

AN EXPERIMENTAL VERIFICATION OF THE LAW OF VARIATION OF MASS WITH VELOCITY FOR CATHODE RAYS.

BY LLOYD T. JONES.

INTRODUCTION.

THE mass of the electron has been shown by W. Kaufmann¹ to be electromagnetic in origin. This mass is shown by the elementary laws of electromagnetism to be constant for small velocities of the electron. M. Abraham² has developed an electro-dynamic theory of moving electrons by which he accounts for the falling off of the ratio e/m for electrons moving with high velocities. If β is the ratio of the velocity of the electron to that of light, the ratio of the mass of the electron moving with the velocity v to its mass, m_0 , when moving with a slow velocity is

$$\frac{m}{m_0} = \frac{3}{4\beta^2} \left[\frac{1 + \beta^2}{2\beta} \log \frac{1 + \beta}{1 - \beta} - 1 \right].$$

The Lorentz-Einstein formula, which satisfies the principle of relativity, gives the ratio of the masses as

$$\frac{m}{m_0} = [1 - \beta^2]^{-\frac{1}{2}}.$$

Kunz³ has discussed the bearing of these formulæ in connection with an electromagnetic emission theory of light and has developed three forms of the formula based on possible changes of form of the electron.

Stark⁴ has found that the mass of the cathode particle increases as the velocity increases. The maximum velocity employed by him, 1.14×10^{10} cm. per sec., was, however, not great enough to cause more than a small per cent. increase in the mass.

Later Guye and Ratnosky⁵ carried out an experiment employing rays of 14.7×10^{10} cm. per sec. velocity and obtained an increase of nearly twenty per cent. in the mass.

Each of the investigators has employed a method in which the cathode

¹ W. Kaufmann, *Gott. Nachr.*, 1901, Heft 2; 1902, Heft 5; *Phys. Zeitschr.*, 4, 54, 1902.

² M. Abraham, *Gott. Nachr.*, 1902, Heft. 1.

³ J. Kunz, *Arch. des Sci.*, Jan. 1913; *PHYS. REV.*, p. 464, 1914.

⁴ H. Stark, *Verh. d. Deut. Phys. Gesell.*, 5, p. 241, 1903.

⁵ Guye and Ratnosky, *Arch. des Sci.*, 31, p. 293, 1911. Guye and Lavanchy, *Comptes Rendus*, p. 52, July, 1915.

beam traverses nonuniform electric and magnetic fields and the deflection is shown on a phosphorescent screen placed perpendicular to the path. This necessitated a homogeneous cathode beam.

The conclusions of the experimenter must be based on a large number of observations taken at each of a number of different velocities. The method that has been developed for the present research lessens materially the difficulties encountered in a verification by cathode rays and is applicable equally well for the β particles of radium. Perhaps the best feature of the method is that it is desired to have rays of all possible velocities present in the discharge rather than a homogeneous beam. This allows one to use the discharge from a high potential transformer without any additional pieces of apparatus to operate during the time of exposure. Since from a single photograph calculations may be made of e/m for all the velocities present it is possible to obtain the desired results by a single exposure.

THE APPARATUS.

In a previous determination of e/m and v for cathode rays¹ an apparatus was used involving the same principles as this; the high discharge potentials used in the present investigation, however, necessitated a change in the manner of introducing the electrodes and more effective insulation guarding against the ionizing and direct effect of the discharge.

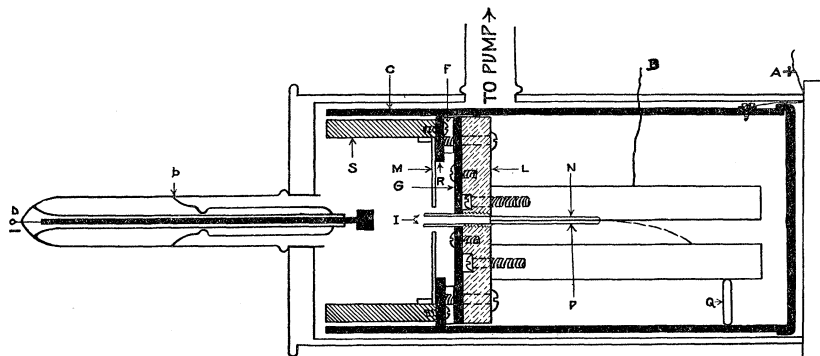


Fig. 1.

A glass jar 11.5 cm. in diameter and about 35 cm. long had a 2.2 cm. hole bored in its base through which the cathode was introduced. The cathode was an aluminum disc about .8 cm. in diameter carried on a small brass rod encased in a small glass tube and connected with one terminal of the transformer through a platinum wire sealed in the glass.

¹L. T. Jones, *PHYS. REV.*, N.S., Vol. III., p. 317, 1914.

The glass tube encasing the cathode rod was supported at two places by a second glass tube sealed, as shown at *b* in Fig. 1, to a tube of 2 cm. diameter which passed through the hole in the base of the jar. This tube was fastened to the base by sealing wax. A brass cylinder, *C*, of 10.2 cm. diameter and about 32 cm. long was fastened rigidly (manner not shown) to the glass jar; and a brass ring, *R*, of 1.5 cm. width and .4 cm. thickness was soldered inside it with its plane perpendicular to the axis of the brass cylinder. An iron cylinder, *S*, of 7.5 cm. inside diameter and .8 cm. thickness was fastened by screws to the ring *R*. An ebonite ring, *F*, of nearly the same dimensions as *R* was fastened to *R* by screws whose heads were sunk well beneath the surface next *G*. *G* was a brass disc of .3 cm. thickness fastened by brass screws to the ebonite disc, *L*, which was of .8 cm. thickness and carried the electrostatic plates. To increase the insulation discs of mica were placed between *G* and *L*, *F* and *G* and *R* and *F*. To prevent trouble due to the heavy discharge a brass disc, *M*, was held against *R* by a slip ring in *S*. A few millimeters' space was left around the small iron tube, *I*, which passed through directly in front of the cathode.

The two electrostatic plates were brass plates $20.5 \times 7.5 \times 1.2$ cm. In the upper one was inlaid a piece of soft iron, *N*, $5.13 \times 1.4 \times .1$ cm. A similar piece of iron, *P*, was held against *N* by eight short iron screws. After the iron piece, *N*, was inlaid and all necessary holes had been made in the plates they were annealed and then one side of each was surfaced to within .001 cm. of plane. The slip *P* also had its face next *N* made plane. A scratch of about .05 cm. width was drawn full length on the plane side of *P*. The ends of this scratch, for about 1 mm. of their length, were closed with solder and the solder cut off flush with the surface. A small cut was then made in each of the bits of solder and these cuts determined the path of the electron immediately before its entrance into the deflecting fields. The electron then takes the path indicated by the dotted line in Fig. 1. The electron is thus protected from the fields until it leaves the constricting canal. Care was taken that the small cut marking the entrance of the electron in the fields was perfect to the ends of *N* and *P* and that the ends of *N* and *P* were exactly even. As a final precaution a small bit of solder was placed in the middle of the canal as well and a small cut made in it. This insured a straight beam through the tube. Each of these cuts was .01 cm. in width and of about the same depth. The softest iron obtainable was used throughout and the brass was free of magnetic material.

The ebonite disc, *L*, with its plate, *G*, was held against the ring, *F*, by four heavy brass screws threaded into *R*. They were insulated from *G* by an air space of about 2 mm.

The two electrostatic plates were held at a fixed distance apart by four porcelain blocks placed one at each corner. Under the back end of the lower plate was placed a brass leg, Q , of adjustable height which served as an additional support for the plates and at the same time connected the lower plate electrically with the brass cylinder, and through it with M and S .

A hole 2.2 cm. in diameter was bored in the side of the glass jar at a suitable position and a glass tube waxed to the jar here connected the vessel to the molecular pump. A wire, A , was connected to the brass cylinder near the back where it could be easily reached from the outside. A was connected to earth and to the second terminal of the transformer. The surfaces of M and S served as anode. The upper of the electrostatic plates was connected electrically to the outside by a wire, B , passing through an insulating plug in the brass cylinder and through a small hole in the glass cylinder. The hole was made vacuum tight by sealing wax.

The photographic chamber was made light tight by closing the ends of the cylinder with a brass cap and the jar was made vacuum tight by closing with a glass plate sealed on with a mixture of beeswax and resin.

The electrostatic potential was applied to A and B and the transformer connected to A and D .

THE SPACING BLOCKS.

Each of the four spacing blocks placed between the corners of the electrostatic plates was a length of a porcelain tube of 1.2 cm. external diameter. After the sections had been cut from the tube they were waxed inside a short piece of brass tubing whose outside was accurately round so it could be chucked in the grinding machine. They were ground down till they were of nearly the same length and the end planes parallel. They were then finished by hand on an iron plate with emery till they were very accurately the same length and the end planes parallel as the measurements showed. The length of the blocks was measured by an optical lever of 24.415 cm. length with a scale 3 meters distant. The lengths of the blocks were $.9822 \pm .0008$ cm.

THE ELECTROSTATIC POTENTIAL.

A high potential storage battery, T , was used to send a small current through two high resistances, M and R , shown in Fig. 2. M consisted of two Wolff boxes aggregating 2×10^6 ohms and R was an adjustable resistance of 10^4 ohms. The potential drop across a part of M was compared by means of the potentiometer, P , with a Weston standard cell of 1.0183 volts at 24° C. The standard cell checked with one recently received

from the Bureau of Standards. By adjusting R the value was easily kept constant to within 1 volt and the value thus determined to less than .1 per cent. The two electrostatic plates were connected directly to the terminals of M .

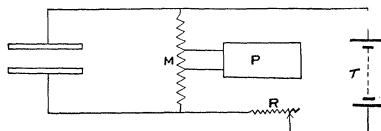


Fig. 2.

THE MAGNETIC FIELD.

The magnetic field was due to a constant current through 240 turns of wire wound on a rectangular wooden frame about 160×60 cm. The cross-section of the coil of wire was about 2×2 cm. Calculation showed the field to be uniform over a range greater than that used.

The field was calibrated by the aid of a solenoid of 1,141 turns and 149.83 cm. length wound uniformly on a wooden frame of about 6×9 cm. cross-section. The solenoid was placed in the geometrical center of the rectangular frame so that the fields either coincided or opposed each other. A small coil of about 200 turns of very fine wire wound on an ebonite rectangle 2×8 cm. was then placed in the center of the solenoid. This coil was connected to a Grassot fluxmeter whose scale was about 4 meters distant. A known constant current was sent through the coil to be calibrated and the current through the solenoid adjusted until the fluxmeter showed no deflection when the two currents were broken simultaneously. The ratio of the currents, 70 to 13.55, gave the value of the field to be 1.854 gauss per ampere. A field of .002 gauss produced a deflection of .3 mm. on the fluxmeter scale.

THE MEASUREMENT OF THE CURRENT.

The current in the magnetic field was measured with a Siemens & Halske ammeter of 150 scale divisions which, with the shunt used, had a range of 0 to 3 amperes. The ammeter was calibrated by passing a current through it in series with two Hartmann & Braun standard resistances of .1 and 1 ohm. The potential drop across each resistance was measured by the Wolff potentiometer against the Weston standard cell and the current calculated. The standard resistances were kept in an oil bath at constant temperature. The Reichsanstalt certificates showed the resistances to be sufficiently correct. The calibrations by the two resistances checked. Throughout the calibrations and experiment an adjustable resistance was used to set the ammeter needle exactly on a

scale mark in order that any variation in the current could be more easily detected. With special care taken for good contacts little difficulty was experienced in keeping the ammeter needle exactly on the division mark.

THE VACUUM.

The Gaede molecular pump, with the Gaede mercury pump as a preliminary, was used for the exhaustion. The molecular pump was connected by 30 mm. tubing directly to the vessel to be exhausted with no stopcock or other constriction intervening. The mercury pump was connected to a McLeod gauge and a large tube of cocoanut charcoal. The order of starting the pumps assures freedom of mercury vapor in the discharge tube. The construction of the apparatus with its sealing-wax joints made it quite impossible to heat the vessel to rid it of moisture. Such a proceeding proved unnecessary with the wide connecting tubes used, however, as an hour of pumping was usually sufficient to produce a vacuum that caused the transformer to spark 20 cm. between its point terminals rather than pass through the discharge tube. This degree of rarefaction was usually produced without the aid of liquid air on the charcoal. To be sure the equivalent spark length of the tube always dropped a few centimeters during the time of discharge, but a half or at most one minute of pumping was sufficient to restore the vacuum.

It may be of interest to some users of the molecular pump to know that considerable trouble was experienced with the pump due to the creeping in of oil from the bearings. Once in about six weeks the pump became stiff and the half H. P. motor was unable to drive it at the normal speed used, 8,000 R. P. M. The oil was then taken from the bearings and the whole pump thoroughly washed with filtered gasoline and dried by drawing air through it. This operation usually required three days.

THE ELECTRIC DISCHARGE.

The transformer used to produce the cathode beam was one built for the ratio 110-40,000 volts operating on a 60-cycle circuit. For a number of photographs it was used on a 440-volt 60-cycle circuit. The rays thus produced were of rather a slow velocity although the vacuum was so high that the transformer sparked across a 20 cm. gap between points outside. In order to lessen the amount of energy used and still retain the potential the transformer was operated on 110-volts D.C. with a Wehnelt interrupter. This arrangement, with or without a capacity across the interrupter, gave rays of a much higher velocity. Under these conditions, however, the equivalent spark gap of the vacuum was only about 8 to 12 cm.

THE FORMULA.

The beam passes through uniform electrostatic and magnetic fields, whose lines are parallel to each other, and strikes the photographic plate which is lying on the lower electrostatic plate.

Let the particle be subjected to the simultaneous action of the electric and magnetic fields. The particle will be bent downward by the electric field and strike the photographic plate at a distance l (measured along the direction of the undeflected beam) from the source. It will at the same time be bent aside by the magnetic field a distance z (measured at right angles to l). Since many velocities are present they will show themselves in a long trace on the photographic plate and e/m may be calculated for any point in the trace and hence for that velocity. If the electrostatic field alone acts the resultant trace will be straight down the center of the plate. If the magnetic field also acts, then for each value of the current a trace will appear at the side and, when the current is reversed, a similar trace on the opposite side and at nearly the same distance from the center one. In photograph 58, Plate I., two values of the current were used which, with the central magnetically undeflected exposure, make five traces on the plate. The magnetic deflection, z , was taken as half the distance between two corresponding points of corresponding traces. The electrostatic plates were mounted horizontally. Each particle then describes an arc of a parabola in the vertical plane and an arc of a circle in the horizontal plane.

THE ELECTROSTATIC DEFLECTION.

Let d be the distance from the upper electrostatic plate to the upper surface of the photographic plate and let t be the thickness of the photographic plate, Fig. 3. If K is the dielectric constant of the photographic

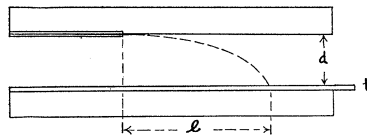


Fig. 3.

plate and V the potential difference in volts of the two electrostatic plates the force on unit charge due to the electric field is

$$E = \frac{V \times 10^8}{d + t/K}. \quad (1)$$

This force produces a downward acceleration of the electron such that

$$Ee = ma, \quad (2)$$

where e is the charge, m the mass and a the acceleration of the electron. If t_1 is the time required for the electron to fall to the photographic plate we shall have

$$vt_1 = \sqrt{l^2 + z^2}, \tag{3}$$

where v is the velocity of the electron. Equation (3) is true, since z is small enough in comparison with l that the chord may be considered equal in length to the arc. Then

$$t_1^2 = \frac{l^2 + z^2}{v^2}, \tag{4}$$

and, since

$$d = \frac{1}{2}at_1^2, \tag{5}$$

we get

$$d = \frac{Ee}{2m} \frac{l^2 + z^2}{v^2}. \tag{6}$$

Substituting the value of E from equation (1) and rearranging we have

$$\frac{mv^2}{e} = \frac{V(l^2 + z^2)10^8}{2d(d + t/K)}. \tag{7}$$

THE MAGNETIC DEFLECTION.

If the plane of the photographic plate be considered as in the plane of this page with the source of cathode rays at the origin, O , of the set of axes shown in Fig. 4, the arc of the circle shown will be the projection on the photographic plate of the electron's path and will show accurately the curvature of the path due to the magnetic field. Let z be the magnetic deflections, measured as previously mentioned, and let l again be the x distance to where some electron of velocity v strikes the plate. If r is the radius of curvature of the circular path due to the magnetic field, the length of the projection on the photographic plate of the actual path, or the arc shown, will be

$$r\theta = vt_1, \tag{8}$$

where θ is the angle at the center of the circle subtended by the arc.

From Fig. 4 we see that

$$\tan \frac{\theta}{2} = \frac{z}{l} \tag{9}$$

and that

$$\tan \theta = \frac{l}{r - z}. \tag{10}$$

But

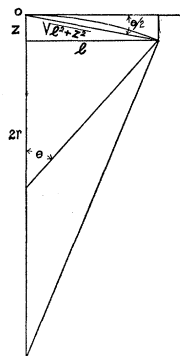


Fig. 4.

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \quad (11)$$

and from Fig. 4

$$\cos \theta = \frac{r - z}{r}. \quad (12)$$

Then

$$\frac{z}{l} = \sqrt{\frac{z}{2r - z}} \quad (13)$$

and

$$\frac{z^2}{l^2} = \frac{z}{2r - z}, \quad (14)$$

whence

$$r = \frac{l^2 + z^2}{2z}. \quad (15)$$

The magnetic force on the particle, due to the field H , is perpendicular to the direction of motion of the particle and hence has only its component, $Hev \cos \theta$, in the y direction. Since only this component produces the deflection z , we shall find the average force, \bar{f} , on the particle and use this value for the magnetic force.

$$\bar{f} = \frac{Hev \int^{\theta} \cos \theta d\theta}{\theta} = Hev \frac{\sin \theta}{\theta}. \quad (16)$$

This force gives the electron an acceleration a_1 in the y direction. Then

$$\bar{f} = ma_1 = Hev \frac{\sin \theta}{\theta}, \quad (17)$$

and

$$z = \frac{1}{2}a_1 t_1^2. \quad (18)$$

From equations (8) and (18) we have

$$t_1^2 = \frac{r^2 \theta^2}{v^2}, \quad (19)$$

and from (17)

$$a_1 = \frac{e}{m} Hv \frac{\sin \theta}{\theta}. \quad (20)$$

On substitution of the values from (19) and (20) in equation (18) we get

$$z = \frac{e}{m} \frac{Hr^2 \theta \sin \theta}{2v}. \quad (21)$$

From Fig. 4

$$\sin \theta = \frac{l}{r}, \quad (22)$$

and, with an approximation,

$$r\theta = vt_1 = \sqrt{l^2 + z^2}. \quad (23)$$

Substituting these values in (21) we find

$$z = \frac{e}{mv} \frac{H}{2} \sqrt{l^2 + z^2} \quad (24)$$

or

$$\frac{e}{mv} = \frac{2z}{H\sqrt{l^2 + z^2}}. \quad (25)$$

From equations (7) and (25) we obtain the desired expressions,

$$v = \frac{zV\sqrt{l^2 + z^2} \times 10^8}{Hld(d + t/K)} \quad (26)$$

and

$$\frac{e}{m} = \frac{z^2 V^2 \times 10^8}{H^2 l^2 d(d + t/K)}. \quad (27)$$

For any single photograph taken with constant deflecting fields equation (27) may be written in the form

$$z = C\sqrt{e/m}, \quad (28)$$

where C is a constant.

This equation shows the traces to be straight lines for constant values of e/m and that the outer traces should curve toward the central one for the higher velocities. From the way e/m enters the equation one would expect only slight curvature of the traces unless e/m diminished very rapidly. The equation shows that only the ratio z/l or the slope of the straight lines need be obtained from the photographic plates. This method is thus made one of particular value for the determination of e/m_0 , for slow velocities, as it permits easy averaging of values.

THE EARTH'S FIELD.

In fastening the apparatus to the stone pier it was carefully placed so that the undeflected beam travelled horizontally in the direction of the earth's field. The effect of the vertical component of the earth's field was then to increase the one deflection of the magnetic field and to lessen the deflection when the current was reversed. From an inspection of the equation it is seen that the effect of this vertical component may be neglected as it cancels due to the method used in measuring z .

The beam of electrons travelled from north to south so that when only the electrostatic field bends it downward it cuts the horizontal component of the earth's field at a small angle. The central trace is thus thrown a little to one side.

Let H_1 be the value of the horizontal component of the earth's field.

When the electron is deflected magnetically it has a component velocity at right angles to the magnetic field H_1 and therefore has a small force acting on it. This force will aid or oppose the force of the electrostatic field depending on the direction of the magnetic deflection. This small force due to H_1 was found to be negligible.

It follows then that a small error made in placing the apparatus such that the beam would travel neither quite horizontally nor exactly in the magnetic meridian would have no appreciable effect on the results of the experiment.

The dielectric constant, K , of the photographic plate was taken as 6. Since the plate is in contact with the lower electrostatic plate and the electrostatic field is on for ten to thirty minutes before the exposures are made the value of the constant chosen must not be that obtained by a method not allowing for the accumulation of a charge by the glass. It should be pointed out, however, that if the value of e/m_0 is measured from the same photograph the value of K will in no wise affect the value of the ratio m/m_0 if only K remain constant. It will enter, however, in the determination of the velocity of the electron but the error thus introduced is relatively small.

The deflecting magnetic field was kept at values sufficiently small that z^2 could be neglected compared with l^2 . The equation for the velocity then becomes

$$v = \frac{zV \times 10^8}{Hd(d + t/K)}. \quad (29)$$

If the value of e/m is calculated from the smaller deflections, z_0 and l_0 , on a photograph the ratio m/m_0 for the higher velocities is given by

$$\frac{m}{m_0} = \frac{z_0^2 l^2}{l_0^2 z^2}. \quad (30)$$

The individual values of e/m as calculated from the photographs are shown in Fig. 5, in which (as well as in Figs. 6, 7 and 8) the full line curves marked "A" and "L" correspond to the theoretical values of Abraham and Lorentz respectively. Of the points lying above both of these curves all except three are due to a single photograph.

The ratio of the masses was also calculated by means of the preceding equation. To test which of the theoretical curves the points collectively best fit it was assumed that the value for the slowest velocity electrons showing on each of the photographs was a value exactly fitting the Lorentz curve and the other values were plotted by using only the ratio of the masses as calculated from the photographs. These values are set down in Fig. 6. Similarly Fig. 7 shows the results assuming the

value for the slowest velocity showing on each of the photographs to lie exactly on the Abraham curve. Now by a comparison of Figs. 6 and 7 it is seen that in either case the points fit the Lorentz curve more nearly than the Abraham curve.

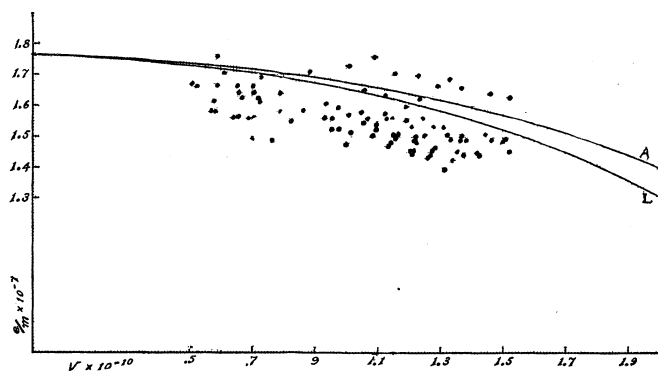


Fig. 5.

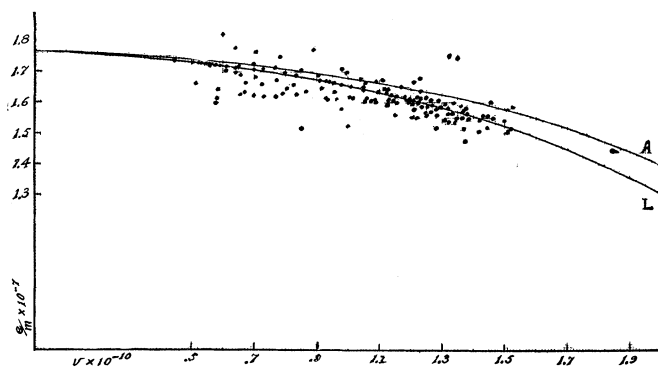


Fig. 6.

Table I. gives the data and results taken from one pair of traces on one of the photographs.

TABLE I.

l Cm.	$2z$ Cm.	z/l Cm.	$e/m \times 10^{-7}$.	$v \times 10^{-10}$.	Remarks.
3.93	.1443	.01836	1.708	.6089	Photograph 58, 1,926 volts, $d = .8137$ cm., $d + t/K = .8418$ cm.
4.43	.1587	.01791	1.625	.6696	
6.93	.2503	.01806	1.653	1.056	
7.43	.2667	.01795	1.632	1.125	
7.93	.2817	.01796	1.599	1.189	
8.43	.2957	.01754	1.558	1.247	
8.93	.3107	.01740	1.533	1.311	
9.43	.3250	.01723	1.504	1.371	
9.93	.3427	.01726	1.508	1.446	
10.43	.3583	.01718	1.495	1.512	

Fig. 8 represents graphically the results shown in Table I.

On each of the photographs the lines seen crossing the electron paths were drawn between the jaws of a pair of vernier calipers. The photographic plate while in position touched the ebonite disc, *L*, Fig. 1, and hence the length of the iron slip, *P*, determined the distance of the opening

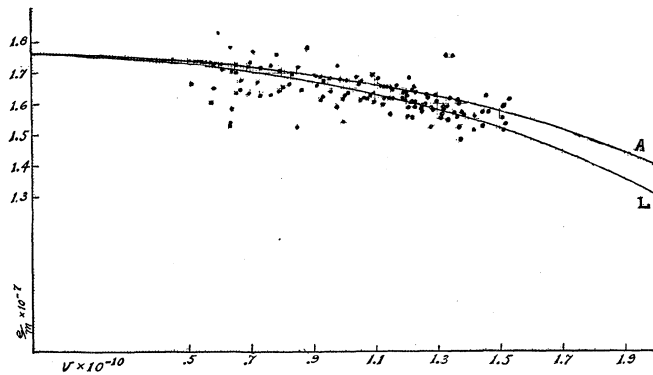


Fig. 7.

from the end of the photographic plate. On each of the photographs the line near the right is that marking the opening and the others show the successive values of *l* for which the values of *e/m* and *v* were calculated.

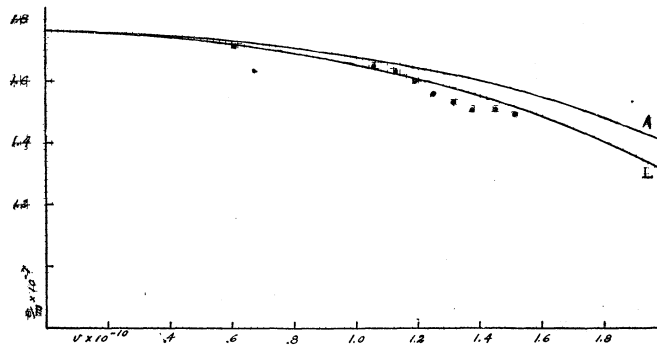
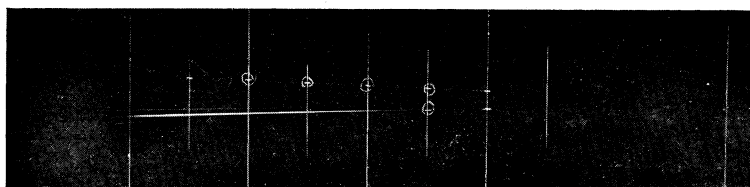


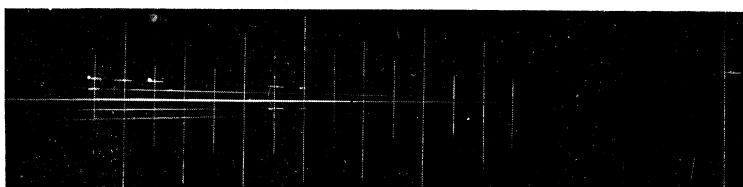
Fig. 8.

In several of the photographs, 54 for instance, the lines could be seen nicely with the unaided eye but were too dim when seen through the comparator microscope. These lines were touched with a sharp pencil and these marks used to determine the position of the lines. The photographs show very easily which of the lines were so treated.

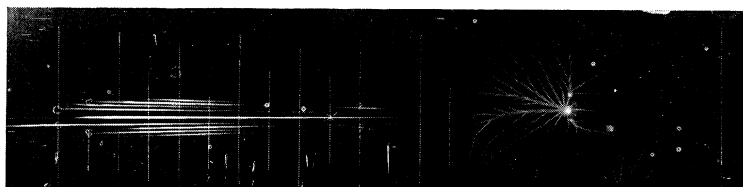
The distance apart of the traces, $2z$, was measured by a comparator reading to .005 mm. The value of $2z$ for each distance was measured



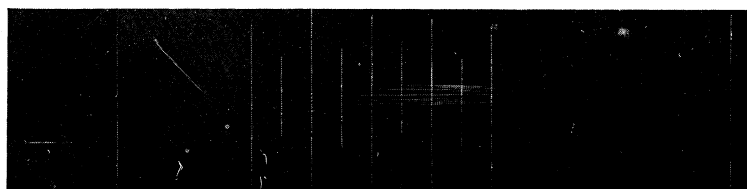
Photograph 54.



Photograph 58.

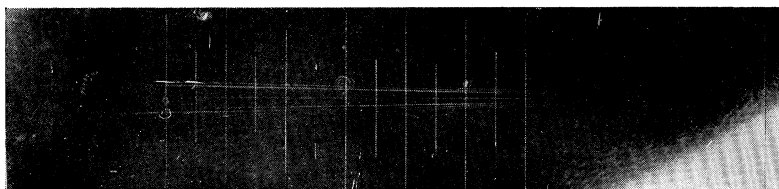


Photograph 59.

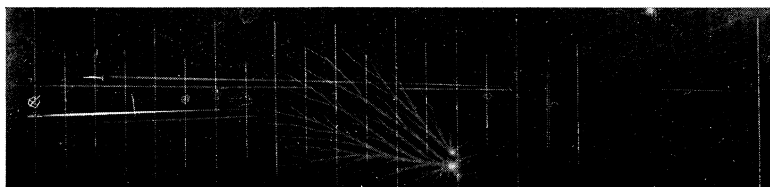


Photograph 60.

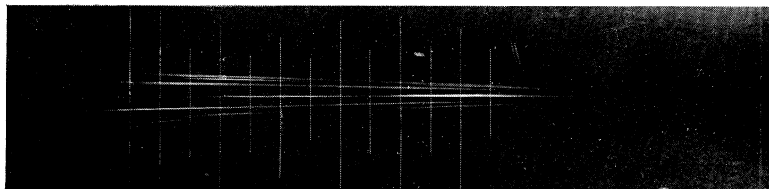
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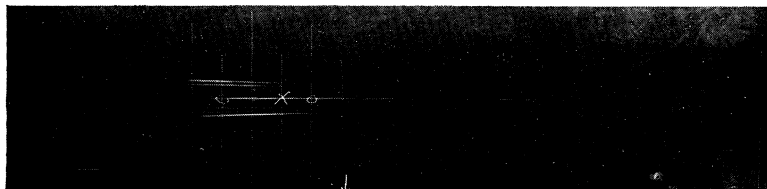
Photograph 63.



Photograph 64.



Photograph 66.



Photograph 68.

LLOYD T. JONES.

five times, usually on different days, and the average taken. Usually no measurement differed more than .001 cm. from the average and almost never did one differ more than .002 cm. The comparator screw was calibrated.

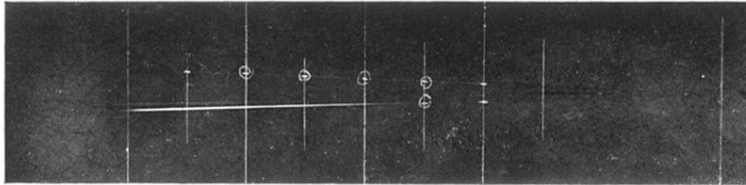
It was found experimentally that the distortion of the magnetic field due to the iron of the constricting canal had the effect of making the magnetic deflection smaller. It may be considered as zero for a small distance p further and then constant for the remainder of the path. This has the effect of making l smaller and hence e/m and v larger. The length l does not enter directly into the value of v unless the two values of l for the distance of travel in the two fields, electrostatic and magnetic, are different. The value 1.765×10^7 was assumed as the correct value of e/m_0 and the two curves in Figs. 5, 6 and 7 accordingly point to this value for slow velocities. The magnitude of the factor p was calculated and this value, $p = .07$ cm., was used to correct all the values of l used. This correction was assumed to be the same for the electrostatic field.

CONCLUSIONS.

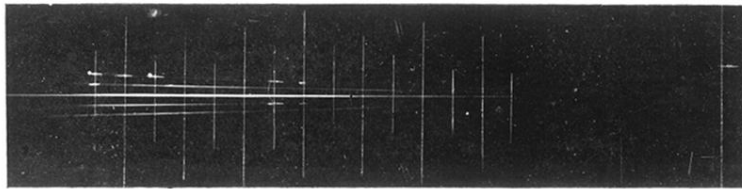
1. The method used in the present investigation does not necessitate a homogeneous cathode beam.
2. Each photograph gives a trace of all velocities present and makes possible a verification of the law from a single photograph.
3. The cathode beam never leaves the region between the electrostatic plates. The uncertain field distribution at the ends of the plates is thus avoided.
4. The present investigation has been carried out with rays of a velocity v little greater than any previously employed.
5. The results favor the Lorentz-Einstein rather than the Abraham formula.

In conclusion I wish to express my appreciation to Dr. C. T. Knipp for the enthusiasm with which he has followed the progress of the work. Also I wish to express my thanks to Prof. A. P. Carman, director of the laboratory, for the facilities he so kindly placed at my disposal.

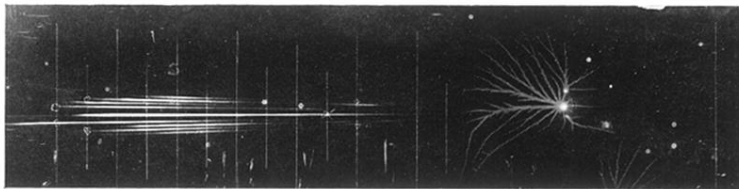
LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
March 1, 1916.



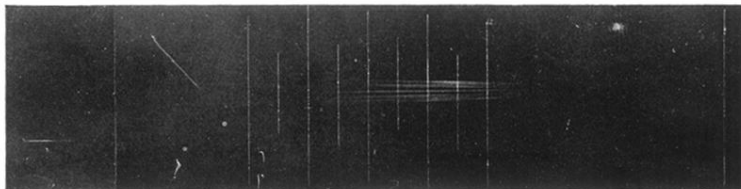
Photograph 54.



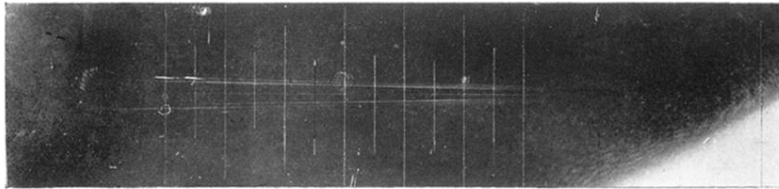
Photograph 58.



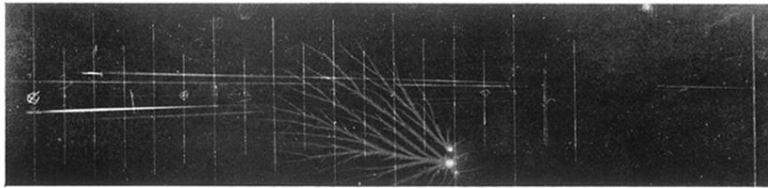
Photograph 59.



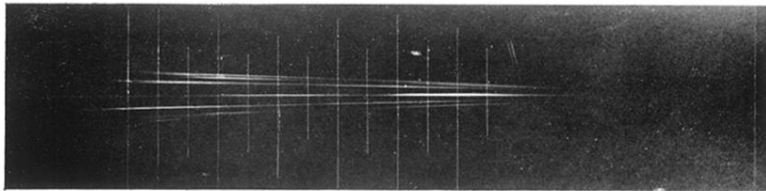
Photograph 60.



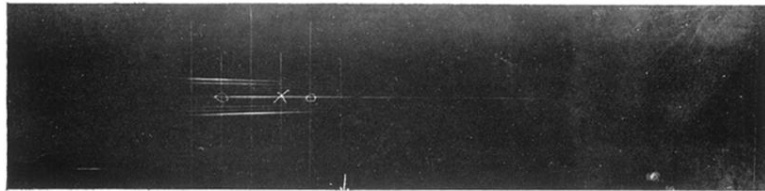
Photograph 63.



Photograph 64.



Photograph 66.



Photograph 68.