A TEST FOR X-RAY REFRACTION MADE WITH MONOCHROMATIC RAYS.

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ONE of the best-known properties of X-rays, and the most important for their application to radiography, is their ability to penetrate all known substances without being refracted. This has been verified by Roentgen and others, and was for a long time accepted as a good argument for the pulse theory in a form involving the assumption that atoms contain no oscillators whose natural frequencies are those of X-rays. Since we now know that this assumption is incorrect, the reason for the absence of refraction is not evident. Chapman[1] seems to have been the first to appreciate this. He tried a prism of ethyl bromide vapor, using heterogeneous rays which included the characteristic frequencies of bromine, but obtained no refraction, although a refractive index of 1 ± .0003 could have been detected. Similar results were obtained in a more exact experiment on potassium bromide prism by Barkla,[2] who could have detected an index differing from 1 by only .000005.

These results show conclusively that rays of most frequencies are refracted very little, if at all. The appreciable refractive effect of a resonance frequency in optical spectra does not always extend far along the spectrum, however, and if this is true of X-rays, the refracted part of a heterogeneous beam might contain so small a part of the energy and be so widely dispersed as to escape detection. This paper presents an analysis showing in what part of the spectrum the refraction, if it exists, ought to be greatest; and showing that under reasonable conditions this greatest value should be extremely small. It then describes experiments made to test the theory. Monochromatic rays were used in these experiments. No evidence of refraction was found.

ANALYSIS OF CONDITIONS FOR REFRACTION.

As a starting point for this analysis we may take the formulas for the refraction and absorption of electromagnetic waves in matter given by Lorentz, Theory of Electrons, Chap. IV., which we may write in the form

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[1] Chapman, Proc. Camb. Phil. Soc., p. 574 (1912).

[2] Barkla, Phil. Mag., Apr., 1916.

$$\mu(n) = 1 + \sum_i \frac{2\pi N_i e_i^2 m_i (n_i^2 - n^2)}{m_i^2 (n_i^2 - n^2)^2 + n^2 g_i^2}, \tag{1}$$

$$k(n) = \frac{1}{c} \sum_i \frac{4\pi N_i e_i^2 n^2 g_i}{m_i^2 (n_i^2 - n^2)^2 + n^2 g_i^2}. \tag{2}$$

These differ in several respects from the forms given by Lorentz. They express the summation of effects due to different resonance frequencies. Ordinary electrostatic units are used. k represents the logarithmic decrement of intensity rather than of amplitude. $\mu(n)$ and $k(n)$ are the coefficients of refraction and absorption respectively at the "radian frequency" n, N_i the number of electrons with resonance at n_i, g_i their damping coefficient, e_i and m_i their charge and mass.[1]

These formulas give the results, familiar in the anomalous dispersion of light, that the strongest refraction occurs on the sides of the absorption bands, and that any resonance frequencies in or near the visible can have no great effect on X-rays. This latter result is confirmed by Chapman's experiments and those of Barkla.

To apply these formulas to an actual case we must make use of the resonance frequencies given by the absorption spectrum (not the fluorescent emission spectrum), and from them predict the refraction. This prediction, which would seem at first sight to involve many doubtful assumptions, may be made without ambiguity by the use of the approximate equation,

$$\mu(n) - 1 = \frac{c}{\pi} \int_0^\infty \frac{k(n_j)\,dn_j}{n_j^2 - n^2}, \tag{3}$$

which connects the values of μ and k given in (1) and (2).

To prove this formula, the value of $k(n_j)$ may be taken from equation (2) and in each term of the sum the variable of integration may be transformed to $x = n_j^2 - n_i^2$. The approximations used in the integration are as follows: (*a*) n is substituted for a factor n_j (to the first power) in the numerator of the integrand; (*b*) n^2 is substituted for the n_j^2 appearing as a coefficient of g_i in the denominator; (*c*) the lower limit of the integration with respect to x is shifted from $-n_i^2$ to $-\infty$. The errors introduced by the first two approximations cannot readily be evaluated, but are certainly small unless g_i is large. This is shown by the fact that if n_i is substituted, instead of n, in each case, the result will be only a similar change in the g_i term in the value of $\mu(n)$. The error

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[1] Ot course the conditions of the X-ray problem do not permit a rigorous use of Lorentz's "physically infinitesimal" elements of volume and the analysis based on them, but since the retardation and absorption per wave length are very small, it seems improbable that the rays are deformed by single atoms enough to make any error in this analysis in other problems than those of scattering.

introduced by (*c*) consists in dropping some small additive terms, all of which contain $g_i/m_i n$ as a factor. Therefore it also is very small if the influence of any one resonance frequency does not extend far along the spectrum.[1]

For the present case, using the results of Duane,[2] Siegbahn,[3] Bragg,[4] and Hull and Rice[5] we may describe the X-ray absorption spectrum of any element by the formula

$$k(n) = A_M\left(\frac{n_M}{n}\right)^3 + A_L\left(\frac{n_L}{n}\right)^3 + A_K\left(\frac{n_K}{n}\right)^3. \tag{4}$$

A_K shall be zero at all frequencies below n_K, that of the eight-fold rise in $k\ (n)$ at the K series of the absorbing substance, and constant for all frequencies well above this. A_L is similarly defined with reference to the four-fold rise at the L series, while A_M is constant for all ordinary frequencies. This form of representation is derived from the fact that each rise is closely related to the phenomena of fluorescence of the corresponding series. Since it is improbable that there is any actual discontinuity in nature, we shall assume the rise in A_K to be continuous through a very short range n_K to n_K'. The exact value of n_K for the case of rhodium appears from our preliminary photographs to be that of the $K\gamma$ line itself; and the rise occurs so suddenly that no definite value can be assigned to the difference $n_K' - n_K$, but it is probably not much over 0.001 n_K and may be very much less. Wagner[6] obtained similar results using palladium and other metals.

Applying (3) to this spectrum, and considering only the refraction due to oscillators associated with the K series, and neglecting the fraction $(n_K' - n_K)/(n - n_K)$ we derive the following expression:

$$\{\mu(n) - 1\}_K = -\frac{A_K\lambda_K}{4\pi^2}\left\{\frac{n_K^2}{n^2} + \frac{n_K^4}{n^4}\log\left|1 - \frac{n^2}{n_K^2}\right|\right\}. \tag{5}$$

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[1] It is interesting to notice that we can obtain some information about these quantities for the case of a continuous spectrum containing oscillators resonating to any frequency by substituting an integration for the summation in (1) with $f(n_i)dn_i$ for N_i, and obtaining the approximate equation

$$k(n) = \frac{2\pi^2 e^2}{cm} f(n),$$

which is independent of g_i. This integration, however, involves the approximation (*c*) of (3) and also the assumption that e_i, m_i, g_i, and $f(n_i)$ do not vary much within the region of the spectrum absorbed by oscillators of any one resonance frequency. This equation, combined with an approximate form of (*c*), in which $g_i = 0$, gives (3), but with these unnecessary restrictions.

[2] Duane, Amer. Phys. Soc., Dec., 1914, stating results of calculation from absorption measurements by Barkla and Sadler and wave-length measurements by Moseley.

[3] Siegbahn, Phys. Zeitschr., Aug. 15, 1914.

[4] Bragg, Phil. Mag., March, 1915.

[5] Hall and Rice, PHYS. REV., Sept. 1916.

[6] Wagner, Ann. d. Phys., March, 1915, p. 868.

Now since $A_K\lambda_K$ is the increase at λ_K of the fraction of the rays absorbed in going a single wave-length, it is very small. For rhodium, for example, it is about 3.5×10^{-5}. Consequently the refraction cannot be measurable unless n has a value so near that of n_K that the logarithm is very large. For such a value we cannot neglect the difference $n_K' - n_K$. Letting $p = (n_K' - n_K)/n_K$ and $q = (n - n_K)/n_K$, and neglecting all terms but the logarithms, we find

$$\{\mu(n) - 1\}_K = -\frac{A_K\lambda_K}{4\pi^2}\left\{\frac{p-q}{p}\log|p-q| + \frac{q}{p}\log|q|\right\}. \qquad (6)$$

This has a maximum at the middle of the range n_K to n_K', and is nearly equal to

$$\frac{A_K\lambda_K}{4\pi^2}\log\frac{1}{p}$$

throughout this range. In this part of the spectrum the terms due to the L and M series may be shown by a brief calculation, confirmed by Barkla's experiment, to be very small. Consequently greatest refraction ought to be found at the discontinuity in the absorption, that is, at the $K\gamma$ line of the refracting substance. It can scarcely be measurable, however, unless the value of p is extraordinarily small, or unless this theory has a fundamental defect. We must therefore expect a negative result in the following test for refraction.

Experimental Test for Refraction.

In these experiments we used the rays from the rhodium target of a Coolidge tube operated by a storage battery at about 40,000 volts. This voltage was not high enough to give general radiation of such frequency that its second-order spectrum could confuse the results.[1] The spectrum was obtained by reflection from the (100) face of a crystal of calcite. A small rectangular cell of brass, in which a prism of rhodium was mounted as shown in Fig. 1, was placed in the path of the reflected rays and behind this at some distance was placed a photographic plate. The brass cell being opaque to the rays cast a shadow upon the plate. It was not, however, sufficiently wide to obscure the rays completely, so the photographs show the ends of each spectrum line projecting above and below the shadow.

Fig. 2 shows the general appearance of a photograph with the projecting ends of the spectrum lines at a, a. The prism faces were carefully ground and were plane down to within a very short distance from their

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[1] Duane & Hunt, Amer. Phys. Soc., April, 1915, Phys. Rev., Aug., 1915. Hull, Amer. Phys. Soc., Nov., 1915, Phys. Rev., Jan., 1916. D. L. Webster, Proc. Nat. Acad., Feb., 1915, Phys. Rev., June, 1916, p. 599.

line of intersection. The cell was so adjusted that no rays were allowed to reach the prism[1] at a point where it was less than .06 mm. thick. At this place the faces were nicely plane. This limiting position will be referred to as the *working edge* of the prism.

Although rays fell on nearly all parts of the prism, very few were transmitted save near its *working edge*. With a properly timed exposure only a very narrow line was recorded opposite the prism, as shown at *b*, Fig. 2, and so it was unnecessary to use any slits to determine the width of the transmitted ray. Since an increase in the time of exposure would have made the line broader, it was faintly recorded on the plate at best. For this reason it would have been difficult to reproduce the photographs themselves in print.

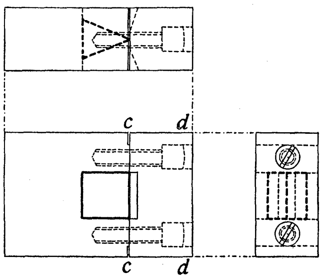

Fig. 1.

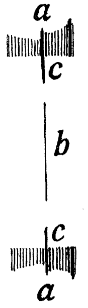

Fig. 2.

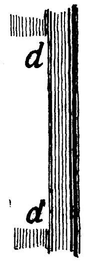

Fig. 3.

A pair of slots, *c*, *c*, Fig. 1, about .1 mm. wide were cut in the brass cell so that one edge of each slot was in line with the *working edge* of the prism. The rays which came through these slots fell upon the plate, as shown at *c*, *c*, Fig. 2. These marks at *c*, *c* showed by comparison with *a*, *a* just what part of the spectrum had been transmitted, and by their alignment with line *b*, whether or not the transmitted rays had been refracted.

To test the regularity of the reflecting crystal many of the photographs taken as described above were followed by others in which the cell was withdrawn toward the left by a micrometer screw through such a

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[1] Barkla (note 2 above) worked with potassium bromide prisms the faces of which were cleavage planes. That such surfaces are more perfectly smooth than ground ones is not quite evident, for these crystals are usually very irregular. It should be noted, however, that a very smooth surface is not essential when working with X-rays. A surface is generally said to be *optically polished* when its irregularities are small compared to the wave-length of light. To speak strictly, however, it is not the surface of the substance, but rather the transmitted or reflected *wave* whose irregularities must be small. The refractive effect of any substance for X-rays, if it exist, is certainly very small and as a result the waves are but slightly distorted by comparatively great irregularities of the surfaces through which they are transmitted.

distance that the edge *d*, *d* (Fig. 1) stood exactly in the position formerly occupied by the working edge of the prism. The result is indicated in Fig. 3. The edge *d*, *d* of the shadow of the cell was found to be straight in all cases.

The sharpness of the photographs makes it reasonable to assume that a deviation of the transmitted ray amounting to as little as .2 mm. or even .1 mm. could have been detected. With the plate 80 cm. from the prism, as we used it in most of the trials, this would correspond to a refractive index of 1.0004 or 1.0002. Many trials were made, using various parts of the spectrum, including several parts of the γ line, the β line, both members of the α doublet, and the general radiation between β and γ. In none of these was there any trace of refraction.

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