THE PHOTO-ELECTRIC CURRENT AS A FUNCTION OF THE ANGLE OF EMISSION AND THE THICKNESS OF THE EMITTING FILM.

BY WILLARD GARDNER.

RUTHERFORD[1] has observed that for a thin deposit of radioactive matter the distribution of the active discharge about the normal to the surface indicates a uniformity of emission at all angles from any given very small element of volume. Schmidt,[2] on the other hand, has taken into consideration the effect of absorption in the case of a thick film of active substance and finds a resultant effective intensity roughly proportional to the cosine of the angle of emergence of the active particles.

In the case of photo-electric emission, it would seem from the work of Stuhlman[3] and Robinson,[4] with thin metallic films, that the activity of the exciting light is not confined to the immediate surface of the metal, but that a finite thickness is effective in the production of photo-electrons. Ladenberg[5] concludes that the effective light penetrates to a depth of eight wave-lengths. In his Physical Optics, Wood has given figures indicating that throughout the ultra-violet region a large percentage of the incident light is absorbed by metals.

On account of their high velocities, the β and cathode particles are able to penetrate a considerable distance through absorbing media. For photo-electrons, Lenard[6] has recorded velocities ranging from 10^7 to 10^8 cm. per second from carbon, platinum, and aluminum. The penetrating power is thus much less than that of β and cathode particles, but no doubt the photo-electrons emerge from a finite thickness of metal and present a similar case to that of the emission from thick films of radioactive matter. The velocity distribution curves of Richardson and Compton,[7] considered in connection with their theory, would also indicate a falling off in velocity due to a penetration of varying thickness of material.

[1] Rad. Sub. and Their Rad., p. 312.
[2] Phys. Zeit., 9, p. 537, 1908.
[3] PHYS. REV., 32, p. 621, 1911; Phil. Mag., 20, p. 331; 22, p. 854.
[4] Phil. Mag., 23, p. 542, 1912; 25, p. 115, 1913.
[5] Ann. d. Physik, 12, 3 p. 573.
[6] Allen, Photo-Electricity, p. 38.
[7] Phil. Mag., Vol. 24, pp. 575–594 (1912).

Lenard[1] has observed a variation of intensity at different angles of emission. Hughes[2] concludes that the emission is the same at all angles, though his conclusion is an inference from an indirect observation. Robinson[3] has attempted to ascertain the approximate limiting angle (maximum angle from the normal) at which photo-electrons are emitted, in order to test the formula given by Riecke. Riecke assumes that within a given cone about the normal the intensity of emission is independent of the angle and that outside of this cone there is a sudden drop in the intensity. Robinson's observations indicate only an approximate verification of this assumption. He finds from his method that the limiting angle for zinc and silver is in the neighborhood of 30°, whereas, in order to satisfy the equation of Riecke, a much larger angle is demanded. It is quite evident, however, from the data recorded, that there is a falling off in intensity with increasing angle, although the investigation gives only approximate values and for a small angular range about the normal.

The present investigation was undertaken with the view of making a more accurate determination of the angular distribution of the photo-electric current from a small flat metal plate, with varying thickness of effective substance.

The accompanying sketch (Fig. 1) gives an outline of the experimental arrangement. A small silver plate A, connected with the negative ter-

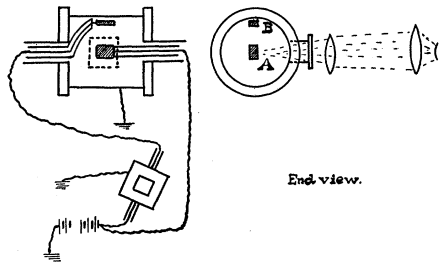

Fig. 1.

minal of a battery of cells of about 320 volts, was introduced through a ground glass joint at one end of a small metallic cylinder, through the other end of which was similarly introduced another small insulated electrode ("feeler") B, which was connected with the leaf of a Wilson tilted-leaf electroscope, and so adjusted as to rotate about A at a distance of approximately 4 cm. A beam of light from a Fuess mercury arc was

[1] Ann. d. Phys., 2, p. 359 (1900); 8, p. 159 (1902).
[2] Ann. d. Phys., 12, pp. 453, 777 (1903).
[3] Ann. d. Phys., 31, p. 769 (1910).

admitted through a quartz window at the side of the cylinder and allowed to fall normally upon the plate A. Before each observation electrode B was connected to earth and then insulated and the intensity of the electron stream at any given angle was determined by measuring the rate of fall of the electroscope leaf. The cylinder was connected to earth, and, as electrode B was made to rotate close to the walls of the cylinder, the initial field within was practically radial in the section containing the electrodes. The inside of the cylinder was coated with soot to avoid the photo-electric effect from reflected light.

For the work with thin films, a quartz plate was substituted for the metal and the films were deposited by the method of cathode sputtering with an apparatus designed by Prof. Lewis for silvering his interferometer plates. The films used were of a beautiful uniformity in appearance.

In Fig. 2 (a) is shown a curve representing the mean of a large number of observations for a thick plate at pressures ranging from 0.1

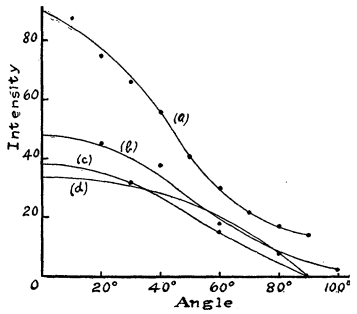

Fig. 2.

to 0.01 mm. of mercury. The precision with which the points fall upon the line is sufficient indication of the degree of accuracy or the mean values.

Curve (b), Fig. 2, represents the mean of several observations taken at one time, for pressures below 0.008 mm. Curve (c) represents the mean value of a series of five observations taken at an extremely high vacuum obtained by the use of liquid air. As judged by the length of spark gap in air in parallel with a gap through the vessel, the pressure was considerably below 0.001 mm. In fact, it was almost impossible to get a discharge to pass through the vacuum. The photo-electric current for these low values of the pressure was extremely small, but the results were perfectly definite. The probable error of the mean for each point was calculated and for the three points given was less than 8 per cent.,

the probable error of any single observation being from 10 to 14 per cent. Owing to the difficulty of maintaining extremely high vacuum over long intervals of time and to the fact that the currents were so extremely small, three points only were determined. The general form of the curve is shown by (*a*), and the particular purpose of the observations at low pressures was to verify the supposition that the finite ordinate in (*a*) at 90° was due to scattering. The relative values of the total current for varying pressures were found to harmonize very well with values reported by Varley.[1] For convenience in plotting, accurate relative values of the ordinates are not given. The intensities of the current for curve (*a*) were much greater in comparison with (*b*) and (*c*) than the plot would indicate.

It is true of course that the greater part of the current measured at the higher pressures was due to ions produced by the emitted electrons. Inasmuch, however, as the field was practically radial, it is to be expected that the direction of the resulting total current would correspond in general with the initial direction of the emitted electrons. Under the influence of a uniform radial field any modification of the character of the angular distribution that might occur would be toward uniformity at all angles, and the finite ordinate at 90° in the curves for higher pressures might be thus accounted for. It may also be observed that the slight departure from a uniform radial field, due to the manner of construction of the apparatus, would also account for a slight distortion, the field being somewhat greater in the neighborhood of 90°.

In Fig. 3 (*a*), is shown the result of a large number of observations taken with thin films of varying thickness, ranging from 20 to 200 $\mu\mu$, taken under conditions of pressure ranging from 0.1 to 0.01 mm. Accurate determination of the film thickness was not attempted. A number of films were weighed and the approximate thickness also determined by means of the spectrophotometer, a rough calibration of the sputtering apparatus being thus accomplished.

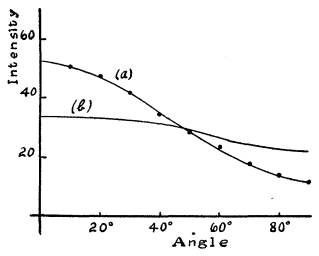

Fig. 3.

In order to explain upon theoretical grounds the results recorded above, it will be assumed that the electrons are emitted equally in all directions from the molecule or infinitesimal element. While the work of Stuhlman and Robinson[2] would seem to indicate that such may not be the case,

[1] Allen, Photo-Electricity, p. 58.

[2] Loc. cit.

yet, in the light of work reported by Hallwachs[1] showing corresponding asymmetry in the actual amount of light absorbed, it would seem that the matter is still an open question, and for the lack of a more definite knowledge regarding this point, it is perhaps not inappropriate to make this assumption as a working hypothesis.

It will also be assumed that the law of absorption of the electrons is logarithmic, as well as the law of absorption of the light.

It should be observed that the method of measurement used in this work gives the total current converging at the " feeler " from the entire volume of emitting substance, and the angular dimension of the converging beam varies with the position of the " feeler." The results reported do not, therefore, purport to be a measurement of what may be called a " parallel " beam in each case, but a measurement of the number of electrons converging at the " feeler " from the entire volume of emitting substance. Therefore, in the formation of the equation, it will be necessary to integrate over the entire volume of metal and not over a given solid angle at the " feeler."

It has been shown by various investigators, viz., Elster and Geitel, Lenard, Hermann and Richtmyer, and others, that for any given wavelength the number of electrons emitted is directly proportional to the intensity of the exciting light, although slight departure from this law has been noted by Ives.[2]

Let us, therefore, assume an equal emission in all directions from the molecule, a logarithmic absorption of the electrons with absorption coefficient k_1, a logarithmic absorption of the light with absorption coefficient k_2, and an actual number of electrons emitted from the volume element proportional to the volume and to the intensity of light within the volume, with proportionality factor C, *i. e.*, the number of electrons emitted per unit intensity of light per unit volume.

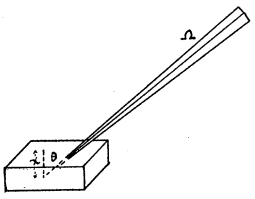

Fig. 4.

Then, if I_0 is the intensity of the light at the surface, the intensity I at a point at depth x in the metal is given by the equation,

$$I = I_0 e^{-k_2 x}.$$

The number of electrons n emitted per unit volume will be

$$n = CI_0 e^{-k_2 x},$$

and for the radiation from a volume element $dxdydz$ through a given solid angle Ω (see Fig. 4), we have,

[1] Allen, Photo-Electricity, p. 47.

[2] PHYS. REV., Vol. III., p. 68, 396 (1914).

$$dN = CI_0 \frac{\Omega}{4\pi} e^{-k_2 x} dxdydz,$$

Under the experimental conditions the angle Ω subtended by the "feeler" at the point of emission was practically constant over the entire volume and the only variable involved in the integration is the depth x measured normally into the metal.

The length of path traversed in the metal before emergence for a beam Ω traveling in a direction defined by the angle θ measured from the normal (azimuth angle = 0) is equal to $x \sec \theta$, and the number dN_θ emerging through Ω at an angle θ will be given as follows:

$$dN_\theta = CI_0 e^{-(k_1 \sec \theta + k_2)x} \frac{\Omega}{4\pi} dxdydz,$$

and the total for the entire plate becomes,

$$N_\theta = \int_0^{x2} CI_0 e^{-(k_1 \sec \theta + k_2)x} \frac{\Omega}{4\pi} dxdydz.$$

Writing,

$$CI_0 \int \frac{\Omega}{4\pi} dydz = A,$$

the integral becomes,

$$(A) \qquad N_\theta = \frac{A \cos \theta}{k_1 + k_2 \cos \theta} (1 - e^{-(k_1 \sec \theta + k_2) x_1}).$$

For a thick plate the exponential term vanishes, giving

$$N_\theta = \frac{A \cos \theta}{k_1 + k \cos \theta}$$

from the illuminated side of the plate. For thin films a similar development gives for the emission at the rear of the film,

$$(B) \qquad N_\theta = \frac{A \cos \theta}{k_2 \cos \theta - k_1} (e^{-k_1 x_1 \sec \theta} - e^{-k_2 x_1}).$$

In Fig. 2 (*d*) is represented the distribution based upon the equation given for a thick plate with an arbitrary selection of constants ($k_1 = 2$, $k_2 = 1$), and there is seen to be a fairly close agreement in the shape of the curve with the experimental curve for high vacuum, except that the point of inflection, though less prominent than in the curves for higher

pressures, still remains in the latter. Allowing for an experimental error of the magnitude noted above, this change of curvature may be made to disappear. Inasmuch, however, as this feature is present in all the curves, it may be significant in this case.

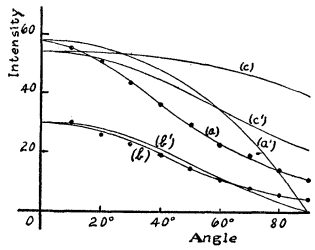

Fig. 5.

In Fig. 5 (a) is shown the experimental curve obtained with a thin film, the distribution measured on the illuminated side; (b) represents the distribution measured on the other side. Curve (c) shows the ratio of the ordinates of (b) and (a). Curves (a'), (b') and (c') represent the corresponding curves plotted from equations (A), (B) and the ratio $(B)/(A)$, where $k_1 = 2$, $k_2 = 1$, and $x_1 = 2$. As stated, the unit of intensity for each curve is arbitrary and the value of the ratio of the ordinates represents only approximately the actual experimental results.

It may be readily shown by forming the derivatives that the functions (A), (B) and $(B)/(A)$ are all diminishing functions of θ. The derivatives are somewhat cumbersome and are therefore omitted, but the curves (a'), (b') and (c') of Fig. 5 will serve to illustrate this point, and are seen to harmonize in general with the corresponding experimental curves.

If we grant that there is a greater tendency for emission from the molecule or volume element in the direction of the light, such a tendency would no doubt result in a relatively larger effect normal to the surface for the " emergent " electrons, but as observed, this is also to be expected from the equations given.

In this connection it may be noted that for the thinnest film used in this work a lack of symmetry in " incident " and " emergent " effect was not observed. For this case the total current was found to be about one per cent. higher for the " incident " effect. However, the total current was measured only in one case and the observation was merely incidental. In view of the fact that so much experimental evidence has been obtained showing that the velocity of emission of the photo-electrons depends entirely upon the frequency of the exciting light,[1] it is very difficult to conceive of any intrinsic influence other than the pressure of the light that would give higher velocities to the " emergent " electrons. It cannot, for example, be conceded that no electrons are emitted from the surface molecules in the direction opposed to the direction of the light, and if the initial velocity is dependent upon the frequency of the light alone, the ultimate potentials for the " incident " and the " emergent "

[1] Millikan, PHYS. REV., March, 1916, p. 355.

effect can differ only by the amount due to the pressure of the light, combined with causes due to the difference in the optical conditions for the " incident " and " emergent " effect.

As may be seen from the manner of the derivation of equations (A) and (B), the rate of change of the intensity of the photo-electric current with the thickness of the film involves the angle θ. Forming the derivative with respect to x_1 and then again taking the derivative with respect to θ, we have,

$$\frac{dN_\theta}{dx_1} = Ae^{-(k_1 \sec\theta + k_2)x_1},$$

and

$$\frac{d\,(dN_\theta)}{d\theta(dx_1)} = -Ak_1x_1 \sec\theta \tan\theta e^{-(k_1 \sec\theta + k_2)x_1}.$$

The change of N_θ with film thickness is seen to be a positive function of x_1 and θ, but the derivative of the latter function with respect to θ is a negative function of x_1 and θ, indicating that the intensity of the photo-electric current increases faster with the film thickness at small values of θ. We should expect, therefore, that the ratio of the ordinates for the thick plate to those for the thin film should decrease with θ. In Fig. 3 (b) is plotted the ratio of the ordinates of (a) Fig. 2 (thick plate) to those of (a) Fig. 3 (thin film), and the results are seen to be in perfect harmony with the equations.

In plotting the theoretical curves the constants and the film thickness are arbitrarily assumed and general qualitative agreement is all that can be expected. The results cannot be taken as conclusive evidence against the claims of Stuhlman and Robinson, but the qualitative agreement in the general shape of the calculated and experimental curves would seem to indicate that the fundamental assumptions made are at least not far from the truth. On the other hand, if there is an actual lack of symmetry in the emission from the molecule, we must await further experimental work before an accurate theory can be proposed.

It is interesting to note the form assumed by the function if we consider the intensity of light constant throughout the metal and take account only of a logarithmic absorption of the consequent radiation (corpuscular or otherwise), a case realized for incandescent substance emitting temperature radiation. In this case the function may be represented in the form of a converging series,

$$N_\theta = Ax_1\left(1 - \frac{1}{2}k_1x_1 \sec\theta + \frac{1}{3}\frac{k_1^2x_1^2 \sec^2\theta}{\underline{|2}} - \frac{1}{4}\frac{k_1^2x_1^2 \sec^2\theta}{\underline{|3}} + - \cdots\right),$$

which is seen to be independent of θ for small values of x_1. Again, if k_1 is also equal to zero, the resultant radiation converging at a point is independent of the angle from the surface normal.

In conclusion, I wish to thank Prof. E. P. Lewis, under whose direction the work has been carried on, for his many valuable suggestions.

PHYSICAL LABORATORY,
UNIVERSITY OF CALIFORNIA,
May, 1916.