as well as those of the intermediate ones, are the same. In fact a number of pairs, such as $_{21}Sc^{45}-{}_{23}V^{51}$, $_{31}Ga^{71}-{}_{35}Br^{81}$, $_{31}Ga^{71}-{}_{37}Rb^{87}$, $_{35}Br^{81}-{}_{37}Rb^{87}$, $_{35}Br^{79}-{}_{37}Rb^{87}$, probably show this regularity.

No explanation is offered for this empirical rule; it seems not to be directly connected with the magic numbers which unfortunately do not as yet make their appearance at all in the data of magnetic moments of nuclei.

I should like to thank Dr. D. C. Peaslee for helpful discussions.

[1] R. Wangsness, Phys. Rev. **78**, 620 (1950).
[2] See the table given by J. E. Mack, Rev. Mod. Phys. **22**, 64 (1950).
[3] See, for instance, the graphs given in L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1948), p. 394, or L. W. Nordheim, Phys. Rev. **75**, 1894 (1949).

Natural Spread of the Conic Distribution of the Čerenkov Radiation

YIN-YUAN LI
Department of Physics, University of Illinois, Urbana, Illinois
June 27, 1950

A VISIBLE radiation with remarkable spatial asymmetry was first observed by Čerenkov (1934)[1] during his observations on high speed electrons penetrating transparent materials. Frank and Tamm[2] showed theoretically that the Čerenkov radiation is propagated along the generatrices of a cone that has the direction of the motion of the electron as the axis and a half-angle θ_0 given by

$$\cos\theta_0 = c/\kappa u. \tag{1}$$

u and c/κ are, respectively, the velocity of the fast electron and the phase velocity of the emitted light in the medium, and κ is the refractive index of the medium. They drew this conclusion by simply considering the enforced interference of coherent rays emitted along the electron path. In order to treat the problem in greater detail it is necessary to take into account the fact that the speed of an electron decreases by a small jump whenever a photon is emitted. With this modification we find that the spatial asymmetry of the radiation is given[3] by

$$I(\theta) = I(\theta_0)(\sin^2 y)/y^2, \tag{2}$$

where

$$y = 2\pi(L/\lambda)(c/u\kappa - \cos\theta). \tag{3}$$

λ is the wave-length of the radiation, θ the angle between the direction of the motion of the electron and the direction of the radiation, and L the length of a free path. In the Frank-Tamm theory the finiteness of L is overlooked. By assuming an infinite L we obtain a maximum at $\theta_0 = \cos^{-1}(c/u\kappa)$ as sharp as a δ-function. Since this will not be correct for a finite L, we have a half-breadth $\Delta\theta(\frac{1}{2})$ for the maximum at θ_0 determined by

$$\sin y = y/\sqrt{2}, \tag{4}$$

i.e.,

$$(2\pi\kappa L/\lambda)\,\sin\theta_0 \Delta\theta(\tfrac{1}{2}) \simeq 160°. \tag{5}$$

We see that $\Delta\theta(\frac{1}{2})$ decreases remarkably with the increase of the free path. It is very interesting to notice that the natural width of the spatial distribution due to the finiteness of the free path of an electron shows quite an analogy to the natural width of a spectral line due to the finite lifetime of an excited atom. In order to estimate the half-breadth which should be actually observed we must substitute the mean free path L_{Av} of the electrons in the medium into (5). From the Frank-Tamm theory we obtain an estimate of $L_{Av} \simeq 10^{-3}$ cm in the case of a medium with $\kappa_{Av} = 1.5$, a transmission region extending from 2000A to 20,000A, and an electron beam with $\frac{1}{2}$ Mev kinetic energy. For the visible radiations the half-breadth is

$$\Delta\theta(\tfrac{1}{2}) \simeq 1°/(1 - c^2/u^2\kappa^2)^{\frac{1}{2}} \simeq 2°. \tag{6}$$

This conservative estimate of $\Delta\theta(\frac{1}{2})$ serves only as a lower limit, since other radiative as well as non-radiative collisions[4] inevitably cause a shorter free path. Consequently, even if a carefully collimated homogeneous beam and a thin plate are used, we still have an observable spread of the visible radiation in its angular distribution.[5] For electrons much faster than those mentioned above ($u \sim c$) both the number of emissions per cm and $\sin\theta_0$ increase slowly with the energy of the incident beam, and so the change of $\Delta\theta(\frac{1}{2})$ from (6) must be very slight. On the other hand, when the velocity of the incident electrons is not far from the threshold c/κ the variation of $\sin\theta_0$, which appears in (5), becomes much greater than that of L_{Av}. The widening of the angular spread with the decreasing of the energy of the incident beam should be noticeable.

The half-breadth of the conic distribution of the Čerenkov radiation is in all cases of the order of several degrees. It is of the same order of magnitude that Ginsburg[6] found theoretically for the angle of separation between two cones of the Čerenkov radiation from a double refractive crystal. His prediction has never been verified, though in recent experiments[5,7,8] mica, with its optical axis in various orientations with respect to the incident beam, has been extensively used. We have made clear that each cone has its natural spread; they closely overlap and cannot be resolved. Probably the double-cone radiation can never be actually observed, even if substances with large deviation among the refractive indices, such as calcite, are used. Furthermore, we can now understand that even if an ideally collimated homogeneous beam and a thin plate of *high dispersive power* are used, no rainbow spectrum with red inside violet (see Eq. (1) and κ increases with frequency) should be observable. Each color has a natural spread of several degrees, and so a close overlap of different colors is inevitable.

In passing we should point out that the Frank and Tamm prediction of the vanishing of radiation when $u \leq c/\kappa$ is merely due to the incorrect assumption of an infinite free path. From Eq. (2) and a finite L we see that when $u < c/\kappa$, it is the distinct maximum of intensity that disappears but not the radiation. The total intensity decreases continuously with the electron velocity decreasing through the threshold c/κ.[9] The non-vanishing radiation intensity when $u < c/\kappa$, as well as the finite spread of radiation also can be easily understood by considering the Huygens construction of coherent rays[10] emitted along a *short* electron free path.

[1] P. A. Čerenkov, Comptes rendus U.S.S.R. **8**, 451 (1934); **12**, 413 (1936); **14**, 102, 105 (1937); Phys. Rev. **52**, 378 (1937).
[2] I. Frank and Ig. Tamm, Comptes rendus U.S.S.R. **14**, 109 (1937).
[3] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949). The derivation of Eq. (2) is given in detail, pp. 261–265.
[4] A quantum theory of the radiative process which produces the photons of the Čerenkov effect will appear in a forthcoming paper.
[5] H. O. Wyckoff and J. E. Henderson, Phys. Rev. **64**, 1 (1943). The observed spread of radiation was interpreted by the authors as *simply* due to the finite width of slits and range of electron energy.
[6] V. L. Ginsburg, J. Phys. U.S.S.R. **3**, 101 (1940).
[7] G. B. Collins and V. G. Reiling, Phys. Rev. **54**, 499 (1938).
[8] J. M. Harding and J. E. Henderson, Phys. Rev. **74**, 1560 (1948).
[9] See Eqs. (36) and (37) of E. Fermi, Phys. Rev. **57**, 485 (1940).
[10] See Fig. 1 of reference 5.

Isomers and Shell Structure

PETER AXEL
*University of Illinois, Urbana, Illinois**
July 26, 1950

THERE are 59 genetically related isomers whose transitions can be classified into the $l = 4$ or $l = 5$ forbiddenness groups by lifetime-energy considerations.[1] The data that have appeared since the last classification was published[1] have not changed the general conclusions. While the reliable internal conversion measurements are consistent with l assignments, they do not independently define the l values. More internal conversion measurements could both check internal conversion and isomeric transition theory.

The simple spin-orbit coupling shell model, as presented by Maria Mayer,[2] makes specific spin and parity predictions for the

TABLE I. List of isomers. For comparison with shell structure the odd mass number isotopes (odd A) are separated into odd proton and odd neutron.

$l=4$ Odd A Odd P	Odd N	Even or unknown A		$l=5$ Odd A Odd P	Odd N		Even or unknown A
Rh^{103}	Se^{77}	Sc^{46}	Sb^{124}	Y^{91}	Zn^{69}	Te^{131}	Sc^{44}
Ag^{107}	Se^{81}	Co^{58}	Sb^{124}	Nb^{91}	Sr^{87}	Xe^{131}	In^{114}
Ag^{109}	Kr^{79}	Co^{60}	Cs^{134}	Nb^{95}	Sn^{117}	Xe^{135}	Pb^{204}
	Kr^{81}						
Au^{197}	Kr^{83}	Br^{80}	Ta^{182}	Tc^{97}	Sn^{119}	Ba^{133}	Pt
	Sr^{85}	Nb^{94}	Ir^{192}	In^{113}	Te^{121}	Ba^{135}	
	Cd^{111}	Tc^{94}	Er	In^{115}	Te^{123}	Ba^{137}	
	Xe^{127}	Rh^{104}	Yb		Te^{125}	Pt^{197}	
	Dy^{165}	In^{112}	Yb		Te^{127}	Hg^{199}	
	Hf^{179}	Sb^{122}	W		Te^{129}		

energy levels of odd A nuclei. A comparison of these predictions with the lifetime-energy l value assignments demonstrates some major inconsistencies. The presently accepted isomers are arranged in Table I according to l value.

The odd nucleon shells of interest are those from 29 to 50, 51 to 82, and 83 to 126. In agreement with theory, none of these isomers are found below nucleon number 39. From 39 to 50, shell structure predicts both the $p_{1/2}$ and $g_{9/2}$ levels. A mixture of magnetic 2^4 and electric 2^5 pole radiation ($l=5$) is therefore possible. The nearby $p_{3/2}$ level would give electric 2^3 pole radiation ($l=3$). Thus, while the $8l=5$ isomers in this region are consistent with shell theory, the $9l=4$ isomers are not. The $f_{7/2}$ subshell, which could combine with $p_{1/2}$ to give $l=4$, is in the shell below 28 and shell binding energies are much greater than the excitation energy of these $l=4$ isomers (<200 kev).

From 51 to 82 nucleons the available subshells are: $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $s_{1/2}$, and $h_{11/2}$. At 63 neutrons, the $g_{7/2}s_{1/2}$ combination could give an $l=4$ transition. However, the ground-state spin of Cd^{111} is $\frac{1}{2}$ and the isomeric transition occurs between two excited states. In the upper part of the shell, $h_{11/2}$ can combine with either $d_{3/2}$ or $s_{1/2}$ to give $l=5$; no $l=4$ is predicted. Therefore, Xe^{127} and Au^{197} are inexplicable. The 13 $l=5$ isomers in this region do fit shell theory. However, in 4 cases (Sn^{117}, Te^{121}, Te^{123}, and Te^{125}), the isomeric transition is followed by a gamma-ray. In each case the cross-over transition is absent in contradiction to predictions based on shell theory.

In the final shell, there is consistency for both $l=4$ and $l=5$. At 99 and 107 neutrons, $l=4$ is consistent with the $h_{9/2}p_{3/2}$ or $h_{9/2}p_{1/2}$ combinations. At 119 neutrons, both $l=5$ transitions can be explained by the $i_{13/2}p_{3/2}$ or $i_{13/2}f_{5/2}$ combinations.

To summarize, the simple spin-orbit coupling shell model would predict no $l=4$ transitions below nucleon number 50 and none in the upper parts of higher shells. Of the 14 odd A nuclei only the 2 with nucleon number above 82 are consistent with this prediction. On the other hand, the 23 odd A isomers in the $l=5$ group would fit nicely into this shell model if the absence of cross-over transitions could be explained. Internal conversion measurements can provide valuable additional tests of both isomeric classification and shell structure models.

* This work was assisted by the joint program of the ONR and AEC.
[1] P. Axel and S. Dancoff, Phys. Rev. **76**, 892 (1949).
[2] M. G. Mayer, Phys. Rev. **78**, 16 (1950).

On the Determination of the P-P Interaction from Scattering Experiments

CARL-ERIK FRÖBERG
Department of Mechanics and Mathematical Physics, Lund University, Lund, Sweden
August 1, 1950

AS is well known[1,2] low energy $P-P$ scattering can be accounted for by several different potential functions. On the other hand, it has recently been proved by Jauho[3] that the nuclear $P-P$ phase shift under certain physically reasonable conditions determines the potential function uniquely. Thus it seems to be of some interest to make a direct attempt at a calculation of the $P-P$ interaction from available experiments.

Starting from the radial equation

$$y''+\left(1-\frac{2\alpha k}{r}\right)y=-V(r)y \tag{1}$$

we can easily prove the relation

$$k\sin\eta(k)=\int_0^\infty uVydr \tag{2}$$

[$y(0)=0$, $y(\infty)\sim\sin(\xi+\eta)$; u is the regular Coulomb wave function, determined by $u(0)=0$, $u(\infty)\sim\sin\xi$, where $\xi=kr-\alpha\ln 2kr+\arg\Gamma(i\alpha+1)$]. Replacing y by u we can solve for V

$$V=-\frac{2}{\pi}\kappa^2\cdot\int_0^\infty\frac{\varphi(\alpha,\beta)\sin\eta}{\alpha^3}d\alpha.$$

Here

$$2\alpha=k/\kappa;\quad \kappa r=\beta;\quad kr=\rho;$$
$$\varphi(\alpha,\beta)=\{(d/d\rho)[u(\alpha,\rho)v(\alpha,\rho)]\}_{\rho=\beta/2\alpha};$$

v is the irregular Coulomb wave function, determined by $v(\infty)\sim\cos\xi$.

Since we know practically nothing of $\sin\eta$ for small values of α (i.e., for high energies), it is natural to use a cut-off convention. Choosing the lower limit in the integral (3) proportional to β, which seems to be reasonable, we find that V can be represented very nicely by Yukawa functions, and we obtain the following values of the range r_0 and the strength J using the cut-off radius a:

a	r_0(cm)	J(Mev)
$\beta/2$	$1.21\cdot10^{-13}$	18.2
β	$1.00\cdot10^{-13}$	22.0

A fuller account of the investigation will be published elsewhere (*Arkiv. f. Fysik*) together with a table of irregular Coulomb wave functions.

[1] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1948).
[2] J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. **22**, 77 (1950).
[3] Jauho, Ann. Acad. Sci. Fenn. (to be published).

Theory of Natural Alpha-Radioactivity

BERNARD L. COHEN*
Department of Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
June 21, 1950

RECENT measurements of cross sections of (α,n), $(\alpha,2n)$, and $(\alpha,3n)$ reactions[1-3] give excitation functions which, when compared with Weisskopf's calculation of alpha-capture cross sections,[4] require that the parameter r_0 in the expression for the nuclear radius

$$r=r_0A^{\frac{1}{3}}\times10^{-13}\text{ cm} \tag{1}$$

be taken as 1.5 in the calculation of the Coulomb barrier penetration factor.[5] Using this in the expression for the decay constant, λ,

$$\lambda=\Gamma_\alpha/\hbar=\Gamma_{0\alpha}P(r_0)/\hbar \tag{2}$$

(where Γ_α is the width for alpha-emission, $\Gamma_{0\alpha}$ is the alpha-width without barrier, and $P(r_0)$ is the Coulomb barrier penetration factor which is a very sensitive function of the assumed value of r_0), requires, in order to obtain the observed values for λ,

$$\Gamma_{0\alpha}\sim10^5\text{ ev.} \tag{3}$$

Condition (3) is in considerable disagreement with the value given by Bethe in his many-body theory of alpha-decay[6]

$$\Gamma_{0\alpha}\sim1\text{ ev.} \tag{4}$$

This discrepancy may perhaps be explained as follows. Bethe assumed that neutron widths and alpha-particle widths without