

Effect of Inclusions on Coercive Force of Iron

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A theory is developed for the contribution to the coercive force of iron of randomly distributed non-magnetic inclusions. Two contributing effects are examined: (1) the surface tension effect, and (2) the effect of internal magnetic poles. It is found that the coercive force depends both on the total volume fraction of inclusions and the state of dispersion. For a given composition of alloy, for particles both much larger and much smaller than the thickness of the domain wall, δ , the net contribution to the coercive force is small. The largest effect occurs for particles whose diameter is about equal to δ . Measurements have been made using a dispersion in iron of Fe_3C in the shape of spheres, the diameters of which could be varied over the desired range. The maximum effect on coercive force is found to occur for particles of size about 1200 angstroms, indicating a value for δ of about this magnitude. For this value of δ the measured value of the coercive force is found to agree well with the calculated value.

I. INTRODUCTION

IN recent years considerable progress has been made in understanding the origin of the coercive force in ferromagnetic materials. The effect on coercive force of non-magnetic inclusions was first clearly recognized by Kersten.¹ He proposed that the increase in coercive force caused by the presence of inclusions was a simple surface tension effect; the domain wall has a definite surface tension and therefore a strong tendency to stick to the inclusions in order to reduce the total wall energy (Fig. 1a and 1b).

Kersten's work was criticized by Néel,² who expressed the idea that the main effect was caused by the presence of internal poles at the interface between inclusions and matrix. In passing an inclusion the domain wall causes a redistribution of these magnetic poles. This results in a lowering of magnetostatic energy (Fig. 1c and 1d) and hence a tendency of the wall to stick to the inclusions.

The experimental work described in this paper deals with the effect of inclusions in iron. The paper itself has two aims: (1) to re-examine the various contributions to the coercive force which non-magnetic inclusions can make, and (2) to interpret the results of the experimental work in the light of this analysis.

II. EFFECT OF INCLUSIONS

In examining the effect of inclusions we shall follow a different procedure from that used by Néel.² He used a three-dimensional Fourier series method to represent the lattice inhomogeneities, both inclusions and lattice strains. In this paper we use a less elaborate mathematical treatment, but shall keep closely from the very outset to the physical picture of inclusions which we set up. Lattice strains we neglect entirely, even those strains set up by the inclusions themselves.

The assumptions and conventions used are the following. The ferromagnetic matrix in the demagnetized

state is divided into a larger number of domains, which for simplicity are assumed to have the shape of cubes with edges of length L . The coordinate system x, y, z has its axes along the cube edges. The x direction is a preferred direction of magnetization.³ The 180° domain wall, which we consider as moving through a particular domain, is parallel (except for a possible slight curvature) to the xy -plane. On the $-z$ side of the wall the magnetization I_s is in the $-x$ direction; on the $+z$ side, in the $+x$ direction. Under the action of an applied field H in the $+x$ direction the domain wall thus has a tendency to shift in the $+z$ direction. Each domain contains n non-magnetic inclusions distributed at random in the matrix. The exact shape of these particles is immaterial so long as their dimensions in all directions are about the same. In carrying out the calculations we shall assume that the particles are spherical with diameter d .

III. GENERAL ANALYSIS

First we consider the domain wall as being rigid and plane. The wall, to which we attribute the thickness δ ,

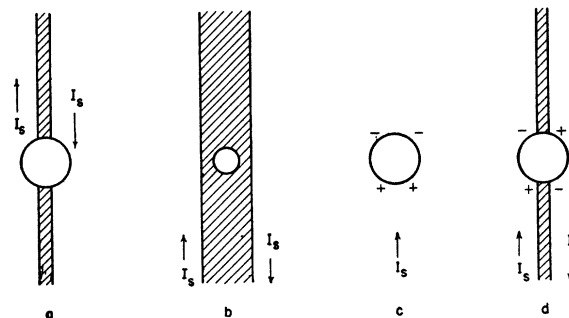


FIG. 1. (a) and (b) show how the total wall energy is reduced by non-magnetic inclusions. In Fig. 1a the diameter of the inclusion is larger, in Fig. 1b smaller than the wall thickness δ . (c) and (d) demonstrate the re-distribution of internal poles which occurs when a domain wall passes the inclusion. The magnetostatic energy in Fig. 1(d) is lower than in Fig. 1(c). I_s is the intensity of magnetization.

¹ M. Kersten, *Ferromagnetische Hysterese u. Koerzitionskraft* (S. Hirzel, Leipzig, 1943).

² L. Néel, *Ann. Univ. Grenoble* **22**, 299 (1945-1946).

³ For a general picture of the present view of domain theory the reader is referred to C. Kittel, *Rev. Mod. Phys.* **21**, 541 (1949).

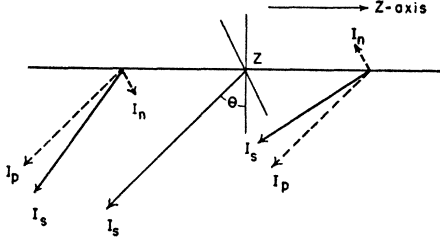


FIG. 2. Model for the calculation of the magnetostatic energy for an inclusion inside the domain wall.

is located at z_0 . The average number of particles cut or absorbed (partially or completely) by the domain wall and which have their centers on either the $+z$ or $-z$ side of the wall is $n(d+\delta)/2L$. The mean deviation of this number [which is of order $(n(d+\delta)/2L)^{1/2}$] gives rise to the coercive force H_c . We introduce the function $\phi(z-z_0)$ as being the reduction in energy of the system due to the interaction between the domain wall and an inclusion centered at (x, y, z) ; $\phi(z-z_0)$ will be specified later. If we distinguish among the various inclusions by the index i , then the equilibrium value of H necessary to hold the wall at z_0 is given by

$$2H(z_0)I_sL^2 = \sum_i \partial[\phi(z_i - z_0)]/\partial z_0. \quad (1)$$

In this expression I_s is the intensity of magnetization and the \sum_i is to be taken over all particles in the domain for which $\phi \neq 0$.

Since the equilibrium value of $H(z_0)$ determined by Eq. (1) depends on z_0 , there is for each domain a particular value of z_0 for which $H^2(z_0)$ is a maximum; this maximum we denote as H_{\max}^2 . We will then define the coercive force for the specimen as a whole to be $H_c = \langle H_{\max}^2 \rangle^{1/2}$; the double brackets indicating the average over the domains. If $\langle H^2(z_0) \rangle$ is the mean value of $H^2(z_0)$ over z_0 , then the relation between $\langle H_{\max}^2 \rangle$ and $\langle H^2(z_0) \rangle$ is, according to Néel,² most conveniently given as

$$\langle H_{\max}^2 \rangle \approx 2\langle H^2(z_0) \rangle \ln p, \quad (2)$$

where p is the number of essentially independent values $H(z_0)$ can assume over a distance L . With H_c defined in this way we are left with the problem of determining the *rms* values of $H(z_0)$. This we do by analysis of Eq. (1).

From Eq. (1)

$$4\langle H^2(z_0) \rangle I_s^2 L^4 = \sum_i \sum_j \left\{ \frac{\partial}{\partial z_0} \phi(z_i - z_0) \right\} \left\{ \frac{\partial}{\partial z_0} \phi(z_j - z_0) \right\} \\ = n \left\langle \left\{ \frac{\partial}{\partial z} \phi(z - z_0) \right\}^2 \right\rangle, \quad (3)$$

since $\partial\phi(z_i - z_0)/\partial z_0$ and $\partial\phi(z_j - z_0)/\partial z_0$ are independent if $i \neq j$, and $\partial\phi(z_i - z_0)/\partial z_0 = 0$. The probability that the center of a particle lies between z and $z+dz$ is dz/L , so

from Eq. (2) we obtain

$$4\langle H^2(z_0) \rangle I_s^2 L^4 = n \int_{z_0 - \frac{1}{2}(d+\delta)}^{z_0 + \frac{1}{2}(d+\delta)} \left\{ \frac{\partial}{\partial z} \phi(z - z_0) \right\}^2 \frac{dz}{L}. \quad (3a)$$

The factor p in Eq. (2) must yet be determined in terms of the geometry of our picture. Clearly, for values of z_0 for which $|\Delta z_0|$ is at least of order $\frac{1}{2}(d+\delta)$, the distribution of particles will show no correlation. This means that p is of the order $2L/(d+\delta)$. Hence from Eqs. (2) and (3a)

$$H_c = \left[2n \ln \left(\frac{2L}{d+\delta} \right) \right. \\ \left. \times \int_{z_0 - \frac{1}{2}(d+\delta)}^{z_0 + \frac{1}{2}(d+\delta)} \left\{ \frac{\partial}{\partial z} \phi(z - z_0) \right\}^2 dz \right]^{1/2} / 2I_s L^{5/2}. \quad (4)$$

With this expression the problem of computing the value of H_c is reduced to that of finding the function $\phi(z-z_0)$. The remainder of this section will be devoted to an evaluation of this quantity for the "surface tension" effect as well as for the "internal magnetic poles" effect.

The Surface Tension Effect

The reduction in energy of the system in this case is considered to be caused simply by the lowering of the net wall "volume" when the wall contains an inclusion.¹ It is assumed that the presence of the inclusion in no way affects the direction of I_s in the matrix or in the wall. For calculation of this effect it is convenient to consider the two extreme cases: (1) for $d \gg \delta$ and (2) for $d \ll \delta$.

For the first case

$$\phi(z-z_0) = \pi\gamma_{180^\circ} \left[\frac{1}{4}d^2 - (z-z_0)^2 \right] \text{ for } |z-z_0| < \frac{1}{2}d, \quad (5)$$

where γ_{180° is the wall energy per unit area for a 180° wall. Direct substitution of this expression into Eq. (4) gives for the coercive force for this case

$$H_c \approx 1.75(\gamma_{180^\circ}/I_s L) \alpha^{1/2} (\ln 2L/d)^{1/2} \text{ for } d \gg \delta, \quad (6)$$

where n has been eliminated by means of the expression $n\pi d^3/6L^3 = \alpha$, the volume fraction of inclusions. Equation (6) is identical with the result of Néel,² if one takes his "distance de correlation" η to be of the order of d .

For $d \ll \delta$

$$\phi(z-z_0) = \frac{1}{6}\pi d^3 E(z-z_0) \text{ for } |z-z_0| < \frac{1}{2}\delta. \quad (7)$$

Here $E(z-z_0)$ is the energy per unit volume of the wall at a point z inside the wall. $E(z-z_0)$ can be calculated exactly but since we are interested only in $\langle (\partial\phi/\partial z)^2 \rangle$ (see Eq. (3)), the approximation $\partial E/\partial z = \text{const.}$ must lead to a reasonably accurate result for H_c . By this procedure H_c is underestimated somewhat. Since

$$\int_{z_0 - \frac{1}{2}\delta}^{z_0 + \frac{1}{2}\delta} E dz = \gamma_{180^\circ},$$

we approximate E by

$$E = (2\gamma_{180^\circ}/\delta)[1 \pm 2(z-z_0)/\delta] \text{ for } z < z_0 \text{ or } z > z_0. \quad (7a)$$

It is a simple calculation to show that

$$H_c \approx 2.8(\gamma_{180^\circ}/I_s L)(d/\delta)^{3/2} \alpha^{1/2} (\ln 2L/\delta)^{1/2} \text{ for } d \ll \delta. \quad (8)$$

Analogous calculations can be made for any value of d/δ . For the limiting case $d \approx \delta$, (6) and (8) give the same value apart from a small numerical factor. A more exact calculation gives

$$H_c \approx -\frac{7}{8} \frac{\gamma_{180^\circ}}{I_s L} \alpha^{1/2} \left(\ln \frac{L}{\delta} \right)^{1/2} \text{ for } d \approx \delta. \quad (9)$$

Curve a in Fig. 4 gives a plot of H_c vs. d for constant α according to Eqs. (4) to (9). For this purpose we have used the values $\alpha = 3 \times 10^{-3}$, $\gamma_{180^\circ} = 2$ ergs/cm², $I_s = 1700$ gauss, $L = 10^{-3}$ cm,⁴ and $\delta = 10^{-5}$ cm. For $d \approx 5\delta$, H_c passes through a faint maximum of about 0.2 oersted.

“Internal Magnetic Poles” Effect

A spherical non-magnetic inclusion present in the matrix corresponds to a magnetic dipole of self-energy

$$E_{\text{dip}} = \frac{1}{2}(4\pi/3)I_s^2 V, \quad (10)$$

where V is the volume of the particle (Fig. 1c). As soon as a particle is partially or completely enclosed by a domain wall a redistribution of the internal magnetic poles takes place. This results in a lowering of the magnetostatic energy. In principle this effect can be understood from the symmetrical situation shown in Fig. 1d. A 180° wall passing through the center of an inclusion gives rise to a magnetic quadrupole of self-energy⁵

$$E_{\text{quad}} \approx \frac{1}{2} E_{\text{dip}} = \pi I_s^2 V/3. \quad (10a)$$

In treating the effect of internal poles it is in general not correct to assume that I_s remains unchanged in the vicinity of these inclusions. The magnetic field origi-

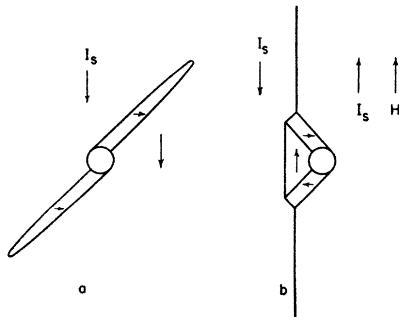


FIG. 3. (a) Observed domain pattern around a non-magnetic inclusion. (b) Structure after a 180° wall has passed the inclusion. The lowest energy state in the absence of a magnetic field is obtained for the wall passing through the center of the inclusion.

⁴ Bozorth and Dillinger, Phys. Rev. **41**, 345 (1932).
⁵ L. Néel, Cahiers phys. **25**, 21 (1944).

TABLE I. Dependence of E_{dip} on particle size.

d	10^{-3} cm ($\sim 100\delta$)	10^{-4} cm ($\sim 10\delta$)
k	30	5
$E_{\gamma_{90^\circ}}$	0.8×10^{-4} ergs	1.2×10^{-7} ergs
E_d	1×10^{-4} ergs	2.5×10^{-7} ergs
E_{dip}	30×10^{-4} ergs	30×10^{-7} ergs

nating from the internal magnetic poles tends to rotate I_s in the vicinity of an inclusion out of the original direction in such a way that the magnetostatic energy is partially released. In neglecting this interaction one would obtain for particles for which $d \gtrsim \delta$ a value for the coercive force which is much too high.

For the case in which $d \ll \delta$, however, the interaction between I_s and the internal magnetic poles can be safely neglected since the exchange forces oppose a rotation of I_s on such a local scale. The main term of $\phi(z-z_0)$ in this case is obtained in the following way. For any point z inside the wall the vector I_s makes a certain angle θ with the preferred direction. In a small region about z we split I_s up into the two components I_n and I_p , respectively, normal and parallel to I_s at z . (Fig. 2.) For a region in which the total change of direction of I_s , $\Delta\theta$, is small we can write

$$I_p = I_s [1 - \frac{1}{2}(\Delta z d\theta/dz)^2]; \quad I_n = I_s \Delta z d\theta/dz.$$

It is easy to see that for a particle with center located at z the component I_p gives rise to a dipole, I_n to a quadrupole. Because of the symmetry, there is no mutual interaction. Taking average values for I_p and I_n over Δz , we find for the corresponding energy terms

$$E_{\text{dip}}' = \frac{2\pi}{3} I_s^2 V \left[1 - \frac{1}{12} \left(\frac{d\theta}{dz} \right)^2 \right],$$

$$E_{\text{quad}}' = -\frac{\pi}{48} I_s^2 V \left(\frac{d\theta}{dz} \right)^2.$$

Hence

$$\begin{aligned} \phi(z-z_0) &= E_{\text{dip}} - (E_{\text{dip}}' + E_{\text{quad}}') \\ &= \pi V \left(\frac{1}{18} - \frac{1}{48} \right) I_s^2 \left(\frac{d\theta}{dz} \right)^2. \end{aligned} \quad (11)$$

The function $\theta(z)$ can be calculated.² However, to simplify the calculation we shall follow the same procedure as we used previously; namely, since in Eq. (2) $\langle (\partial\theta/\partial z)^2 \rangle$ occurs, we take

$$\left(\frac{d\theta}{dz} \right)^2 = (2\pi^2/\delta^2) [1 \pm (2/\delta)(z-z_0)] \text{ for } z < z_0 \text{ or } z > z_0. \quad (11a)$$

By carrying through the remainder of the calculation indicated by Eq. (4), one readily finds that

$$H_c \approx 2.8(I_s d^{7/2}/L\delta^{5/2})\alpha^{1/2}(\ln 2L/\delta)^{1/2} \text{ for } d \ll \delta. \quad (11b)$$

It is instructive to make a comparison here. By comparison of Eqs. (8) and (11b) it is seen that for

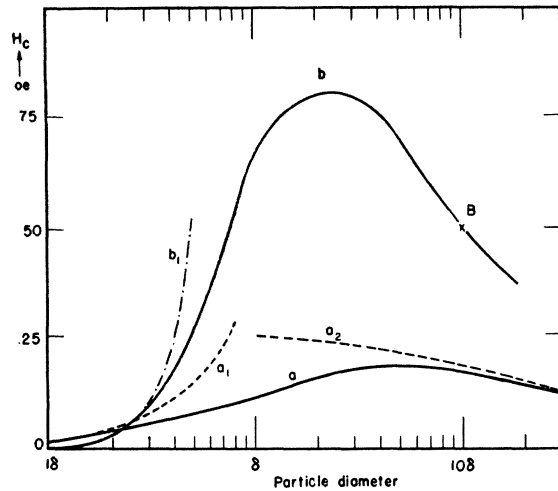


FIG. 4. Expected behavior of the coercive force H_c in iron caused by a volume fraction of a random distribution of non-magnetic inclusion equal 3×10^{-3} as a function of the particle diameter d . Curve (a) is for the surface tension effect, curve (b) for the "internal poles" effect; (a_1), (a_2), and (b_1) are the extrapolated curves according to Eqs. (8), (6), and (11b), respectively. Point B is calculated from Eq. (13). The curves give only orders of magnitude.

small particles ($d \ll \delta$) the effect of "internal magnetic poles" is $I_s^2 d^2 / \gamma_{180^\circ} \delta$ times as large as the "surface tension" effect. For extremely small particles the surface tension effect, though itself small, is the larger of the two. For particles of the order $\frac{1}{4} \delta$ the two effects are about equal; for particles larger than this the magnetic pole effect starts to predominate.

For $d \cong \delta$ the interaction between the internal poles and the vector I_s can no longer be neglected. For $d \gg \delta$ almost complete release of magnetic energy takes place with the appearance of a secondary domain structure as drawn in Fig. 3a.^{5,6} The relation between d and the equilibrium value of l ($2 \times$ the length of a secondary domain) follows essentially from an early study by Döring.⁷ The usual approach is to assume that the domains have the shape of an ellipsoid of revolution with short axis d and long axis l . Then the total wall energy

$$E_{\gamma_{90^\circ}} = \gamma_{90^\circ} \times \text{surface area} = \frac{1}{4} \pi^2 \gamma_{90^\circ} l d.$$

The magnetostatic energy, E_d , stored in the demagnetizing field is given by

$$E_d = a N I_s^2 \times \text{vol} = \frac{1}{6} \pi a N I_s^2 l d.$$

The demagnetizing factor $N = (4\pi^2/k^2)(\ln 2k - 1)$ if $k = l/d \gg 1$. The factor a has been introduced by Williams, Bozorth, and Shockley⁶ to take into account the permeability of the matrix. For pure iron $a = \frac{1}{16}$. Finally the energy term due to the applied field H ,

$$E_H = H I_s \times \text{vol} = \frac{1}{6} \pi H I_s l d^2.$$

Minimizing $E = E_{\gamma_{90^\circ}} + E_d + E_H$ with respect to l one easily finds

$$k^2 = \frac{8\pi a I_s^2 d (\ln 2k - 2)}{3\pi \gamma_{90^\circ} - 2H I_s d}. \quad (12)$$

Taking $H = 1$ oersted, $\gamma_{90^\circ} = 1$ erg/cm², one can calculate k for values of d up to 3×10^{-5} cm when the denominator goes to zero, which means that the secondary domains grow until they join the domain walls. To show that by this secondary domain structure an almost complete release of magnetostatic energy E_{dip} does take place Table I shows values calculated neglecting the term $2H I_s d$ in Eq. (7). We see from Table I that for very large particles the release of energy is almost complete, but that less is released as the size of the particle decreases. As d approaches δ a yet smaller fraction of the dipole energy is released, especially when the inclusion becomes too small for the formation of well-developed secondary domains.

It is not difficult to show that when well-developed secondary domains are formed the average equilibrium field is given by

$$2\langle H^2 \rangle^{\frac{1}{2}} I_s L^2 = (l/L) n \pi d \gamma_{180^\circ}.$$

Since $l = kd$

$$H_c \approx \langle H^2 \rangle^{\frac{1}{2}} \approx (3\gamma_{180^\circ} k / I_s d) \alpha \text{ for } d \gg \delta. \quad (13)$$

According to Eqs. (12) and (13) H_c will decrease rather slowly with increasing d . In case $L \gg kd$ the right member of Eq. (13) must be multiplied by $(2 \ln 2L/kd)^{\frac{1}{2}}$ to account for the difference between $\langle H^2(z_0) \rangle$ and $\langle H_{\max}^2 \rangle$. The application of Eq. (13) to practical cases is rather limited, since, in general, for a well-developed domain structure kd is of the same order as L . For $\alpha = 3 \times 10^{-3}$, formal substitution of $d = 10^{-4}$ cm yields $H_c \sim 0.4$ oersted.

For $d \approx \delta$, the secondary domains have shrunk to a disturbed region of dimension δ around the inclusion. A calculation of the energy terms involved in this case is rather difficult, and we shall only make a rather crude estimate by interpolating the results for $d \gg \delta$ and $d \ll \delta$. According to the preceding paragraphs, the release in magnetostatic energy E_{dip} as given by Eq. (1) due to the interaction between the internal charges at the interface and the vector I_s is complete for $d \gg \delta$ and negligible for $d \ll \delta$. For $d \sim \delta$ it seems reasonable to assume a reduction of E_{dip} by this interaction of about $\frac{1}{2}$. To find the magnitude of ϕ we must make a further estimate; neglecting the foregoing interaction between dipole and matrix we find by interpolation of Eqs. (10a) through (11) that for a particle, for which $d \approx \delta$, located symmetrically in the domain wall E_{dip} is reduced by a factor of about $\frac{1}{4}$. Hence the magnitude of ϕ is about $E_{dip}/8$. We compare this to the surface tension effect, where the corresponding value of ϕ is of the order $\gamma_{180^\circ} V / \delta$. Hence the "magnetic pole effect" will be of the order of $\frac{1}{8} E_{dip} / (\gamma_{180^\circ} V / \delta) \sim 4$ times as large

⁶ Williams, Bozorth, and Shockley, Phys. Rev. **75**, 155 (1949).

⁷ Becker and Döring, *Ferromagnetismus* (Verlag Julius Springer, Berlin, 1938), p. 192.

as that due to the surface tension effect. (It is clear that this may be in error by a factor of 2 or more.) The expected behavior of H_c is shown in Fig. 4 by the curve b taking $\alpha = 3 \times 10^{-3}$. After a rapid increase for $d < \delta$, H_c tends for $d > \delta$ to approach a behavior as expressed by Eq. (13). For $d \sim \delta$, H_c passes through a maximum of the order of one oersted.

IV. EXPERIMENT

It is apparent from the foregoing analysis that one might expect inclusions to have a pronounced effect on the coercive force; for a given value of α the most striking effects might be observed for particles of the same order of size as the thickness of the Bloch wall. We have examined experimentally this possibility in iron using the compound Fe_3C in finely dispersed form. Since for iron the 180° Bloch wall has a theoretical thickness of about 1000A, it was necessary to have particles of controllable dimensions of about this size. By the metallurgical treatment described below it was possible for us to prepare samples of iron containing inclusions of Fe_3C in the form of randomly distributed spheres, the diameters of which were varied between 400 and 3000A. With these samples we were able (1) to find the "critical size" for maximum effect for given α on the coercive force, and (2) to determine the magnitude of H_c for a given volume fraction of inclusions of this critical size.

The samples were prepared and the measurements were made in the following manner. The specimens were Puron iron wires drawn to a size of 0.03 in. diameter by one foot long. Highly super-saturated solid solutions of C in α -Fe were prepared by quenching to room temperature from 720°C an alloy of 0.02 percent in α -Fe. These specimens were then tempered at selected temperatures in the range 125°C to 350°C , which tempering resulted in the gradual disappearance of C from solid solution into a second phase, Fe_3C , in α -Fe. This phase change was traced by means of the internal friction found in this alloy. During the phase change the coercive force was also measured at intervals at 20°C .

The detailed analysis of the results depends entirely

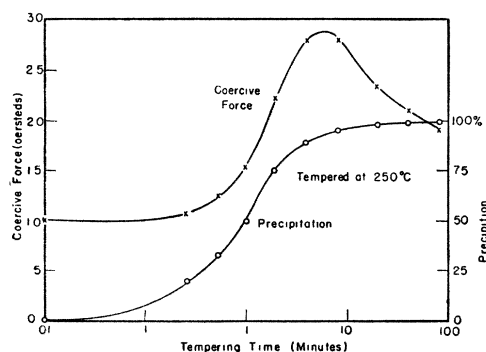


FIG. 5. Coercive force and precipitation as a function of time for an alloy of 0.02 percent of carbon in iron which was tempered at 250°C .

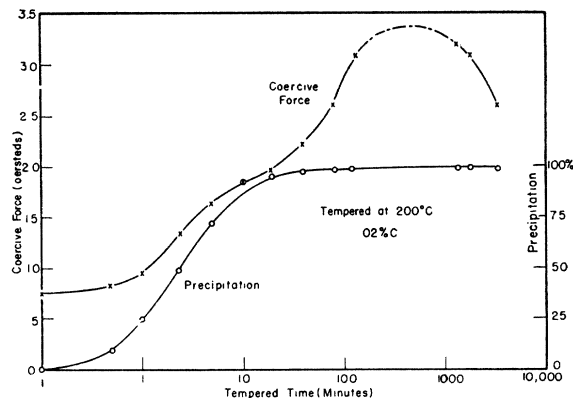


FIG. 6. Coercive force and precipitation for an alloy tempered at 200°C .

upon certain facts which we believe applies to this phase change. These facts are the following: (1) the Fe_3C precipitates in the form of spheres, and (2) the number of these spheres per unit volume remains constant during the major part of the precipitation at a given temperature. Originally these proposals were supported only by some rather indirect analyses of earlier experimental work.⁸ More recently, however, these proposals have been given additional support by examination of specimens using an electron microscope.⁹

Since we wish to know the manner in which the coercive force varies with the particle size, we must have some way of computing the particle diameter. Clearly this is given by the expression

$$\alpha = \frac{1}{6} \pi d^3 N', \quad (14)$$

where N' is the number of particles per unit volume. This expression, of course, is valid if the particles are all the same size; a condition which is a consequence of the two proposals made above. Since α can be calculated readily, d can be computed once N' is known. N' itself may be calculated from the expression,¹⁰

$$N' = (3/4\pi)(2D\tau)^{-3} [(n_0 - n_1)/(n_\infty(0) - n_1)]^{3/2}. \quad (15)$$

In this expression n_0 , n_1 and $n_\infty(0)$ are carbon concentrations referring to (1) concentration of C in Fe_3C , (2) equilibrium solid solubility at the temperature of tempering and (3) initial concentration in solid solution, respectively. D is the diffusion coefficient of C in α -Fe, which has been determined accurately as a function of temperature by one of the authors.¹¹ τ is an empirically measured time constant, it is the time required for the phase transformation to become 60

⁸ C. Wert, J. App. Phys. **20**, 943 (1949); C. Zener, J. App. Phys. **20**, 950 (1949).

⁹ Unpublished results of an investigation made by J. Radavich, Purdue University.

¹⁰ C. Wert and C. Zener, J. App. Phys. **21**, 5 (1950). This expression was given incorrectly in this reference. All calculations in this paper and in reference 10 were made with the correct expression, Eq. (15).

¹¹ C. Wert (to be published).

percent complete. N' is found to vary rapidly with temperature, decreasing by about a factor of 10 with each 100°C increase in temperature of tempering.

A typical measurement is shown in Fig. 5. Here the carbon was precipitated at 250°C . At intervals during the tempering the amount of precipitate formed and the coercive force were both measured; the time dependence of both of these quantities is shown. At this temperature $D=1.3\times 10^{-10}$ cm²/sec., τ is seen to be about 70 sec., $n_x(0)$ was about 0.02 wt percent, $n_1\approx 0$ and n_0 is about 6 wt percent, hence N'_{250° was calculated using Eq. (15) as about 1.8×10^{12} particles/cc. Since we have concluded that this number, N' , is about constant during the tempering, d can be computed at any stage of the phase change by means of Eq. (14). For example, at the 95 percent precipitation point $\alpha\approx 0.003$, hence at this point $d\sim 1400\text{\AA}$. The increase in coercive force due to this volume fraction of precipitate in this state of dispersion is about 1.9 oersteds. (The initial value of about one oersted is apparently caused by other inclusions (perhaps oxides), internal stress or grain boundaries.)

There is one aspect of the curves in Fig. 6 which must yet be explained; this is the decrease in the coercive force when the precipitation is slowly drawing to a close. We can offer at present only a qualitative explanation of this phenomenon; we believe it to be due to the coalescence of precipitate particles, which coalescence takes place only after the precipitation is essentially complete. It results in a decrease in N' and an increase in d ; as we shall show shortly, particles of diameter 1400A are already "oversize;" a further increase in d can only lessen the coercive force. Hence in this case H_c drops upon coalescence. It must not be supposed, however, that this coalescence after precipitation always results in a decrease in coercive force; Fig. 6 shows this not to be the case. At 200°C for 95 percent precipitation $d\sim 1000\text{\AA}$; this, as it turns out, is "undersize" for maximum effect. Hence coalescence will at first increase the coercive force until the particles reach an optimum size, after which further coalescence causes a decrease as the particles become now "oversize." Since we have no way even of estimating the rate

of coalescence, we can offer at present no quantitative arguments to support our view.

Data and calculations pertinent to the theory developed in Sec. II taken from data shown in Figs. 6 and 7 and from numerous other measurements not shown in detail are presented in Fig. 7. Here is plotted the coercive force as a function of particle size for a given value of α , namely, 0.003 (which we have seen occurs at the 95 percent transformation point). The curve in Fig. 8 shows that the critical size for maximum effect on coercive force is about 1200A. Since the theory developed earlier indicates a maximum effect for $d\sim\delta$, this means that δ for α -Fe must be about 1200A also. The magnitude of H_c at the maximum, two oersteds is also in reasonable agreement with the order of magnitude of one oersted predicted in Sec. II.

Unfortunately, Fe_3C is ferromagnetic itself with a saturation magnetization of about 1200 gauss and a Curie point of 200°C . If the magnetic anisotropy of Fe_3C would be very small compared with Fe, the internal poles effect would be smaller by a factor $I_s^2(\text{Fe})/I_s^2(\text{Fe}_3\text{C})\approx 10$ below the Curie point of Fe_3C . At the Curie point a rise in H_c could then be expected. On the other hand, for a very large anisotropy particles of this small size will approximately give the same contribution as if they were non-magnetic.

It is not clear then how to take into account the ferromagnetic properties of Fe_3C in consideration of either the "surface tension" or "magnetic poles" effect. To try to throw a little light on this problem we have made one further measurement. A specimen of iron containing carbon in solid solution was tempered at 210°C to give nearly complete precipitation. The Fe_3C in this state was dispersed in spheres of diameter 3200A. The coercive force of this sample was measured as a function of temperature up to 300°C . That part of the coercive force due to the inclusions is shown plotted as a function of temperature in Fig. 8. It is to be observed that no sudden change in the coercive force occurs at 200°C , the Curie point of Fe_3C . The simplest way to account for this effect is to consider the small Fe_3C particles to behave as non-magnetic inclusions.

APPENDIX

To allow for a small curvature of the domain wall we represent the plane of the wall by a Fourier series as follows:

$$z = z_0 + \sum_q z_q \frac{\sin\left(\frac{2\pi q y}{L}\right)}{\cos\left(\frac{2\pi q y}{L}\right)}. \quad (\text{A1})$$

where z_0 is the mean distance of the plane above the xy plane. The x -components in the Fourier development can be neglected for, as Néel² pointed out, since I_s is along the x -direction these components would give rise to very large extra energy terms due to the appearance of internal free magnetic poles at the surface of the wall. The energy E of the system can be written as $E = E_1 + E_2 + E_3$. E_1 is the energy of the slightly curved wall if no inclusions were present and is equal to

$$E_1 = L^2 \gamma_{180^\circ} + \frac{1}{2} \pi^2 \gamma_{180^\circ} \sum_q q^2 z_q^2.$$

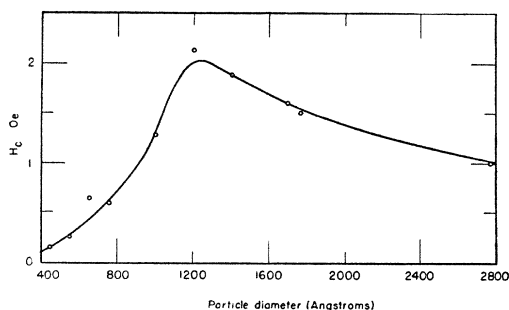


FIG. 7. Coercive force as a function of particle size for $\alpha\approx 0.003$.

E_2 is the decrease in E by the inclusions and is given by

$$E_2 = -\sum_i \phi[z_i - z(x_i, y_i)].$$

E_3 is the magnetic energy terms, hence

$$E_3 = -2HI_s I^2 z_0.$$

For equilibrium of the wall at z_0 , H , and z_q have to satisfy $\partial E/\partial z_0 = 0$ and $\partial E/\partial z_q = 0$. The first condition gives

$$2HI_s I^2 = -\sum_i \frac{\partial}{\partial z} [z_i - z(x_i, y_i)]. \tag{A2}$$

The second condition gives

$$\pi^2 \gamma_{180^\circ} q^2 z_q = -\sum_i \left\{ \frac{\partial}{\partial z} \phi[z_i - z(x_i, y_i)] \sin(\cos) \frac{2\pi q y_i}{L} \right\}. \tag{A3}$$

Equations (A2) and (A3) determine one or perhaps more solutions for H and z_q . Since these equations are difficult to solve we shall approximate $\phi(z_i - z)$ by the following linear expression

$$\phi(z - z_i) = \phi(0) \left[1 \pm \frac{2}{d + \delta} (z_i - z) \right]; \quad \pm \text{ for } z_i \leq z. \tag{A4}$$

From (A3) and (A4) we find

$$\pi^2 \gamma_{180^\circ} q^2 z_q = \sum \mp \frac{2\phi(0)}{d + \delta} \sin(\cos) \frac{2\pi q y_i}{L}. \tag{A5}$$

As we see the main contribution in (A5) comes from the two terms with $q=1$. Substitution of z_q given by (A5) in (A1) gives

$$z - z_0 = \frac{2\phi(0)}{\pi^2 \gamma_{180^\circ} (d + \delta)} \left[\sum_q \frac{1}{q^2} \sum \mp \cos \frac{2\pi q}{L} (y_i - y) \right]$$

and the "mean" value

$$\{ \langle (z - z_0)^2 \rangle \}^{\frac{1}{2}} = \frac{\phi(0)}{6\gamma_{180^\circ} (d + \delta)} \left(\sum_i i^0 \right)^{\frac{1}{2}}. \tag{A6}$$

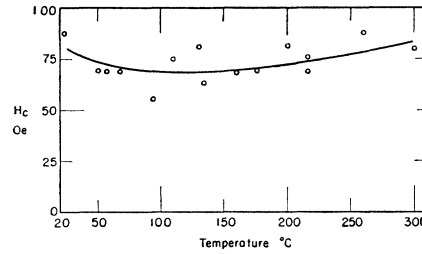


FIG. 8. H_c vs. temp. for a stable alloy consisting of 0.003 volume fraction inclusions dispersed in particles about 3200A in diameter.

Consider, for example, the surface tension effect.

In the case that $d \gg \delta$ the factor $\phi(0) = \frac{1}{4} \pi d^2 \gamma_{180^\circ}$ and for a slightly curved wall $\sum_i i^0 \approx nd/L$. This gives $\{ \langle (z - z_0)^2 \rangle \}^{\frac{1}{2}} = 0.17 \alpha^{\frac{1}{2}} L$. In the experimental work the volume fraction of inclusions $\alpha = 3 \times 10^{-2}$. For this value

$$\{ \langle (z - z_0)^2 \rangle \}^{\frac{1}{2}} \approx 10^{-5} \text{ cm} \approx \delta.$$

For $d \ll \delta$ the value of $\phi(0) = \pi d^3 / 24 \delta$ and $\sum_i i^0 = n \delta / L$. From these values and Eq. (6) we find

$$\{ \langle (z - z_0)^2 \rangle \}^{\frac{1}{2}} = 0.23 (d/8)^{\frac{1}{2}} \alpha^{\frac{1}{2}} L \approx \delta (d/\delta)^{\frac{1}{2}}.$$

These results justify the calculations for the given value of α .

From Eqs. (A2) and (A4) we see that in this approximation the coercive force H_c is the same as for a rigid, plane boundary. One remark must be made. For the usual particle shape quadratic terms enter in the expression for $\phi(z - z_0)$. Because of these terms value of $\langle H^2(z_0) \rangle$ and of H_c when the domain wall adjusts itself to the particle distribution is somewhat smaller than in case of a rigid, plane wall.