TABLE I. Effect of the induced quadrupole moment.*

Element	Z	Orbital	R	$\langle V\rangle a_H^3$	$\langle 1/r^3\rangle a_H^3$	$Q/(10^{-24}\,\text{cm}^2)$	$Q_{\text{corr}}/(10^{-24}\,\text{cm}^2)$
B	5	$2p$	0.220	0.860	1.18	0.06^a, 0.03^b	0.077^a, 0.038^b
Al	13	$3p$	0.121	1.04	2.57	0.156	0.177
Sc	21	$3d$	0.452	1.89	1.26	——	——
Ga	31	$4p$	0.055	2.21	12.1	0.232^c, 0.146^d	0.245^c, 0.155^d
In	49	$5p$	0.038	1.66	13.2	1.144^e, 1.161^f	1.189^e, 1.207^f
Eu	63	$4f$	0.405	9.29	6.88	1.2^g, 2.5^h	2.0^g, 4.2^h
Lu	71	$5d$	0.125	4.52	10.9	5.9^i, 7.0^j	6.7^i 8.0^j
Ac	89	$6d$	0.116	1.28	3.33	——	——

* In the last two columns of this table the superscripts refer as follows to different sotopes: ^{a} B^{10}, ^{b} B^{11}, ^{c} Ga^{69}, ^{d} Ga^{71}, ^{e} In^{113}, ^{f} In^{115}, ^{g} Eu^{151}, ^{h} Eu^{153}, ^{i} Lu^{175}, ^{j} Lu^{176}.

where χ is the Thomas-Fermi function at a point in the electron cloud, r is the length of the vector from the nucleus to this point, and θ is the angle included by this vector and the axis of the nuclear quadrupole moment Q. The density of electrons ρ is $8\pi p^3/3h^3$. Let $\Delta\rho$ be the density due to the second term of (1). Thus,

$$\Delta\rho = 8\pi p_0^2 \Delta p/h^3, \tag{2}$$

where Δp is the change of momentum associated with the term containing Q, and p_0 would be the maximum momentum, p, for $Q=0$. We have

$$(p_0\Delta p)/m = e^2Q(3\cos^2\theta - 1)/4r^3. \tag{3}$$

From (1), (2), (3) we obtain

$$\Delta\rho = \pi(2me^2/h^2r^2)^{3/2}(Z\chi/r)^{1/2}Q(3\cos^2\nu - 1). \tag{4}$$

The potential due to $\Delta\rho$ is that of a quadrupole moment ΔQ:

$$\Delta Q = 2\pi\int_0^\pi\int_0^\infty r^4(3\cos^2\theta-1)\Delta\rho\,\sin\theta d\theta dr$$
$$= (16\pi^2/5)(2me^2/h^2)^{3/2}QZ^{1/2}\int_0^\infty(\chi r)^{1/2}dr. \tag{5}$$

Upon substituting $r=(0.88534a_H/Z^{1/3})x$, where x is the Thomas-Fermi variable (a_H = Bohr radius), we obtain

$$\Delta Q = [2(1.7707)^{3/2}/5\pi]Q\int_0^\infty(\chi x)^{1/2}dx. \tag{5a}$$

We shall consider the case of a single valence electron; its radial wave function times r will be called v. The energy of interaction E_Q with the nuclear moment can be written:

$$E_Q = -AQ\int_0^\infty(v^2/r^3)dr, \tag{6}$$

where A is a constant. For the interaction $E_{\Delta Q}$ with the induced moment, the penetration of the electron inside the core leads to:

$$E_{\Delta Q} = \frac{2(1.7707)^{3/2}AQ}{5\pi}\int_0^\infty drv^2\left\{\frac{1}{r^3}\int_0^x(\chi x')^{1/2}dx' + r^2\int_x^\infty[(\chi x')^{1/2}/r'^5]dx'\right\}, \tag{7}$$

where $r'=(0.88534a_H/Z^{1/3})x'$, and the limit x of the x' integrals pertains to r. The difference in sign of E_Q and $E_{\Delta Q}$ reflects the fact that the electrons concentrate in the region where the potential due to the nuclear Q is positive, thus tending to compensate the effect of the nucleus. If we let $R=-E_{\Delta Q}/E_Q$, then Q is $1/(1-R)$ times the value previously obtained without the induced effect. We write

$$R = 0.2998\langle V\rangle/\langle 1/r^3\rangle, \tag{8}$$

where

$$\langle V\rangle = \int_0^\infty drv^2\left\{\frac{1}{r^3}\int_0^x(\chi x')^{1/2}dx' + r^2\int_x^\infty[(\chi x')^{1/2}/r'^5]dx'\right\}, \tag{9}$$

$$\langle 1/r^3\rangle = \int_0^\infty(v^2/r^3)dr, \tag{10}$$

with: $\int_0^\infty v^2dr = 1$.

Table I gives the values of R for eight elements. The values of $\langle V\rangle$ and $\langle 1/r^3\rangle$ are also listed, together with the quadrupole moments[1] as determined at present and the corrected values, in the cases where data are available. The valence electron functions were obtained by means of the Thomas-Fermi potential,

$$[(Z-1)\chi+1]e/r.$$

A more detailed discussion will be given in a forthcoming paper.

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[1] J. E. Mack, Rev. Mod. Phys. 22, 64 (1950).

Nuclear Magnetic Resonance for K^{39}

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NUCLEAR magnetic resonance has been found for K^{39} at 1.59 Mc in a field of 8000 gauss using a recording oscillating-detector spectrometer. The sample was 1 ml of saturated aqueous solution of KNO_2. The frequency of the K^{39} resonance was compared with the frequency of the N^{14} resonance in concentrated nitric acid by repeatedly substituting the samples in the oscillator coil without otherwise disturbing the apparatus. The frequencies repeated within the accuracy of a General Radio Type 620-A frequency meter. The result is

$$\nu(K^{39})/\nu(N^{14}) = 0.64580\pm0.00006.$$

Using the measurements of Proctor and Yu[1] on N^{14} in nitric acid, we find

$$\mu(K^{39})/\mu(H^1) = 0.13999\pm0.00002$$

with no diamagnetic correction or allowance for possible chemical shift. This value agrees with molecular beam measurements.[2]

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[1] W. G. Proctor and F. C. Yu, Phys. Rev. 77, 716 (1950).
[2] Kusch, Millman, and Rabi, Phys. Rev. 55, 1176 (1939).

Magnetic Moments of Odd Nuclei

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IT has been pointed out by Wangsness[1] that a certain regularity seems to be associated with the magnetic moments of odd nuclei differing by two neutrons. There are, however, certain difficulties with the proposed classification. Besides failing to explain the pair $_{47}Ag^{107,109}$ as mentioned by Wangsness, it fails also in the cases of $_{81}Tl^{203,205}$, $_{55}Cs^{133,135}$, and $_{55}Cs^{135,137}$, not to mention the case[2] of $_1H^{1,3}$. Moreover, by using the argument given by Wangsness as to the effect of the increase and decrease of electric charge density in the nucleus one should expect the inverse effect to take place by the addition of a pair of protons or an alpha-particle instead of two neutrons. This is confirmed by the two pairs $_0n^1-{}_2He^3$ and $_{78}Pt^{195}-{}_{80}Hg^{199}$, but stands in contradiction to the known cases of: $_{17}Cl^{37}-{}_{19}K^{39}$, $_{55}Cs^{137}-{}_{57}La^{139}$, $_{48}Cd^{113}-{}_{50}Sn^{115}$, $_{36}Kr^{83}-{}_{38}Sr^{87}$, $_{54}Xe^{131}-{}_{56}Ba^{135}$.

A simple and exclusive rule to summarize these data with the exception of the Cs isotopes is the following. An addition of either two protons or two neutrons to an odd nucleus, provided it leaves its spin unchanged, "pushes" its magnetic moment away from a line intermediate to the Schmidt lines.[3]

Except for the cases of $_1H^3$ and $_2He^3$ this actually means that the magnetic moments are pushed toward a better agreement with the naive one-particle model by the addition of such pairs.

Using the above rules inductively one may argue that the nuclei $(Z+2k, N+2m)$ should be closer to the Schmidt lines than the initial (Z, N) nucleus, provided that the spins of the initial,