedly with increasing concentration; e.g., at 1.26°K, the velocity with the 0.80 percent concentration was 26.52 meters/second as compared to 19.08 meters/second for pure He<sup>4</sup>. The results of these experiments are compared with the theoretical predictions of Pomeranchuk.

### I. INTRODUCTION

THE current theories of liquid He II<sup>1,2</sup> regard it as a mixture of two components, the normal fluid and the superfluid. As the temperature decreases from the  $\lambda$ -point, the ratio of the normal fluid to the superfluid density decreases very sharply, until at 1.4°K, e.g., only about six percent of the total liquid is in the normal phase.<sup>3</sup> Now, since it is one of the peculiarities of He II that only the atoms of the normal fluid can exchange momentum with other atoms, any extraneous particles in He II, such as atoms of He<sup>3</sup>, are associated with the normal component. This reasoning has been verified experimentally by different investigators.<sup>4–6</sup> Any concentration of extraneous particles, large or small, of course, remains constant with decreasing temperature, so that even a very small percentage of such particles affects the normal density very strongly at lower temperatures.

Thus, even small amounts of He<sup>3</sup> constitute a very effective parameter in the study of any of those properties of liquid He II which depend strongly on the ratio of normal to superfluid density. One of these properties is second sound, the propagation of a temperature wave through liquid He II. A heat pulse sent into the liquid in one place creates there an excess of normal fluid. The equilibrium ratio of normal fluid and superfluid densities is re-established by a counterflow of the two fluids, which in turn disturbs the equilibrium elsewhere, and in this fashion the disturbing heat pulse is propagated through the liquid. Extensive experimental investigations of second sound<sup>7–9</sup> have shown it to be one of the most clear-cut and most accurately measurable phenomena of liquid He II. Because of this, and because of the strong dependence of the second sound velocity

on the ratio of normal to superfluid densities and thus also on the concentration of He<sup>3</sup>, we have felt that significant results could be obtained even with the small amount of He<sup>3</sup> which we had available.

A preliminary report of measurements of the second sound velocity with a 0.09 percent concentration of He<sup>3</sup> in He<sup>4</sup> has already appeared.<sup>10</sup> This article discusses in greater detail the experimental procedure used, and lists results for various concentrations between 0.09 and 0.80 percent, the highest concentration which we could achieve with the 3 cm<sup>3</sup> NTP of He<sup>3</sup> obtained from the AEC.

### II. APPARATUS

The choice of the method and the design of the apparatus used in this series of experiments were determined primarily by two considerations. First of all, only a very small amount of He<sup>3</sup> was available, and thus every attempt had to be made to minimize the volume of the apparatus in order to obtain concentrations of reasonable magnitude. Furthermore, the isotope equilibrium in the liquid mixture could easily have been upset by any appreciable heat flux setting up internal convection of the normal and superfluid components of He II.<sup>5,6</sup> In such a process the normal component flows away from the source of heat and, since it alone can exchange momentum with other atoms, carries along most of the He<sup>3</sup>. To avoid this, the heat input both at the transmitter and the receiver of the second sound had to be kept to a very low value.

As is shown schematically in Fig. 1, the cavity assembly consisted of a set of closely fitting Lucite parts, *L*, holding two circular carbon strip resistors,<sup>11</sup> *R*<sub>1</sub> and *R*<sub>2</sub>, at either end of a cavity 3.0 mm in diameter and 10.0 mm in length. To each resistor were soldered two Kovar wires, one of which in turn was connected to a Kovar tube which, together with the other wire, formed a coaxial lead. The wire and the coaxial tube were separated by glass, and both in turn passed through Kovar-glass seals, *K*<sub>1</sub> and *K*<sub>2</sub>. The whole assembly was made helium-tight by a stainless steel sleeve, *S*, which was soldered at both ends to the Kovar-glass seals, and the cavity was filled by means of a copper tube, *F*, 0.025" in diameter, which was soldered to a 0.015"

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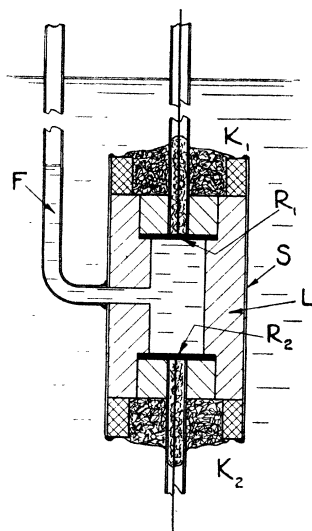


FIG. 1. Schematic diagram of the cavity assembly, showing resistors,  $R_1$  and  $R_2$ , lucite pieces,  $L$ , Kovar-glass seals,  $K_1$  and  $K_2$ , stainless steel sleeve,  $S$ , and filling tube,  $F$ .

stainless steel tube just above the cavity. The diameter of the cavity was smaller than that of the resistors; this difference ensured that the second sound propagation was essentially one-dimensional.

The second sound measurements were carried out by the pulse method,<sup>9,12</sup> in which the time of flight of a heat pulse between a transmitter and a receiver a known distance apart is measured by standard radar techniques. By feeding single pulses of heat into the transmitter and simultaneously pulsing the current through the receiver, it was possible to keep the input of heat to a very low value. The actual electronic arrangement is shown in block diagram in Fig. 2. A pulse generator triggered the sweep of an  $A/R$  radar range scope and, after an adjustable delay, sent into the transmitter a 60-volt signal of length variable up to 250  $\mu$ sec. Simultaneously, a small steady current of about 1 milliamper through the receiver was pulsed manually for about half a second. The values of the transmitter and receiver resistance values were, respectively, about 17000 and 300 ohms. The input signal sent a current of about 3 milliamper through the transmitter, so that the total heat input into both resistances remained very small.

As the temperature pulse reached the receiver, the voltage signal resulting from the resistance change was sent to the vertical plates of the scope through a high gain amplifier. Since direct pick-up within the electronic circuit rendered visible the input pulse into the transmitter, the transit time could be directly determined from the scope picture. This was recorded photographically, superimposed on a calibrating signal of 50 kc, and the final measurements were made on a projection of the developed film.

The cavity was filled with mixtures of  $\text{He}^3$  and  $\text{He}^4$  of varying concentrations by means of a gas-handling system which included bulbs of  $\text{He}^3$  and of  $\text{He}^4$ , manom-

eters, and two Toepler pumps, with which any desired amount of the two isotopes could be mixed together and pushed into the cavity. Temperatures were measured by the vapor pressure of the helium bath using the latest vapor pressure tables.<sup>13</sup> Before a set of readings were taken, the bath was kept at a fixed temperature for about ten minutes to ensure temperature equilibrium, and this was checked by measurement of the vapor pressure of the mixture with a manometer attached to the filling tube of the cavity.

### III. PROCEDURE AND RESULTS

Both previous calculations and preliminary experimentation indicated that with the values of resistance used and the single pulse transmitted to the pulsed receiver circuit, any heat flux effect was small enough to be negligible. Thus, we could take several readings at any one temperature in fairly rapid succession, and this was done for the various mixtures used throughout the temperature range between 1.24°K and 1.70°K. As will be discussed in greater detail below, the results of all the readings at any one temperature agreed very well to within the experimental error. At temperatures above 1.70°K, however, such multiple measurements could not be taken. Since the ratio of the amplitude of the heat pulse to the power input into the transmitter varies inversely with the specific heat of liquid helium, this temperature amplitude began to decrease sharply at about 1.70°K. In this region, therefore, the steady current through the receiver had to be increased to render the signal visible. This also increased the effect of heat flux, and so it was deemed advisable to take only a few readings, at rather long intervals, at each temperature. Here again the results were entirely consistent to within the experimental error, but the reliability of the values is necessarily somewhat lower than at the lower temperatures.

When the temperature for a certain set of readings was reached by raising the bath temperature from a lower value, heat would flow into the cavity. Conversely, when the temperature was attained after a previously higher value, heat would flow out of the cavity. Measurements were taken under both conditions, and their

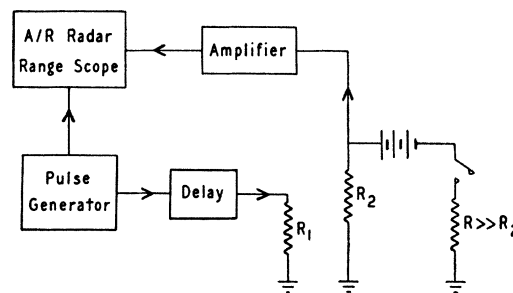


FIG. 2. Schematic block diagram of electronic apparatus.  $R_1$  and  $R_2$  are the transmitter and receiver resistors, respectively.

<sup>12</sup> D. V. Osborne, *Nature* **162**, 213 (1948).

<sup>13</sup> H. van Dijk and D. Shoenberg, *Nature* **164**, 151 (1949).

complete consistency showed that any disturbance of isotope equilibrium due to such heat flow was negligible. As a further check on the reproducibility of our results, some of the measurements with the 0.80 percent concentration were repeated on different days. Here again the values differed by less than the experimental error, indicating that our precautions to attain temperature equilibrium were sufficient. It must be noted, however, that in spite of all precautions isotope equilibrium may not have been established in the cavity. The only way to obtain such equilibrium with certainty is to stir the liquid, and that could not be done with the present apparatus.

The different concentrations of He<sup>3</sup> were obtained by first measuring the volume, pressure, and temperature of the 89 percent pure He<sup>3</sup>, and then diluting it successively with known amounts of pure He<sup>4</sup>. The necessary pressure measurements were made on a mercury manometer in a way which may have introduced an error in the concentration values as large as five percent. More accurate determinations of the concentrations in a mass spectrometer were intended; but, unfortunately, the spectrometer in this laboratory was found to be unsuitable for this type of measurement without

TABLE I. Second sound velocity in liquid helium.

Temperature (°K)	Velocity (m/sec.)	Temperature (°K)	Velocity (m/sec.)
A. Pure He <sup>4</sup>			
1.26	19.08	1.545	20.33
1.30	19.28	1.60	20.50
1.40	19.71	1.65	20.46
1.45	19.97	1.70	20.33
1.50	20.12		
B. 0.09 percent He <sup>3</sup>			
1.26	20.68	1.60	20.92
1.39	20.58		
C. 0.21 percent He <sup>3</sup>			
1.25	22.02	1.50	21.16
1.395	21.29	1.59	21.13
D. 0.41 percent He <sup>3</sup>			
1.24	24.04	1.50	21.90
1.42	22.35	1.60	21.68
E. 0.60 percent He <sup>3</sup>			
1.24	25.71		
F. 0.80 percent He <sup>3</sup>			
1.24	27.09	1.55	23.02
1.26	26.52	1.575	22.96
1.26	26.66	1.58	22.94
1.27	26.44	1.60	22.88
1.28	25.84	1.61	22.72
1.33	25.32	1.65	22.31
1.38	24.63	1.66	22.16
1.40	24.40	1.70	21.87
1.40	24.36	1.75	21.47
1.425	24.22	1.80	21.12
1.46	23.78	1.85	20.42
1.50	23.45	1.895	19.51
1.545	23.13	1.93	18.76

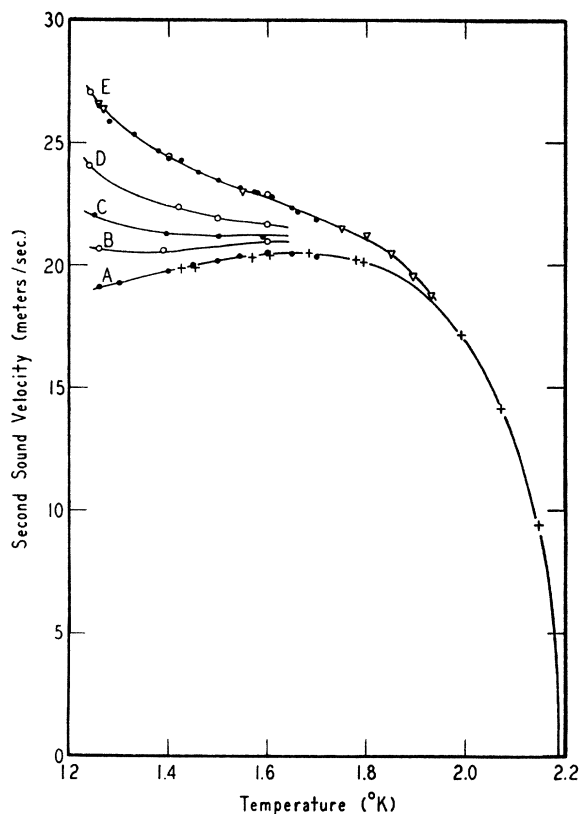


FIG. 3. Second sound velocity as a function of temperature for different concentrations: A, Pure He<sup>4</sup>; B, 0.09 percent He<sup>3</sup>; C, 0.21 percent He<sup>3</sup>; D, 0.41 percent He<sup>3</sup>; E, 0.80 percent He<sup>3</sup>. In curve A the dots are the measurements taken in these experiments, the crosses those of reference 8. In curve E the open and closed dots are measurements taken with the same apparatus on different days. The triangles are the points taken with increased receiver current.

calibration. Since no calibrating mixture was available, this difficulty has not yet been resolved.

The combined results of the various runs are shown in Table I, and are also plotted in Figs. 3 and 4. As mentioned above, all values for temperatures below 1.70°K are the averages of a group of measurements, none of which deviate from their average by more than one-half of one percent. The measurements at 1.70°K with the 0.80 percent concentration were also taken with the low receiver current, allowing a whole series of readings; here, however, the signal received had already become very small with a corresponding decrease in the accuracy of the measurements. The value cited at this point, therefore, is an average from which the individual measurements deviate by almost one percent. In the temperature region between 1.75°K and 2.0°K, with the 0.80 percent concentration, only one or two readings were taken at any one temperature, all sets of two readings agreeing with each other to within one-half of one percent or better.

With the same transmitter pulse width and height and the same receiver current, the receiver signal am-

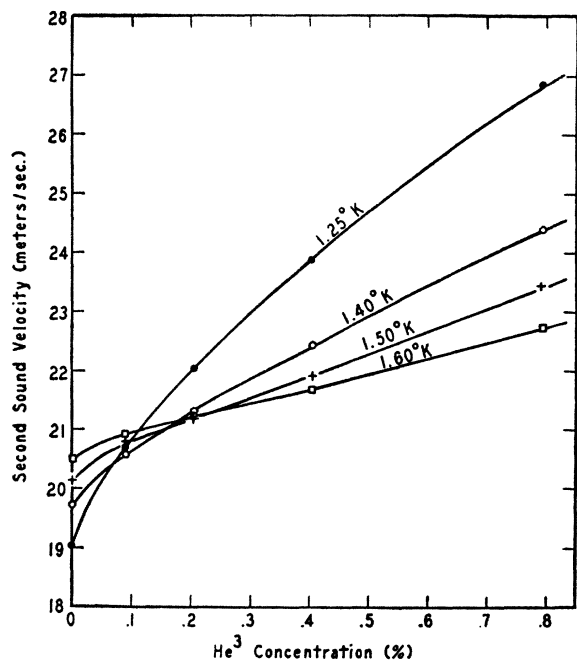


FIG. 4. Second sound velocity as a function of concentration for different temperatures. The points are taken from the curves shown in Fig. 3.

plitude at any one temperature decreased systematically with increasing He<sup>3</sup> concentration. The amplitude,  $T$ , of the temperature pulse is related to the power input,  $W$ , at the transmitter by the relation,

$$T/W = 1/\rho c u_2$$

(see e.g., reference 12), where  $\rho c$  is the specific heat per unit volume of the liquid, and  $u_2$  the second sound velocity. This equation is simply a statement of energy conservation with the assumption of negligible heat capacity in the transmitter.

A comparison of the height of the direct signal with that of the first echo showed that there was no appreciable increase in attenuation with increasing concentration. We therefore could use this equation to obtain values of the specific heat of the mixtures relative to that of pure He<sup>4</sup> from the respective signal amplitudes, and the known values of  $u_2$  and  $W$ . With constant  $W$ , the product of  $T$  and  $u_2$  decreased with increasing concentration by as much as a factor of about two for the 0.80 percent concentration at 1.25°K. Since there is no increase in attenuation to explain this effect, a corresponding increase in specific heat is implied. The magnitude of this increase is quite unexpected, and since the amplitude measurements could be in error by about ten percent, more precise experiments are being carried out before quantitative conclusions are drawn.

#### IV. DISCUSSION

The only theoretical treatment of the problem of second sound velocity in mixtures of He<sup>3</sup> and He<sup>4</sup> is that by Pomeranchuk in a paper<sup>14</sup> in which he treats the effect of small amounts of extraneous particles on the thermodynamic relations for He II and on the second sound velocity. He derives for the second sound velocity under these conditions the expression,

$$u_2^2 = (T/C)(\rho_s/\rho_n)[(S_0 + k\epsilon/m)^2 + k\epsilon C/m],$$

where  $\epsilon$  is the concentration of the extraneous particles, and where  $C$ , the specific heat, and  $\rho_n$ , the normal component density, are themselves functions of  $\epsilon$ . According to Pomeranchuk's theory, the first terms containing  $\epsilon$  to become appreciable as the temperature decreases from the  $\lambda$ -point are those in  $k\epsilon/m$ , so that at first the second sound velocity for mixtures exceeds that for pure He<sup>4</sup>. At lower temperatures the normal component density begins to exceed that of pure He<sup>4</sup> very markedly, reversing the trend of the effect caused by the He<sup>3</sup>, until ultimately the velocity for mixtures is far lower than that for pure He<sup>4</sup>.

Pomeranchuk treats two cases, one for which the minimum energy of the extraneous particles occurs for zero particle momentum ( $p_0=0$ ), and the other where the energy minimum occurs at some value of momentum,  $p_0 \neq 0$ . The main difference between the two cases in relation to the second sound velocity lies in the behavior of the normal component density,  $\rho_n$ ; in the first case Pomeranchuk obtains,

$$\rho_n = \rho_{n0} + \rho\epsilon/m,$$

and in the second,

$$\rho_n = \rho_{n0} + (\rho/m)(p_0/3kT)\epsilon.$$

He calculates second sound velocities for different concentrations at various temperatures for the case  $p_0=0$ , and points out that the difference between the two cases is not very great for  $T > 1.1^\circ\text{K}$ , and concentrations of 0.1 percent or less. For the 0.09 percent concentration our values for the fractional increase of the second sound velocity for mixtures over that for pure He<sup>4</sup> agree very closely with the theoretical predictions, but are lower than Pomeranchuk's values for our higher concentrations. According to the theory this would indicate that for He<sup>3</sup>,  $p_0 \neq 0$ .

We wish to thank Professor Howard L. Schultz for his help in designing the electronic equipment. We are also grateful to Professor William W. Watson and Mr. John Donaldson for devoting much time and effort to the problem of determining the He<sup>3</sup> concentration with the mass spectrometer.

<sup>14</sup> I. Pomeranchuk, J. Exp. Theor. Phys. U.S.S.R. **19**, 42 (1949).