The Angular Distributions of the Alpha-Particles and of the Gamma-Rays from the Disintegration of Fluorine by Protons*

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The theoretical angular distributions of the alpha-particles and of the gamma-rays from the fluorine +proton reactions are given. They are compared with the experimental data, and the following results are obtained: (1) Both angular distributions of the α_π-group and of the α_0-group favor the assumption of even parity of F^{19}. (2) If the parity of F^{19} is taken as even, then (a) the isotropic angular distribution of the gamma-rays at the resonances of 340 and 669 kev can be interpreted as due to the formation of Ne^{20} compound states of $J=1^+$ by the capture of s-protons, (b) the slightly anisotropic angular distribution of the gamma-rays at the resonances of 598, 874, and 1381 kev can be interpreted as due to the formation of Ne^{20} states of $J=1^+$ by a mixture of s- and d-protons, or of $J=2^-$ by p-protons, (c) the highly anisotropic distribution of the gamma-rays observed at 1290 and 1355 kev can be interpreted as due to the formation of Ne^{20} states of $J=3^+$, and (d) the experimental data of the gamma-ray angular distribution and of the ratio $I(\alpha)_{140°}/I(\gamma)_{90°}$ are compatible with the assignment $j'=3^-$ for the γ_1-level of O^{16}. In general it is possible to make spectroscopic assignments consistent with the experimental data but unique assignments will await further experimental results.

I. INTRODUCTION

ENERGY, spin, and parity are three important properties of a nuclear state. The energy of a nuclear state, especially of an excited state, is normally determined by measuring the energies of the products, both particles and gamma-rays, of a nuclear reaction which involves that state. In a similar way, information about the spin and parity of a nuclear state can be obtained by studying the relative angular momenta of the reaction products. A number of methods are available for this purpose. The simplest is the one of observing the level width, which, in favorable cases, reveals the relative orbital angular momentum of the products of one of the nuclear processes, or the multipole nature of the radiation emitted. While this method is simple, it is usually not very conclusive. The most suitable method is no doubt that involving the measurement of angular distributions. The angular distributions of the reaction products and the angular correlations between the successively emitted products are determined theoretically by the symmetry properties of the nuclear states involved. When measured accurately, they give the most extensive information about the spin and parity of the nuclear states. The information is especially unambiguous when only one level of the intermediate nucleus is involved in the reaction.

In the following we shall first give the theoretical formulas and then the result of calculation for the angular distributions of the alpha-particles and of the gamma-rays from the fluorine+proton reactions. Five alpha-groups are known for these reactions:[1] the α_0-group which leads to the ground state of O^{16} and the α_π, α_1, α_2, and α_3-groups, in the order of decreasing energy, corresponding to one electron-pair level and three gamma-ray levels, respectively. The angular distributions of the α_0-group and of the gamma-rays have been measured at several proton energies. For other groups and at other energies information is only available indirectly. Comparison will be made between the calculated result and the experimental data, but the conclusions so far reached are of preliminary nature because of the scarcity of the available data.

While this work was going on, Arnold[2] obtained very interesting results for the angular correlation between the alpha-particles and the gamma-rays of the reaction $F^{19}(p,\alpha\gamma)O^{16}$. His results help to clarify several points which were unsettled in our original analysis.

II. THEORETICAL FORMULAS

The theoretical formulas for the calculation of the angular distributions of the reaction products have been given in a number of papers.[3] In some of the papers, the formulas were developed by assuming Russell-Saunders coupling. Since there is no definite evidence that such coupling is prevalent in nuclear reactions, we adopt the general treatment which includes the Russell-Saunders coupling as a special case. In the general treatment the spin of the initial nucleus is first combined with that of the incident particle giving a resultant **S**. **S** is then combined with the relative orbital angular momentum **l** of the incident particle to give the total angular momentum of the system. A transition from any sub-state of the two-particle system with an angular momentum $(\mathbf{l}+\mathbf{S})$ to a sub-state of the compound nucleus with an angular momentum **J** is considered possible if the parity is conserved and if $\mathbf{J}=\mathbf{l}+\mathbf{S}$ and $M=\lambda+\sigma$, where M,

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[1] A summary of the experimental results is to be found in a recent article by Chao, Tollestrup, Fowler, and Lauritsen, Phys. Rev. **79**, 108 (1950).

[2] W. R. Arnold, Phys. Rev. **80**, 34 (1950). The author is also indebted to Professor D. R. Inglis for this information.

[3] R. D. Meyer, Phys. Rev. **54**, 361 (1938); E. Gerjuoy, Phys. Rev. **58**, 503 (1940); D. R. Inglis, Phys. Rev. **74**, 21 (1948); also E. R. Cohen, Ph.D. thesis at California Institute of Technology (1949).

λ, and σ are the projections of $\mathbf{J}$, $\mathbf{l}$, and $\mathbf{S}$ in the direction of the incident particles, which is taken as the direction of quantization. In this section, we shall give the formulas to be used in convenient forms and supplement them with necessary explanations.

(A) Angular Distributions of the Emitted Particles

Suppose that a bombarding particle P is captured by a target nucleus A with the result of formation of a compound nucleus C and that a particle Q subsequently escapes leaving a residual nucleus B. Consider the general case in which several levels of the compound nucleus contribute to the capture process. Let $\mathbf{J}$, $\mathbf{l}$, $\mathbf{S}$, M, λ, and σ have the meanings as explained in the foregoing, λ being always zero and $M=\sigma$. Let $\mathbf{l}'$ and λ' be the relative orbital angular momentum and its projection in the direction of quantization of the outgoing particle Q, and $\mathbf{S}'$ and σ' be the combined spin and its projection of the system $B+Q$. Then the angular distribution of the outgoing particle Q is given by:

$$I_Q(\theta)=K\sum_{SS'}\sum_{\sigma\sigma'}\left|\sum_{Jll'}\frac{\overset{CJM}{H}_{l0S\sigma}\overset{CJM}{H^*}_{l'\lambda'S'\sigma'}P_{l'\lambda'}(\theta)}{(E-E_J)+(i\Gamma/2)}\right|^2 \tag{1}$$

with $\mathbf{J}=\mathbf{S}+\mathbf{l}=\mathbf{S}'+\mathbf{l}'$, $M=\sigma=\lambda'+\sigma'$. It is to be noted that amplitudes contributed by different levels or by different $\mathbf{l}$ or $\mathbf{l}'$ interfere among themselves, while amplitudes due to states of different spin orientations give no interference. Both $\mathbf{S}$ and $\mathbf{S}'$ may assume different values and have different orientations. These are taken care of by the summations over S, S', σ, and σ'. It is also to be noted that summation over λ' is not necessary, since $\lambda'=\sigma-\sigma'$. $P_{l'\lambda'}(\theta)$ is the normalized associated Legendre polynomial, which represents the wave of the outoing particle specified by $\mathbf{l}'$ and λ'. We have omitted the factor $\exp(i\lambda'\phi)$, which should accompany $P_{l'\lambda'}(\theta)$, because the interference effect between states belonging to different values of λ', i.e., to different values of σ or σ', vanishes.

The matrix elements H and H^* can be written as:

$$\overset{CJM}{H}_{l0S\sigma}=\overset{JM}{\alpha}_{l0S\sigma}\phi(l)\overset{CJ}{u}_{JlS},$$
$$\overset{CJM}{H^*}_{l'\lambda'S'\sigma'}=\overset{JM}{\alpha}_{l'\lambda'S'\sigma'}\phi(l')\overset{CJ}{u^*}_{Jl'S'}. \tag{2}$$

In the expression for H, α is the transformation coefficient between the representation labeled by J, M and that labeled by l, λ, and S, σ of the same system $A+P$, $\phi(l)$ is the square root of the barrier penetration factor of the incoming particle of relative orbital angular momentum $\mathbf{l}$, and the product $\phi(l)u$ is the matrix element for the transition between the sub-state JM of the compound nucleus C and the sub-state JM of the system $A+P$, formed by the vector addition of $\mathbf{l}$ and $\mathbf{S}$. In the present treatment, u is supposed to be dependent on $\mathbf{l}$ and $\mathbf{S}$ but independent of M. Numerical factors appearing during the expansion of the incident wave function are absorbed in the functions u's. Similar explanations hold for the expression of H^*. The coefficients α's can be found in the book by Condon and Shortley.[4]

We shall now consider the special form of (1) when it is applied to the alpha-particles of the reactions $F^{19}(p,\alpha)O^{16}$ and $F^{19}(p,\alpha\gamma)O^{16}$ and treat the case of α_γ-groups and of α_0-group separately.

1. The α_γ-Groups

Since the resonance levels for the α_γ-groups are narrow and the background is low, we have here a fair approximation for most of these resonances by treating them as a one-level problem. The Ne^{20} compound states which yield the α_γ-groups do not give the α_0-group. Hence they are supposed to have $J=0^-$, 1^+, 2^-, $\cdots$, as are usually assumed, where the $+$ or $-$ sign indicates the even or odd parity of the level. The value of S which can give the above set of J's depends on the spin and parity of F^{19}. The spin of F^{19} is known[5] to be $\frac{1}{2}$. Hence $S=0$ or 1. If the parity of F^{19} is even, the Ne^{20} levels with $J=0^-$, 1^+, 2^-, $\cdots$ can only be formed with $S=1$, i.e., with the parallel spin of F^{19} and H^1; if the parity of F^{19} is odd, these levels can be formed with $S=1$ or 0, i.e., with parallel or antiparallel spin combinations, except the level with $J=0^-$ which only be formed with $S=0$. Since the outgoing alpha-particle has no spin, there is only one value of S' and it is equal to the spin j' of the residual nucleus O^{16}. $\phi(l)$ decreases rapidly as l increases when the energy of the incident particle is below the barrier. In general we consider only the lowest l which satisfies the conservation laws but we do consider admixtures of d-waves ($l=2$) when s-waves ($l=0$) are possible. Throughout most of the calculation, we take the lowest possible value of l'. At exact resonance, $E=E_J$. Writing

$$A_{lS}=\phi(l)\phi(l')\overset{CJ}{u}_{JlS}\overset{CJ}{u^*}_{Jl'S'}/(\tfrac{1}{2}i\Gamma), \tag{3}$$

we have, from (1),

$$I_{\alpha_\gamma}(\theta)=K\sum_{S}\sum_{\sigma\sigma'}\Big|\sum_{l}\overset{JM}{\alpha}_{l0S\sigma}\overset{JM}{\alpha}_{l'\lambda'S'\sigma'}A_{lS}P_{l'\lambda'}(\theta)\Big|^2, \tag{4}$$

where $\sum_{l'}$ is replaced by its value for the minimum allowable l'. The quantity A_{lS} is complex. It may be called the partial amplitude of the compound nuclear state. In the case of narrow resonance, the phase factor in A_{lS} can be evaluated.[6] If the parity of F^{19} is even, Ne^{20} compound states of $J=1^+$ can be formed by combining $l=0$ or 2 with $S=1$. We set, in this case,

$$X=A_{2S}/A_{0S}. \tag{5}$$

If the parity of F^{19} is odd, Ne^{20} compound states of

[4] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (The Macmillan Company, New York, 1935), pp. 76–77.
[5] J. E. Mack, Rev. Mod. Phys. **22**, 64 (1950).
[6] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

$J=1^+, 2^-, 3^+$, etc., can be formed combining $l=1, 2, 3$, etc., with $S=1$ or 0. We set, in these cases,

$$t=|A_{l1}/A_{l0}|^2. \tag{6}$$

X may be called the ratio of the partial amplitudes due to the d-wave and the s-wave components of protons and t the ratio of the contributions due to the triplet combination and the singlet combination. X and t are the arbitrary constants occurring in the final expression of the angular distribution.

2. *The* α_0*-Group*

In this case the resonance levels are not very sharp and the background is appreciable. Hence we have to take into account the effect of several levels at one proton energy. We let, now,

$$\overset{J}{A}_{ll'SS'}=\phi(0)\phi(l')\overset{CJ}{u}_{JlS}\overset{CJ}{u^*}_{Jl'S'}/[(E-E_J)+\tfrac{1}{2}i\Gamma], \tag{7}$$

where $\phi(0)$ is the square root of the barrier penetration factor for $l=0$. Then

$$I_{\alpha_0}(\theta)=K\sum_{SS'}\sum_{\sigma\sigma'} \times\left|\sum_{Jll'}\overset{JM}{\alpha}_{l0S\sigma}\overset{JM}{\alpha}_{l'\lambda'S'\sigma'}\frac{\phi(l)}{\phi(0)}\overset{J}{A}_{ll'SS'}P_{l'\lambda'}(\theta)\right|^2. \tag{8}$$

We have purposely separated $\phi(l)/\phi(0)$ from A in order to facilitate the comparison of different A's. Formula (7) can be simplified further because the spin of the residual nucleus, i.e., of the ground state of O^{16}, is zero and hence $S'=0$, $l'=J$, $\lambda'=M$, and the second transformation coefficient α equals unity. Thus, we need only three indices to label A and have

$$I_{\alpha_0}(\theta)=K\sum_S\sum_\sigma\left|\sum_{Jl}\overset{JM}{\alpha}_{l0SM}\frac{\phi(l)}{\phi(0)}A_{JlS}\exp(i\omega_{JlS})P_{l'\lambda'}(\theta)\right|^2, \tag{9}$$

where we have separated the phase factor $e^{i\omega}$ from A_{JlS}. A_{JlS} and ω_{JlS} are now the arbitrary constants occurring in the final expression of the angular distribution.

(B) Angular Distribution of the Gamma-Rays

The formula for the angular distribution of the gamma-rays from the reaction $F^{19}(p,\alpha\gamma)O^{16}$ is similar to (4). It can be written as

$$I_\gamma(\theta)=K\sum_S\sum_{\sigma\sigma'}\left\{\left|\sum_l\overset{JM}{\alpha}_{l0S\sigma}\overset{JM}{\alpha}_{l'\lambda'S'\sigma'}A_{lS}\right|^2 f_{S'\sigma'}(\theta)\right\}, \tag{10}$$

where A_{lS} is the quantity defined in (3) and $f_{S'\sigma'}(\theta)$ is the angular distribution of the radiation due to the transition $\Delta j'=S'$, $\Delta m'=\sigma'$ of O^{16}. Formula (10) is much simpler than the general formula for the gamma-rays from such reactions because (a) we consider only one level of the Ne^{20} compound nucleus, (b) the outgoing particle has zero spin and hence the spin of the excited O^{16} nucleus is equal to S', and (c) the spin of the ground state of O^{16} is zero. We can use here also the quantities X and t defined in (5) and (6) in the final expression. It is to be noted that the probability for an alpha-particle emitted in the direction (θ, ϕ) and a gamma-ray subsequently emitted in the direction (θ', ϕ') is a function $P(\theta, \phi, \theta', \phi')$ and that the intensity of the gamma-ray in the direction (θ', ϕ') is, in general, given by:

$$I_\gamma(\theta', \phi')=\int P(\theta, \phi, \theta', \phi')d\omega_\alpha. \tag{11}$$

When the outgoing α-particles may have more than one value of l', Eq. (11) is to be used[7] instead of Eq. (10).

The function $f_{S'\sigma'}(\theta)$ gives the angular distributions of the dipole, quadrupole, and octupole radiations for $S'=1, 2,$ and 3, respectively. These angular distributions are given below:[8]

(a) Dipole radiation:

$$\begin{aligned} f_{10}(\theta)&=3(1-\cos^2\theta),\\ f_{11}(\theta)&=3(1+\cos^2\theta)/2. \end{aligned} \tag{12}$$

(b) Quadrupole radiation:

$$\begin{aligned} f_{20}(\theta)&=15(\cos^2\theta-\cos^4\theta),\\ f_{21}(\theta)&=\tfrac{1}{2}5(1-3\cos^2\theta+4\cos^4\theta),\\ f_{22}(\theta)&=5(1-\cos^4\theta)/2. \end{aligned} \tag{13}$$

(c) Octupole radiation:

$$\begin{aligned} f_{30}(\theta)&=(7/32)\cdot 12(1-11\cos^2\theta+35\cos^4\theta-25\cos^6\theta),\\ f_{31}(\theta)&=(7/32)(1+111\cos^2\theta-305\cos^4\theta+225\cos^6\theta),\\ f_{32}(\theta)&=(7/32)\cdot 10(1-3\cos^2\theta+11\cos^4\theta-9\cos^6\theta),\\ f_{33}(\theta)&=(7/32)\cdot 15(1-\cos^2\theta-\cos^4\theta+\cos^6\theta). \end{aligned} \tag{14}$$

TABLE I. Observed angular distributions of the α_γ-groups and of the gamma-rays.

Levels	$I_\gamma(\theta)\sim 1+a\cos^2\theta$ Coefficient a D.H.	Kellogg	$\dfrac{I(\alpha)_{140°}}{I(\gamma)_{90°}}$
340	0		0.88+
598	0.2		0.90+
669	0		0.97
874	0.1	0.10	0.90
935		0	0.88
1290		0.67	0.99
1355		0.25	0.97
1381		−0.07	0.88

III. THEORETICAL ANGULAR DISTRIBUTIONS AND THEIR COMPARISON WITH THE EXPERIMENTAL DATA

(A) Angular Distributions of the α_γ-Groups and of the Gamma-Rays

The angular distribution of the α_γ-groups have not been measured; those of the gamma-rays have been

[7] The author is indebted to Professor R. F. Christy for pointing out this fact.

[8] D. R. Hamilton, Phys. Rev. **58**, 122 (1940); D. S. Ling, Jr. and D. L. Falkoff, Phys. Rev. **76**, 1639 (1949).

TABLE II. Angular distributions of the α_γ-groups and of the gamma-rays under the assumption of even parity of F^{19}.

	$Ne^{20}(J=0^-)$	$Ne^{20}(J=1^+)$	$Ne^{20}(J=2^-)$	$Ne^{20}(J=3^+)$
$O^{16}(j'=1^-)$	$I_\alpha=0$	$I_\alpha\sim 1+\frac{0.95x}{1-0.32x}\cos^2\theta$ $I_\gamma\sim 1-\frac{0.47x}{1+0.16x}\cos^2\theta$	$I_\alpha\sim 1+\cos^2\theta$ $I_\gamma\sim 1+0.64\cos^2\theta$ $I_\alpha'/I_\gamma'=1.4$	$I_\alpha\sim 1+1.2\cos^2\theta+1.0\cos^4\theta$ $I_\gamma\sim 1+0.82\cos^2\theta$ $I_\alpha'/I_\gamma'=1.64$
$O^{16}(j'=1^+)$	$I_\alpha\sim 1$ $I_\gamma\sim 1$ $I_\alpha'/I_\gamma'=1$	$I_\alpha\sim 1$ $I_\gamma\sim 1+\frac{0.95x}{1-0.32x}\cos^2\theta$	$I_\alpha\sim 1+1.62\cos^2\theta$ $I_\gamma\sim 1-0.45\cos^2\theta$ $I_\alpha'/I_\gamma'=1.08$	$I_\alpha\sim 1-0.33\cos^2\theta+3.0\cos^4\theta$ $I_\gamma\sim 1-0.44\cos^2\theta$ $I_\alpha'/I_\gamma'=0.99$
$O^{16}(j'=2^-)$	$I_\alpha\sim 1$ $I_\gamma\sim 1$ $I_\alpha'/I_\gamma'=1$	$I_\alpha\sim 1-\frac{0.19x}{1+0.063x}\cos^2\theta$ $I_\gamma\sim 1-\frac{0.47x}{1+0.16x}\cos^2\theta$	$I_\alpha\sim 1$ $I_\gamma\sim 1+\cos^2\theta$ $I_\alpha'/I_\gamma'=1.33$	$I_\alpha\sim 1+1.6\cos^2\theta$ $I_\gamma\sim 1+2.9\cos^2\theta-2.0\cos^4\theta$ $I_\alpha'/I_\gamma'=2.0$
$O^{16}(j'=2^+)$	$I_\alpha=0$	$I_\alpha\sim 1+\frac{0.95x}{1-0.32x}\cos^2\theta$ $I_\gamma\sim 1+\frac{0.47x}{1-0.16x}\cos^2\theta$	$I_\alpha\sim 1-0.78\cos^2\theta$ $I_\gamma\sim 1+0.43\cos^2\theta$ $I_\alpha'/I_\gamma'=0.83$	$I_\alpha\sim 1+4.6\cos^2\theta-4.7\cos^4\theta$ $I_\gamma\sim 1-1.4\cos^2\theta+1.8\cos^4\theta$ $I_\alpha'/I_\gamma'=1.18$
$O^{16}(j'=3^-)$	$I_\alpha=0$	$I_\alpha\sim 1+\frac{0.95x}{1-0.32x}\cos^2\theta$ $I_\gamma\sim 1+\frac{0.71x}{1-0.24x}\cos^2\theta$	$I_\alpha\sim 1-\frac{2}{3}\cos^2\theta$ $I_\gamma\sim 1+0.24\cos^2\theta$ $I_\alpha'/I_\gamma'=0.83$	$I_\alpha\sim 1-0.9\cos^2\theta$ $I_\gamma\sim 1+1.27\cos^2\theta+0.15\cos^4\theta$ $I_\alpha'/I_\gamma'=0.96$
$O^{16}(j'=3^+)$	$I_\alpha\sim 1$ $I_\gamma\sim 1$ $I_\alpha'/I_\gamma'=1$	$I_\alpha\sim 1-\frac{0.27x}{1+0.09x}\cos^2\theta$ $I_\gamma\sim 1-\frac{0.57x}{1+0.19x}\cos^2\theta$	$I_\alpha\sim 1+\frac{1}{3}\cos^2\theta$ $I_\gamma\sim 1+1.3\cos^2\theta$ $I_\alpha'/I_\gamma'=1.55$	$I_\alpha\sim 1$ $I_\gamma\sim 1+1.2\cos^2\theta+\cos^4\theta$ $I_\alpha'/I_\gamma'=1.6$

measured by Devons and Hine[9] at the resonances below 900 kev of the proton energy and are being studied more extensively in this laboratory.[10] Devons and Hine found that the angular distribution of the gamma-rays so far obtained can be expressed in the form,

$$I_\gamma(\theta)=1+a\cos^2\theta, \tag{15}$$

with a equal to zero or a small fraction (≤ 0.2). The preliminary result of the gamma-ray angular distribution obtained in this laboratory does not show appreciable asymmetry between the forward and backward direction, and it yields a value of a as high as 0.67 at 1290 kev when the intensity is expressed in the form (15). Information about the angular distribution can also be obtained indirectly from the measured ratio of the alpha-intensity at 140° in the center-of-mass system to the gamma-intensity[1] at 90°. Table I gives the experimental values of the coefficient a and of the ratio $I(\alpha)_{140°}/I(\gamma)_{90°}$ at various gamma-ray resonances, the values of the coefficient a marked with D. H. being taken from reference 9.

As was mentioned before, the Ne^{20} compound states giving rise to the α_γ-groups are supposed to have $J=0^-$, 1^+, $2^-\cdots$. We consider only those levels with $J\leq 3$. Of the three gamma-rays, the intensity of γ_1 is strongest. The γ_1-level of O^{16} is expected to have $j'=1^-$ according to the shell structure model of the nucleus and to have $j'=3^-$ or to be a doublet with $j'=2^-$ and 2^+ according to the alpha-particle model. On the other hand, Devons and Hine suggested $j'=1^+$ for the γ_1-level taking the F^{19} parity to be odd. By means of formulas (4) and (10), we have calculated the theoretical angular distributions of the α_γ-group and of the corresponding gamma-rays for various transitions which are derived from different assignments of the compound state of Ne^{20} and of the excited state of O^{16} and with even parity of F^{19}. We shall see later on that the assumption of even parity of F^{19} is favored by the observed angular distributions of the α_π- and α_0-groups. Also even parity is indicated by the filling of 2s-levels between O^{16} and Ne^{20}. The results of our calculation are given in Table II, where I_α and I_γ are the angular distributions of the alpha-particles and of the gamma-rays, respectively, I_α'/I_γ' is the theoretical ratio of the alpha-intensity at 140° to the gamma-intensity at 90° and x is the real part of the quantity X, which is defined in Eq. (5).

[9] S. Devons and M. G. N. Hine, Proc. Roy. Soc. **199A**, 56 (1949).
[10] Day, Chao, Fowler, and Perry, Phys. Rev. **80**, 131 (1950).

Comparison between the experimental data of Table I and the theoretical expressions of Table II now gives the following interpretations:

(a) The isotropic angular distribution of the gamma-rays at the resonances of 340, 669, and 935 kev can be interpreted as due to the capture of protons of $l=0$ or to the formation of Ne^{20} compound nuclei of $J=0^-$.

(b) The slightly anisotropic angular distribution of the gamma-rays at the resonance of 1381 kev can be interpreted as due to the formation of a Ne^{20} compound state of $J=1^+$ by a mixture of *s*- and *d*-protons. The observed value of the coefficient a is -0.07 at this resonance. From the barrier effect and the phase difference between the *s*- and *d*-protons we would expect a value of $x\sim-0.06$ at the proton energy of 1381 kev. This is found to be quite close to the values of x, which the assignments $j'=1^+$, 2^+, and 3^- require in order to have $a=-0.07$. Furthermore, with the values of x corresponding to $a=-0.07$ the assignments $j'=1^+$, 2^+ and 3^- give $I_\alpha'/I_\gamma'=0.98$, 0.94, and 0.95, respectively. They are all close to the observed ratio 0.88 at this resonance.

(c) The resonance at 598 kev gives a gamma-ray angular distribution with $a=0.2$. This can be interpreted by the assignment $J=2^-$ for the Ne^{20} compound nucleus and $j'=3^-$ for the γ_1-level. The resonance at 874 kev can be interpreted in the same way as the one at 598 kev or as that at 1381 kev.

(d) The large values of the coefficient a at the resonances of 1290 and 1355 kev can be interpreted as caused by (i) the formation of Ne^{20} compound states of $J=2^-$, or (ii) the formation of Ne^{20} states of $J=3^+$. It is seen in Table II, that with $J=2^-$ all transitions which give $a>0.5$ lead to values of I_α'/I_γ' greater than unity. Since the observed values of I_α'/I_γ' at these two resonances are 0.99 and 0.97, we must assume in this case that the intensities of the three gamma-ray components are just of the right proportion to make the average value of this ratio approximately equal to unity. If, on the other hand, we take $J=3^+$, the assignment with $j'=3^-$ leads to $I_\alpha'/I_\gamma'=0.96$, which is very close to the observed ratios. The coefficient a given by this assignment is now considerably higher than the observed coefficients at these two resonances. But this is to be expected from the effect of the background and of averaging over the three gamma-ray components. The small $\cos^4\theta$ term in I_γ required by this assignment would be inappreciable in the preliminary measurement in this laboratory.

(e) Arnold[2] found his results of correlation measurement between the alpha-particles and the gamma-rays at 340-kev proton energy to be in good agreement with the assignment $J=1^+$ for the Ne^{20} compound state and $j'=3^-$ for the γ_1-level of O^{16}. The assignment $j'=3^-$ is just the one required in (c) and (d). If we take $j'=3^-$ for the γ_1-level, the interpretation for the isotropic and slightly anisotropic angular distributions of the gamma-rays in (a) and (b) would be the same except that the assignment $J=0^-$ to the Ne^{20} compound state must be dropped for the gamma-ray resonances so far observed, because a state of $J=0^-$ could not transit to one of $j'=3^-$ by alpha-emission only. Hence, in addition to the levels at 340 and 1381 kev, which have been given the assignment $J=1^+$, the levels at 669 and 935 kev would have also $J=1^+$ on this basis. The fact that the α_1-group is present at all observed gamma-ray resonances might be an indication that transitions from a compound state of $J=0^-$ to all the three gamma-levels of O^{16} are forbidden. If this be the case, it might be still possible to detect a compound state of $J=0^-$ by the proton scattering experiment.

We give now in Table III our tentative assignment to the compound nucleus, the relative orbital angular momentum of the captured proton, and the calculated and observed proton widths for each resonance discussed in the foregoing. The values of the calculated proton width are obtained from the curves given by Christy and Latter,[11] and are normalized here to 1 kev at 1-Mev proton energy without barrier. The values of the observed proton width are taken from reference 1. They are calculated from the gamma-ray yields under the assumption that the alpha-width is large compared with the proton width. It is seen in Table III that the ratio of the observed to the calculated proton width varies considerably. Since the assumption $\Gamma_p\ll\Gamma_\alpha$ is only a trial one, some of the irregularities might be due to the breakdown of this assumption.

For some assignments of J and j' more than one value of l' satisfy the conservation laws of the angular momentum and the parity. Furthermore, the square roots of the relative barrier penetration factors of the α_1-group at the proton energy of 874 kev are: $\phi'=0.26$, 0.20, 0.10, 0.03, and 0.006 for $l'=0$, 1, 2, 3, and 4, respectively. Alpha-emission with a value of l' greater by two units than the lowest value might therefore produce an appreciable interference effect. When two or more values of l' are possible, it is found that the corresponding partial amplitudes give interference only on

TABLE III. Assignments to the Ne-levels giving rise to the gamma-ray resonance.

Level (kev)	J(Ne)	l_p	$(\Gamma_p)_{cal}$* (kev)	$(\Gamma_p)_{ob}$ (kev)	$(\Gamma_p)_{ob}$† / $(\Gamma_p)_{cal}$
340	1^+	0	0.0002	0.043	210
598	2^-	1	0.0020	0.044	22
669	1^+	0	0.014	0.13	9
874	2^-	1	0.013	1.13	87
874	1^+	0,2	0.042	1.13	27
935	1^+	0	0.060	0.63	11
1290	3^+	2	0.007	0.14	20
1355	3^+	2	0.008	0.21	26
1381	1^+	0,2	0.230	2.9	13

* $(\Gamma_p)_{cal}$, normalized to 1 kev at 1-Mev proton energy without barrier.
† This column gives widths without barrier in kev, when the proton energy is reduced to 1 Mev.

[11] R. F. Christy and R. Latter, Rev. Mod. Phys. **20**, 185 (1948).

TABLE IV. Angular distribution of the α_π-group.

Proton wave	$Ne(J=1^-)$	$Ne(J=2^+)$
$l=0$	$F^{19}(+)$ $I(\alpha_\pi)=0$ $F^{19}(-)$ $I(\alpha_\pi)\sim 1$	$F^{19}(+)$ $I(\alpha_\pi)=0$ $F^{19}(-)$ $I(\alpha_\pi)=0$
$l=1$	$F^{19}(+)$ $I(\alpha_\pi)\sim\cos^2\theta+(t/2)(1-\cos^2\theta)$ $t=0\rightarrow\infty$ $I_{140°}/I_{Av}=1.8\rightarrow 0.6$ $F^{19}(-)$ $I(\alpha_\pi)=0$	$F^{19}(+)$ $I(\alpha_\pi)=0$ $F^{19}(-)$ $I(\alpha_\pi)\sim 1+3\cos^2\theta$ $I_{140°}/I_{Av}=1.40$
$l=2$	$F^{19}(+)$ $I(\alpha_\pi)=0$ $F^{19}(-)$ $I(\alpha_\pi)\sim 1+3\cos^2\theta$ $I_{140°}/I_{Av}=1.40$	$F^{19}(+)$ $I(\alpha_\pi)\sim 1+(6t-6)\cos^2\theta+(9-6t)\cos^4\theta$ $t=0\rightarrow\infty$ $I_{140°}/I_{Av}=0.8\rightarrow 1.8$ $F^{19}(-)$ $I(\alpha_\pi)=0$

the alpha-intensity but not on the gamma-intensity. Hence, the calculated angular distributions of the gamma-rays, in particular, the coefficients of the $\cos^2\theta$ and $\cos^4\theta$ terms and the coefficients of x in the expressions of I_γ in Table II are not appreciably changed by considering more than one value of l'. The foregoing conclusions (a), (b), (c), (d), and (e) are therefore still valid when two values of l' are taken into consideration. The interference of two partial amplitudes corresponding to different values of l' might give appreciable change to the alpha-intensity. Detailed investigation about this change is, however, omitted because of the scarcity of the observed data.

(B) Angular Distribution of the α_π-Group

The resonance levels for the pair excitation are not very narrow compared to their spacing. Strictly speaking, we ought to consider the interference effect of the neighboring levels. Since, however, no data are available for the angular distribution of the α_π-group except some indirect information, for the sake of simplicity, we made calculation for this group also by means of formula (4). The excitation of the pairs is usually attributed to the formation of Ne^{20} compound states with $J=0^+$, 1^-, 2^+, $\cdots$. The assignment of the pair level is assumed to be $j'=0^+$. The calculated angular distributions of the α_π-group together with the ratio $I(\alpha_\pi)_{140°}/I(\alpha_\pi)_{Av}$ are given in Table IV.

Two sources of information are available for the angular distribution of the α_π-group: (a) the ratio of the yield of the α_π-group per unit solid angle at 140° with the incident beam to that of the pairs at 13° is found to be about 2 at the pair resonances 843 and 1236 kev in this laboratory.[1,12] Since the angular distribution of the pairs is expected to be isotropic, this value can be taken as the observed value of $I(\alpha_\pi)_{140°}/I(\alpha_\pi)_{Av}$. (b) The relative yield of α_0/α_π at the neighborhood of 1236 kev as measured at 90° by Van Patter *et al.*[13] is almost four times larger than the relative yield of α_0/pairs obtained at 90° by Streib *et al.*,[14] or $I(\alpha_\pi)_{90°}/I(\alpha_\pi)_{Av}\sim\frac{1}{4}$. Comparing the results (a) and (b) with the corresponding ratios which are given in Table IV or derivable therefrom, we find that the experimental results can be best interpreted as due to the capture of *p*-protons or *d*-protons by F^{19} with even parity.

(C) Angular Distribution of the α_0-Group

The Ne^{20} compound states giving rise to the high energy α_0-group ought to have $J=0^+$, 1^-, 2^+, $\cdots$. We shall limit our discussion to incident protons with orbital angular momenta up to $l=2$. The angular distributions will be different according to which parity F^{19} has. For convenience in discussion, we give in Table V the values of l and S for each value of J under the assumption of even or odd parity for F^{19}. The combination $(lSJ)=(3\ 1\ 2)$ is omitted under the odd parity of F^{19} because l is greater than 2 in that case.

At low proton energies the angular distribution of the α_0-group has been measured by Mclean *et al.*,[15] and analyzed by Gerjuoy[3] under the assumption of even parity of F^{19}. Rubin[16] has measured the angular distribution of the α_0-group at various proton energies and with great care at the two energies 1155 and 1348 kev.[17] He obtained:

$$I(\theta)=1-\tfrac{1}{3}\cos\theta+\cos^2\theta-\cos^3\theta \text{ at 1155 kev} \quad (16)$$

and

$$I(\theta)=1-\cos\theta+6\cos^2\theta+\cos^3\theta-5\cos^4\theta \text{ at 1348 kev.} \quad (17)$$

He also observed that the result at 1348 kev is in fair agreement with the theoretical angular distribution expected from the formation of a compound state of $J=2$ plus a background under the assumption of even parity of F^{19}.

TABLE V. Compound Ne states for the α_0-group.

$F^{19}(+)$			$F^{19}(-)$		
l	S	J	l	S	J
0	0	0^+	1	1	0^+
1	0	1^-	0	1	1^-
1	1	1^-	2	1	1^-
2	0	2^+	1	1	2^+
2	1	2^+	2	1	3^-

[12] Rasmussen, Hornyak, Lauritsen, and Lauritsen, Phys. Rev. **77**, 617 (1950).

[13] Strait, Van Patter, and Buechner, Phys. Rev. **78**, 337 (A) (1950); Van Patter, Sperduto, Strait, and Buechner, Progress Report of Laboratory for Nuclear Science and Engineering, MIT, p. 57, April (1950) (unpublished).

[14] Streib, Fowler, and Lauritsen, Phys. Rev. **59**, 253 (1941).

[15] Mclean, Ellett, and Jacobs, Phys. Rev. **58**, 500 (1940).

[16] S. Rubin, Phys. Rev. **72**, 1176 (1947).

[17] These values are a little higher than those originally given in Rubin's paper because of a change of the voltage scale.

We have calculated the two angular distributions of the α_0-group corresponding to both parities of F^{19} by means of formula (9). As the ratios of the square roots of the barrier penetration factors in (9), we take their values at the proton energy 1200 kev, i.e.,

$$\phi(0):\phi(1):\phi(2)=1:0.58:0.18. \tag{18}$$

In the following, we shall compare the observed angular distributions at 1155 and 1348 kev with each of the two theoretical angular distributions corresponding to the even and odd parity of F^{19}.

(1) *Angular Distribution of the α_0-Group under the Assumption of Even Parity of* F^{19}

In this case, we consider the contributions from three levels of $J=0^+$, 1^-, and 2^+. J is always equal to l. Hence we need only two indices l and S to specify the partial amplitude A and we can write A_{lS} instead of A_{JlS}. With this convention, we have, by applying formula (9) and dropping the constant K,

$$\begin{aligned}I_{\alpha0}(\theta)=&\{0.50(A_{00})^2+0.021(A_{20})^2+0.25(A_{11})^2\\&-0.20A_{20}A_{00}\cos\omega_{20}\}\\&+\{A_{10}A_{00}\cos\omega_{10}+0.35A_{21}A_{11}\cos(\omega_{21}-\omega_{11})\\&-0.20A_{20}A_{10}\cos(\omega_{20}-\omega_{10})\}\cos\theta\\&+\{0.50(A_{10})^2-0.125(A_{20})^2\\&+0.60A_{20}A_{00}\cos\omega_{20}-0.25(A_{11})^2\\&+0.125(A_{21})^2\}\cos^2\theta\\&+\{0.60A_{20}A_{10}\cos(\omega_{20}-\omega_{10})\\&-0.35A_{21}A_{11}\cos(\omega_{21}-\omega_{11})\}\cos^3\theta\\&+\{0.187(A_{20})^2-0.125(A_{21})^2\}\cos^4\theta,\end{aligned} \tag{19}$$

ω_{00} being set equal to zero. The above formula is similar to the one given by Gerjuoy[3] with only two differences: (1) he used the barrier penetration factors at 400 kev; (2) he had separated the expansion coefficient $(2l+1)^{\frac{1}{2}}$ of the incident wave from the partial amplitude.

Let us now compare the observed angular distribution (16) and (17) with the theoretical expression (19). In doing this we indicate the partial amplitudes at 1155 and 1348 kev by A_{lS}' and A_{lS}'', respectively. The results obtained are as follows:

(a) The coefficient -5 of the $\cos^4\theta$ term at 1348 kev requires $A_{21}''\geq 6.3$ and $A_{21}''>A_{20}''$. It is seen in (19) that this high amplitude A_{21}'' is sufficient to account for the large coefficient of $\cos^2\theta$ at that resonance and hence makes the main contribution of the intensity, indicating a level with $J=2^+$ at 1348 kev.

(b) The presence of the $\cos\theta$ and $\cos^3\theta$ terms at 1348 kev requires an additional level $J=1^-$ to interfere with the level $J=2^+$. The angular distribution (17) could be interpreted as due to two levels $J=1^-$ and 2^+ only. But the position of the level $J=1^-$ cannot be established by (17) alone. Furthermore, (17) does not exclude the possibility of any influence of a third level $J=0^+$.

(c) The $\cos^3\theta$ term at 1155 kev requires the interference of two levels $J=1^-$ and 2^+, while the absence of the $\cos^4\theta$ term indicates small values of A_{21}' and A_{20}'. This is to be expected if we assume that a level $J=1^-$ is at 1155 kev and that interference is produced by the level $J=2^+$ at 1348 kev.

(d) If the assumption made in (c) is true, it is found by comparing the coefficients of the $\cos\theta$ powers of (16) and (19) that the observed angular distribution at 1155 kev can be better explained by assuming another level $J=0^+$ in addition to the two levels $J=1^-$ and 2^+. The level $J=0^+$ might be a broad one which constitutes the background.

(2) *Angular Distribution of the α_0-Group under the Assumption of Odd Parity of* F^{19}

If the parity of F^{19} is odd, four values of J are possible with $l\lesseqgtr 2$. The $\cos^4\theta$ term at 1348 kev can be contributed only by the interference of the levels $J=1^-$ and 3^-. In order to account for the $\cos\theta$ and $\cos^3\theta$ term, there must be at least one level with even parity, which might have $J=0^+$ or 2^+. Because the resonance at 1348 kev is narrow and that at 1155 kev is broad, it seems reasonable to assume a level $J=3^-$ at 1348 kev, a level $J=1^-$ at 1155 kev, and a broad level $J=0^+$ as the background. We take now these three levels to calculate the angular distribution of the α_0-group in this case. Since S has the same value 1 for all the partial amplitudes, we write $A_{J,l}$ (with a comma between J and l) instead of A_{JlS}. By applying formula (9) and Eq. (18), we obtain now:

$$\begin{aligned}I_{\alpha0}(\theta)=&\{1.5(A_{1,0})^2+0.056(A_{0,1})^2+0.005(A_{1,2})^2\\&+0.018(A_{3,2})^2+0.17A_{1,0}A_{1,2}\cos\omega_{1,2}\\&-0.32A_{1,0}A_{3,2}\cos\omega_{3,2}\\&-0.019A_{1,2}A_{3,2}\cos(\omega_{1,2}-\omega_{3,2})\}\\&+\{-0.58A_{1,0}A_{0,1}\cos\omega_{0,1}\\&+0.067A_{0,1}A_{1,2}\cos(\omega_{0,1}-\omega_{1,2})\\&+0.19A_{0,1}A_{3,2}\cos(\omega_{0,1}-\omega_{3,2})\}\cos\theta\\&+\{0.015(A_{1,2})^2-0.035(A_{3,2})^2\\&-0.52A_{1,0}A_{1,2}\cos\omega_{1,2}\\&+0.98A_{1,0}A_{3,2}\cos\omega_{3,2}\\&+0.22A_{1,2}A_{3,2}\cos(\omega_{1,2}-\omega_{3,2})\}\cos^2\theta\\&+\{0.31A_{0,1}A_{3,2}\cos(\omega_{0,1}-\omega_{3,2})\}\cos^3\theta\\&+\{0.088(A_{3,2})^2-0.28A_{1,2}A_{3,2}\\&\times\cos(\omega_{1,2}-\omega_{3,2})\}\cos^4\theta.\end{aligned} \tag{20}$$

By comparing (17) and (20), the coefficients of $\cos^4\theta$ now give

$$0.088(A_{3,2})^2-0.28A_{1,2}A_{3,2}\cos(\omega_{1,2}-\omega_{3,2})=-5$$

or

$$0.088(A_{3,2})^2-0.28A_{1,2}A_{3,2}+5\lesseqgtr 0. \tag{21}$$

The last equation gives a minimum value of $A_{1,2}=4.7$, for which $A_{3,2}=7.5$. Since $A_{1,2}$ and $A_{1,0}$ are the partial amplitudes of the state $J=1^-$ due respectively to d-protons and s-protons with the factor $\phi(l)/\phi(0)$ taken out, $A_{1,2}$ and $A_{1,0}$ are expected to be of the same order of magnitude. Now, a value of $A_{1,0}$ of the order of 5 would result in a very large constant term of the order of 20 in the expression (20). This is incompatible with the observed constant term 1 in (17). The above conclusion is independent of the fact that we have omitted the possibility of a level $J=2^+$, because: (a) Eq. (21) will not change, and (b) the constant term might be even greater, if we introduce a level $J=2^+$ in addition to the three levels already assumed. The only way to avoid the above difficulty is to assume $A_{1,2} \gg A_{1,0}$. This might be possible if the capture proton fills a d-orbit inside the nucleus in forming the compound state $J=1^-$ and if an incoming d-proton has a much larger cross section for this process than an s-proton. The coefficient 6 of the $\cos^2\theta$ term in (17) also requires a high value of $A_{1,2}$ of the order of 20 if the resonance peak at 1348 kev is not mainly produced by interference. This in turn would make the level $J=1^-$ the main contributor of intensity and hence the resonance level at 1348 kev. While all the requirements mentioned above might not be impossible, the explanation given in (1) seems to be much more natural.

IV. CONCLUSIONS

Direct information on the angular distribution of the alpha-particles is available only for the α_0-group. The analysis of the data of this group is, however, rendered difficult by the interference effects. On the other hand, data about the angular distribution of the gamma-rays are still meager. Hence, most of the foregoing conclusions can be considered as only preliminary. We shall now summarize them as follows:

(a) The angular distribution of the α_π-group as well as that of the α_0-group favors the assumption of even parity for F^{19}.

(b) The isotropic gamma-ray angular distribution at some of the resonances can be interpreted as due to the capture of s-protons. The level at 340 kev which gives a strong resonance at low proton energy can be taken as one of $J=1^+$, formed by the capture of an s-proton, capture of d-protons being negligible at this energy. The levels at 669 and 935 kev which also give isotropic angular distribution of gamma-rays are probably formed in the same way.

(c) The gamma-ray angular distribution with a small coefficient of $\cos^2\theta$ can be interpreted as due to the formation of a compound state of $J=1^+$ by a mixture of s- and d-protons. This is probably the case with the level at 1381 kev, and possibly with some other levels.

(d) The level at 598 kev is probably one of $J=2^-$, formed by p-protons. The level at 874 kev can be interpreted as one of $J=1^+$ or of $J=2^-$.

(e) The levels at 1290 and 1355 kev, which give relatively large coefficients of the $\cos^2\theta$ term in the gamma-ray angular distribution, are probably those of $J=3^+$, formed by the capture of d-protons.

(f) The experimental data of the gamma-ray angular distribution and of the ratio I_α'/I_γ' are all in conformity with the assignment $j'=3^-$ for the γ_1-level of O^{16}.

It is to be mentioned that with the assignments of the three gamma-levels of O^{16} limited to the values of j' listed in Table II it is still difficult to explain the observed intensity ratios of the three α_γ-groups as given in reference (1). For further progress, it will be necessary to extend the correlation measurement between the alpha-particles and the gamma-rays and to measure the angular distributions of the separate α-groups.

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