TABLE I. Effect of the induced quadrupole moment.*

Element	Z	Orbital	R	$\langle V\rangle a_H{}^3$	$\langle 1/r^3\rangle a_H{}^3$	$Q/(10^{-24}\,\mathrm{cm}^2)$	$Q_{corr}/(10^{-24}\,\mathrm{cm}^2)$
B	5	$2p$	0.220	0.860	1.18	0.06^a, 0.03^b	0.077^a, 0.038^b
Al	13	$3p$	0.121	1.04	2.57	0.156	0.177
Sc	21	$3d$	0.452	1.89	1.26	——	——
Ga	31	$4p$	0.055	2.21	12.1	0.232^c, 0.146^d	0.245^c, 0.155^d
In	49	$5p$	0.038	1.66	13.2	1.144^e, 1.161^f	1.189^e, 1.207^f
Eu	63	$4f$	0.405	9.29	6.88	1.2^g, 2.5^h	2.0^g, 4.2^h
Lu	71	$5d$	0.125	4.52	10.9	5.9^i, 7.0^j	6.7^i 8.0^j
Ac	89	$6d$	0.116	1.28	3.33	——	——

* In the last two columns of this table the superscripts refer as follows to different sotopes: a B^{10}, b B^{11}, c Ga^{69}, d Ga^{71}, e In^{113}, f In^{115}, g Eu^{151}, h Eu^{153}, i Lu^{175}, j Lu^{176}.

where χ is the Thomas-Fermi function at a point in the electron cloud, r is the length of the vector from the nucleus to this point, and θ is the angle included by this vector and the axis of the nuclear quadrupole moment Q. The density of electrons ρ is $8\pi p^3/3h^3$. Let $\Delta\rho$ be the density due to the second term of (1). Thus,

$$\Delta\rho = 8\pi p_0{}^2\Delta p/h^3, \qquad (2)$$

where Δp is the change of momentum associated with the term containing Q, and p_0 would be the maximum momentum, p, for $Q=0$. We have

$$(p_0\Delta p)/m = e^2Q(3\cos^2\theta-1)/4r^3. \qquad (3)$$

From (1), (2), (3) we obtain

$$\Delta\rho = \pi(2me^2/h^2r^2)^{\frac{3}{2}}(Z\chi/r)^{\frac{1}{2}}Q(3\cos^2\nu-1). \qquad (4)$$

The potential due to $\Delta\rho$ is that of a quadrupole moment ΔQ:

$$\Delta Q = 2\pi\int_0^\pi\int_0^\infty r^4(3\cos^2\theta-1)\Delta\rho\,\sin\theta d\theta dr$$
$$= (16\pi^2/5)(2me^2/h^2)^{\frac{3}{2}}QZ^{\frac{1}{2}}\int_0^\infty(\chi r)^{\frac{1}{2}}dr. \qquad (5)$$

Upon substituting $r=(0.88534a_H/Z^{\frac{1}{3}})x$, where x is the Thomas-Fermi variable (a_H = Bohr radius), we obtain

$$\Delta Q = [2(1.7707)^{\frac{3}{2}}/5\pi]Q\int_0^\infty(\chi x)^{\frac{1}{2}}dx. \qquad (5a)$$

We shall consider the case of a single valence electron; its radial wave function times r will be called v. The energy of interaction E_Q with the nuclear moment can be written:

$$E_Q = -AQ\int_0^\infty(v^2/r^3)dr, \qquad (6)$$

where A is a constant. For the interaction $E_{\Delta Q}$ with the induced moment, the penetration of the electron inside the core leads to:

$$E_{\Delta Q} = \frac{2(1.7707)^{\frac{3}{2}}AQ}{5\pi}\int_0^\infty drv^2\left\{\frac{1}{r^3}\int_0^x(\chi x')^{\frac{1}{2}}dx' + r^2\int_x^\infty[(\chi x')^{\frac{1}{2}}/r'^5]dx'\right\}, \qquad (7)$$

where $r'=(0.88534a_H/Z^{\frac{1}{3}})x'$, and the limit x of the x' integrals pertains to r. The difference in sign of E_Q and $E_{\Delta Q}$ reflects the fact that the electrons concentrate in the region where the potential due to the nuclear Q is positive, thus tending to compensate the effect of the nucleus. If we let $R=-E_{\Delta Q}/E_Q$, then Q is $1/(1-R)$ times the value previously obtained without the induced effect. We write

$$R = 0.2998\langle V\rangle/\langle 1/r^3\rangle, \qquad (8)$$

where

$$\langle V\rangle = \int_0^\infty drv^2\left\{\frac{1}{r^3}\int_0^x(\chi x')^{\frac{1}{2}}dx' + r^2\int_x^\infty[(\chi x')^{\frac{1}{2}}/r'^5]dx'\right\}, \qquad (9)$$

$$\langle 1/r^3\rangle = \int_0^\infty(v^2/r^3)dr, \qquad (10)$$

with: $\int_0^\infty v^2dr=1$.

Table I gives the values of R for eight elements. The values of $\langle V\rangle$ and $\langle 1/r^3\rangle$ are also listed, together with the quadrupole moments[1] as determined at present and the corrected values, in the cases where data are available. The valence electron functions were obtained by means of the Thomas-Fermi potential,

$$[(Z-1)\chi+1]e/r.$$

A more detailed discussion will be given in a forthcoming paper.

It is a great pleasure to thank Professor Edward Teller, who suggested this problem, for many helpful discussions. I am also indebted to Drs. H. M. Foley and H. Snyder for stimulating discussions.

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Nuclear Magnetic Resonance for K^{39}

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NUCLEAR magnetic resonance has been found for K^{39} at 1.59 Mc in a field of 8000 gauss using a recording oscillating-detector spectrometer. The sample was 1 ml of saturated aqueous solution of KNO_2. The frequency of the K^{39} resonance was compared with the frequency of the N^{14} resonance in concentrated nitric acid by repeatedly substituting the samples in the oscillator coil without otherwise disturbing the apparatus. The frequencies repeated within the accuracy of a General Radio Type 620-A frequency meter. The result is

$$\nu(K^{39})/\nu(N^{14}) = 0.64580\pm0.00006.$$

Using the measurements of Proctor and Yu[1] on N^{14} in nitric acid, we find

$$\mu(K^{39})/\mu(H^1) = 0.13999\pm0.00002$$

with no diamagnetic correction or allowance for possible chemical shift. This value agrees with molecular beam measurements.[2]

* Work supported by the National Research Council of Canada.
[1] W. G. Proctor and F. C. Yu, Phys. Rev. 77, 716 (1950).
[2] Kusch, Millman, and Rabi, Phys. Rev. 55, 1176 (1939).

Magnetic Moments of Odd Nuclei

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IT has been pointed out by Wangsness[1] that a certain regularity seems to be associated with the magnetic moments of odd nuclei differing by two neutrons. There are, however, certain difficulties with the proposed classification. Besides failing to explain the pair ${}_{47}Ag^{107,109}$ as mentioned by Wangsness, it fails also in the cases of ${}_{81}Tl^{203,205}$, ${}_{55}Cs^{133,135}$, and ${}_{55}Cs^{135,137}$, not to mention the case[2] of ${}_1H^{1,3}$. Moreover, by using the argument given by Wangsness as to the effect of the increase and decrease of electric charge density in the nucleus one should expect the inverse effect to take place by the addition of a pair of protons or an alpha-particle instead of two neutrons. This is confirmed by the two pairs ${}_0n^1-{}_2He^3$ and ${}_{78}Pt^{195}-{}_{80}Hg^{199}$, but stands in contradiction to the known cases of: ${}_{17}Cl^{37}-{}_{19}K^{39}$, ${}_{55}Cs^{137}-{}_{57}La^{139}$, ${}_{48}Cd^{113}-{}_{50}Sn^{115}$, ${}_{36}Kr^{83}-{}_{38}Sr^{87}$, ${}_{54}Xe^{131}-{}_{56}Ba^{135}$.

A simple and exclusive rule to summarize these data with the exception of the Cs isotopes is the following. An addition of either two protons or two neutrons to an odd nucleus, provided it leaves its spin unchanged, "pushes" its magnetic moment away from a line intermediate to the Schmidt lines.[3]

Except for the cases of ${}_1H^3$ and ${}_2He^3$ this actually means that the magnetic moments are pushed toward a better agreement with the naive one-particle model by the addition of such pairs.

Using the above rules inductively one may argue that the nuclei $(Z+2k, N+2m)$ should be closer to the Schmidt lines than the initial (Z, N) nucleus, provided that the spins of the initial, as well as those of the intermediate ones, are the same. In fact a number of pairs, such as $_{21}Sc^{45}-_{23}V^{51}$, $_{31}Ga^{71}-_{35}Br^{81}$, $_{31}Ga^{71}-_{37}Rb^{87}$, $_{35}Br^{81}-_{37}Rb^{87}$, $_{35}Br^{79}-_{37}Rb^{87}$, probably show this regularity.

No explanation is offered for this empirical rule; it seems not to be directly connected with the magic numbers which unfortunately do not as yet make their appearance at all in the data of magnetic moments of nuclei.

I should like to thank Dr. D. C. Peaslee for helpful discussions.

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3 See, for instance, the graphs given in L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1948), p. 394, or L. W. Nordheim, Phys. Rev. **75**, 1894 (1949).

Natural Spread of the Conic Distribution of the Čerenkov Radiation

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A VISIBLE radiation with remarkable spatial asymmetry was first observed by Čerenkov (1934)[1] during his observations on high speed electrons penetrating transparent materials. Frank and Tamm[2] showed theoretically that the Čerenkov radiation is propagated along the generatrices of a cone that has the direction of the motion of the electron as the axis and a half-angle θ_0 given by

$$\cos\theta_0 = c/\kappa u. \quad (1)$$

u and c/κ are, respectively, the velocity of the fast electron and the phase velocity of the emitted light in the medium, and κ is the refractive index of the medium. They drew this conclusion by simply considering the enforced interference of coherent rays emitted along the electron path. In order to treat the problem in greater detail it is necessary to take into account the fact that the speed of an electron decreases by a small jump whenever a photon is emitted. With this modification we find that the spatial asymmetry of the radiation is given[3] by

$$I(\theta) = I(\theta_0)(\sin^2 y)/y^2, \quad (2)$$

where

$$y = 2\pi(L/\lambda)(c/u\kappa - \cos\theta). \quad (3)$$

λ is the wave-length of the radiation, θ the angle between the direction of the motion of the electron and the direction of the radiation, and L the length of a free path. In the Frank-Tamm theory the finiteness of L is overlooked. By assuming an infinite L we obtain a maximum at $\theta_0 = \cos^{-1}(c/u\kappa)$ as sharp as a δ-function. Since this will not be correct for a finite L, we have a half-breadth $\Delta\theta(\frac{1}{2})$ for the maximum at θ_0 determined by

$$\sin y = y/\sqrt{2}, \quad (4)$$

i.e.,

$$(2\pi\kappa L/\lambda)\sin\theta_0\Delta\theta(\tfrac{1}{2}) \simeq 160°. \quad (5)$$

We see that $\Delta\theta(\frac{1}{2})$ decreases remarkably with the increase of the free path. It is very interesting to notice that the natural width of the spatial distribution due to the finiteness of the free path of an electron shows quite an analogy to the natural width of a spectral line due to the finite lifetime of an excited atom. In order to estimate the half-breadth which should be actually observed we must substitute the mean free path L_{Av} of the electrons in the medium into (5). From the Frank-Tamm theory we obtain an estimate of $L_{Av} \simeq 10^{-3}$ cm in the case of a medium with $\kappa_{Av} = 1.5$, a transmission region extending from 2000A to 20,000A, and an electron beam with $\frac{1}{2}$ Mev kinetic energy. For the visible radiations the half-breadth is

$$\Delta\theta(\tfrac{1}{2}) \simeq 1°/(1-c^2/u^2\kappa^2)^{\frac{1}{2}} \simeq 2°. \quad (6)$$

This conservative estimate of $\Delta\theta(\frac{1}{2})$ serves only as a lower limit, since other radiative as well as non-radiative collisions[4] inevitably cause a shorter free path. Consequently, even if a carefully collimated homogeneous beam and a thin plate are used, we still have an observable spread of the visible radiation in its angular distribution.[5] For electrons much faster than those mentioned above ($u \sim c$) both the number of emissions per cm and $\sin\theta_0$ increase slowly with the energy of the incident beam, and so the change of $\Delta\theta(\frac{1}{2})$ from (6) must be very slight. On the other hand, when the velocity of the incident electrons is not far from the threshold c/κ the variation of $\sin\theta_0$, which appears in (5), becomes much greater than that of L_{Av}. The widening of the angular spread with the decreasing of the energy of the incident beam should be noticeable.

The half-breadth of the conic distribution of the Čerenkov radiation is in all cases of the order of several degrees. It is of the same order of magnitude that Ginsburg[6] found theoretically for the angle of separation between two cones of the Čerenkov radiation from a double refractive crystal. His prediction has never been verified, though in recent experiments[5,7,8] mica, with its optical axis in various orientations with respect to the incident beam, has been extensively used. We have made clear that each cone has its natural spread; they closely overlap and cannot be resolved. Probably the double-cone radiation can never be actually observed, even if substances with large deviation among the refractive indices, such as calcite, are used. Furthermore, we can now understand that even if an ideally collimated homogeneous beam and a thin plate of *high dispersive power* are used, no rainbow spectrum with red inside violet (see Eq. (1) and κ increases with frequency) should be observable. Each color has a natural spread of several degrees, and so a close overlap of different colors is inevitable.

In passing we should point out that the Frank and Tamm prediction of the vanishing of radiation when $u \leq c/\kappa$ is merely due to the incorrect assumption of an infinite free path. From Eq. (2) and a finite L we see that when $u < c/\kappa$, it is the distinct maximum of intensity that disappears but not the radiation. The total intensity decreases continuously with the electron velocity decreasing through the threshold c/κ.[9] The non-vanishing radiation intensity when $u < c/\kappa$, as well as the finite spread of radiation also can be easily understood by considering the Huygens construction of coherent rays[10] emitted along a *short* electron free path.

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Isomers and Shell Structure

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THERE are 59 genetically related isomers whose transitions can be classified into the $l=4$ or $l=5$ forbiddenness groups by lifetime-energy considerations.[1] The data that have appeared since the last classification was published[1] have not changed the general conclusions. While the reliable internal conversion measurements are consistent with l assignments, they do not independently define the l values. More internal conversion measurements could both check internal conversion and isomeric transition theory.

The simple spin-orbit coupling shell model, as presented by Maria Mayer,[2] makes specific spin and parity predictions for the