with the magnetic field should be only those due to the small normal currents existing in the presence of the Meissner currents. A crude estimate suggests that these are far below the limits of accuracy of this experiment.

The results, then, support the London view of superconductivity, but a quantitative measurement of the constant λ requires an accuracy far exceeding that of this work.

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Electron Conductivity and Mean Free Paths

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Mean free path and electron density are calculated from measurements on the complex conductivity of a discharge. Formulas based on two alternative assumptions are tested against experimental facts; one is constancy of mean free path (independence of electron velocity), the other is constancy of mean free time.

In a recent paper,[1] devoted to the measurement of complex electron conductivities in a positive column of a glow discharge, attention has been called to the opportunities which such experiments hold for gaining information about kinetic processes taking place in the conducting gas. It was shown that electron densities can be computed from the imaginary as well as the real part of the conductivity when the mean free path is given independently, and that the densities agree tolerably well. The purpose of the present note is to indicate how the mean free path itself can be determined from such measurements, and what light this sheds on other questions of interest in the kinetic theory of discharges.

The formula[2] used in the analysis of the experiments under discussion was based on the assumption of a constant mean free path of the electrons: λ is not a function of electron velocities v and hence is independent of the temperature, T. This is not generally true. An alternative and equally simple supposition is constancy of the mean free *time* τ, which is λ/v. What these hypotheses mean fundamentally is seen from the general relation

$$Nq\lambda=1, \tag{1}$$

in which N is the number density of molecules intercepting the electrons (in this instance Hg atoms) and q the collision cross section, a function of v. It is clear that constancy of λ implies a constant q, whereas constancy of τ requires q to be proportional to v^{-1}. We shall discuss these two alternatives in sequence.

[1] F. P. Adler, J. App. Phys. **20**, 1125 (1949).
[2] H. Margenau, Phys. Rev. **69**, 508 (1946).

If $\lambda=$ const., the complex conductivity is given by

$$\sigma=\frac{4}{3}\frac{e^2\lambda n}{(2\pi mkT)^{\frac{1}{2}}}[K_2(x_1)-ix_1^{\frac{1}{2}}K_{\frac{3}{2}}(x_1)]\equiv\sigma_r-i\sigma_i, \tag{2}$$

as was shown in reference 2. Here $x_1=(m\omega^2\lambda^2/2kT)$, and ω is the radian frequency of the microwaves. This formula is exact provided the distribution function for the electrons is Maxwellian, and T is the electron temperature. In reference 1, T was measured by probe methods. The ratio σ_i/σ_r is a function of x_1 alone, and it is plotted in reference 2. The experimental ratio leads therefore at once to knowledge of x_1 and, if T is known, to λ. Furthermore if the premise, $\lambda=$ const., is true, the values thus determined should not vary with T.

The situation is somewhat different if $\tau=$ const., and the analysis of reference 2 must be modified. Indeed the conductivity formula is then much simpler.[3] The equations lead to a distribution function which is strictly Maxwellian and corresponds to an electron temperature

$$T''=T+[M\gamma\tau^2/6k(1+\omega^2\tau^2)].$$

The complex conductivity[4] takes the form

$$\sigma=[ne^2\tau/m(1+\omega^2\tau^2)](1-i\omega\tau). \tag{3}$$

Here the ratio σ_i/σ_r, which is $\omega\tau$, leads directly to the determination of τ. If the assumption $\tau=$ const. is true, the ratio test will thus reveal it. Both τ and λ are,

[3] Its derivation is given by J. H. Cahn, Phys. Rev. **75**, 838 (1949) and by E. Everhart and S. C. Brown, Phys. Rev. **76**, 839 (1949).
[4] This is very similar to the Lorentz formula for the conductivity of electrons. It results if the friction constant, g, in the equation of motion is equated to m/τ.

of course, inversely proportional to the pressure of the gas carrying the discharge, as is evident from Eq. (1).

In Fig. 1 the experimental data are plotted against the electron temperature, the upper curve representing $p\tau$, the lower $p\lambda$. The $p\tau$-values were obtained by using the relation $\sigma_i/\sigma_r = \omega\tau$, and it is seen that the upper curve belies its premise, the constancy of $p\tau$. However, $p\lambda$, which was computed from Eq. (2), is scattered in random fashion about a constant value.

Having thus established the approximate validity of our first alternative, we may use the data to determine the electronic mean free path in Hg. It turns out to be 9.5×10^{-3} cm at 1 mm pressure.

This procedure does not prove that λ is truly independent of velocities, a result which would be very surprising in view of other facts. What it means is that at low electron velocities such as those to which our temperatures correspond, the average of λ over the range of velocities comprised in the Maxwell distribution is constant. Brode[5] has measured collision probabilities of electrons in Hg for energies somewhat higher than ours. His values show a rapid drop between energies of 1 and 2 volts; they correspond to an average λ between $\frac{1}{3}$ and $\frac{1}{2}$ of the value here obtained. If these inferences are to be in harmony, the collision probability must have a maximum in the region where Brode's measurements begin and drop to low values at smaller energies, suggesting a Ramsauer effect. Among the data published by other workers one can easily find values for λ that agree with ours (see Fig. 4 of reference 5), but it is perhaps unwarranted to look for exact numerical agreement because the present conductivity measurements were made at much higher pressures than the other observations.

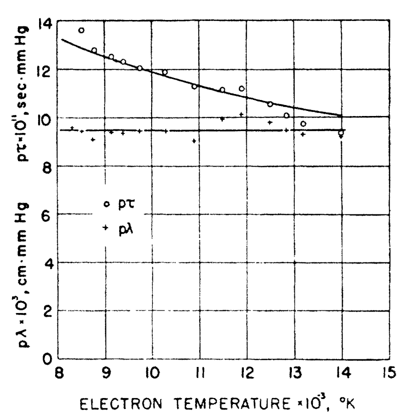

FIG. 1. Experimental data on the dependence of $p\lambda$ and $p\tau$ on electron temperature.

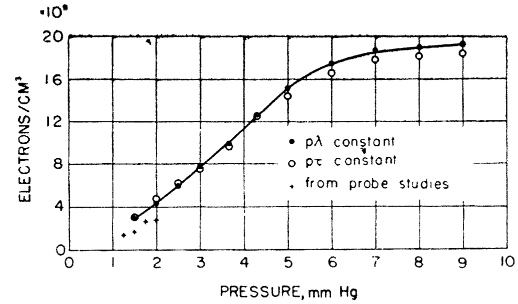

FIG. 2. Electron density as a function of pressure.

There is some internal consistency in the two curves of Fig. 1. To be sure, the upper one is not to be trusted in detail since the theory yielding it is not correct. But the trend is proper: if λ is constant, τ must decrease with v. The curve fitted through the circles corresponds in fact to $p\tau \propto (1/v) \propto T^{-\frac{1}{2}}$; i.e., to $p\lambda = \text{const.}$

When the mean free path is determined from the ratio σ_i/σ_r, the density of electrons follows uniquely from the measurements, and the ambiguity inherent in Fig. 8 of reference 1 is eliminated. Figure 2 shows the results (black dots). If the simpler formula (3) were chosen in analyzing the data, the circles would result. Thus it is seen that the electron density is not very sensitive to the kinetic assumptions from which it is derived.

[5] R. B. Brode, Proc. Roy. Soc. A125, 134 (1929).