Electromagnetic Forces on a Superconductor

W. V. HOUSTON AND N. MUENCH
The Rice Institute, Houston, Texas
(Received June 5, 1950)

Measurements of the eddy current damping of the rotational oscillations of a tin sphere suspended in a magnetic field from a torsion fiber give the expected results as a function of the magnetic field and the conductivity as long as the tin is a normal conductor. Below the transition temperature, however, the damping torque becomes less than 10^{-5} times its value in the normal state and no changes in period are observed except those attributable to residual frozen-in moments.

I. INTRODUCTION

THE ordinary formulation of the force exerted by an electromagnetic field on matter is divided into two parts. There is the force on charges, $\rho\mathbf{E}$, and the force on currents, $\mathbf{i}\times\mathbf{B}$. In case the matter is a conductor, the charges are located at the surface and the force is transmitted to the body by way of other forces that hold the metal together and prevent the electrons from leaving it. The force associated with a current, however, does not require the intervention of a surface, as is shown by Barlow's wheel. The force is transmitted by means of the interaction between the electrons and the crystal lattice.

The force to be expected on a superconductor has been formulated by London[1] and by von Laue.[2] The charge density is divided into a normal charge density, ρ_0, and a super charge density ρ_s. From the field equations one can show that the total charge density $\rho=\rho_0+\rho_s=0$. Since ρ_s is presumably due to electrons and is therefore negative, ρ_0 must be dominated by the positive ions. Although an electric field will not exist in a superconductor in a steady state, such a field may be present as a transient. During its transient existence it will act on the positive ρ_0 but not on ρ_s. The force on ρ_s is not transmitted to the crystal lattice, and its effect in producing a supercurrent may be described entirely in terms of the magnetic field. An experiment reported by Kikoin and Gubar[3,4] in 1940 on the gyromagnetic ratio of a superconductor can be completely described from this point of view.

Similarly, the current in a superconductor may be divided into an ordinary current, $\mathbf{i}_0$, and a supercurrent, $\mathbf{i}_s$. The normal current transmits a force to the crystal lattice in the usual way, but the supercurrent transmits force only to the surface. When no current is entering or leaving a superconductor, the supercurrent exerts an inward tension on the surface amounting to $\frac{1}{2}\lambda\mathbf{i}_s^2$. However, it transmits no force tangent to the surface.

[1] F. London, *Une Conception Nouvelle de la Supra-Conductibilité* (Hermann & Cie, Paris, 1937).

[2] M. von Laue, *Theorie der Supraleitung* (Springer-Verlag, Berlin, 1949).

[3] I. K. Kikoin and S. W. Gubar, J. Phys. U.S.S.R. III, 333 (1940).

[4] W. Meissner, Sitz. Bayerische Akademie der Wissenschaften (1948), p. 321.

To examine further the adequacy of this recent formulation, the torsional oscillations of a superconducting sphere in a magnetic field have been studied, and any electromagnetic forces tangent to the surface have been found to be below the limits of observation.

II. APPARATUS

The apparatus is shown schematically in Fig. 1. The sphere was cast in vacuum from spectroscopically pure tin and carefully machined to an inch in diameter. The departure from a true sphere was found to be less than 0.0004 inch. The sphere was attached by a Lucite holder to a glass rod some 80 cm long, at the top of which was a mirror. This assembly was supported by a 3-mil tungsten wire about 60 cm long which served as a torsion fiber, and gave a period of torsional oscillation in the neighborhood of 16.3 sec.

The sphere was cooled by immersion in liquid helium, which was itself surrounded by liquid air. The temperature was controlled by pumping off the helium vapor and was measured in terms of the vapor pressure. To avoid mechanical disturbances from the boiling helium, the sphere was surrounded by a glass tube open at the top to the vapor. By this means good thermal contact was maintained but mechanical contact with the liquid was avoided.

The magnetic field was provided by a pair of Helmholtz coils giving a field of 11.1×10^{-4} weber per sq. meter per ampere, uniform to better than 0.1 percent over the volume of the sphere. These coils were also used to compensate the horizontal component of the earth's field when cooling the sphere through the transition temperature. By means of such compensation any frozen-in moment was practically confined to the vertical direction. Without compensation of the earth's field, or with an additional horizontal field present while cooling the sphere, only an incomplete Meissner effect was observed, and some permanent moment was frozen-in. This moment led to a change in period in a magnetic field. It was not possible, however, to establish a relationship between the magnitude of the frozen-in moment and that of the external field in which the sphere was cooled.

III. RESULTS

The torsional oscillations were observed by means of a lamp and scale at about a meter. The damping was measured to an accuracy of about 1 percent by recording a series of maximum excursions to each side. The period was obtained to about one part in 1500 by means of a stop watch.

Figure 2 shows the damping coefficient as a function of the square of the magnetic field at a temperature of 4.2°K, and shows the expected linear relationship. The principal source of error is the change in the torsion fiber with time under the weight of the sphere. After the sphere was first suspended the damping due to the fiber steadily decreased, but it was not possible to wait long enough for it to reach a steady value. A relaxation of the stress caused the damping to increase again.

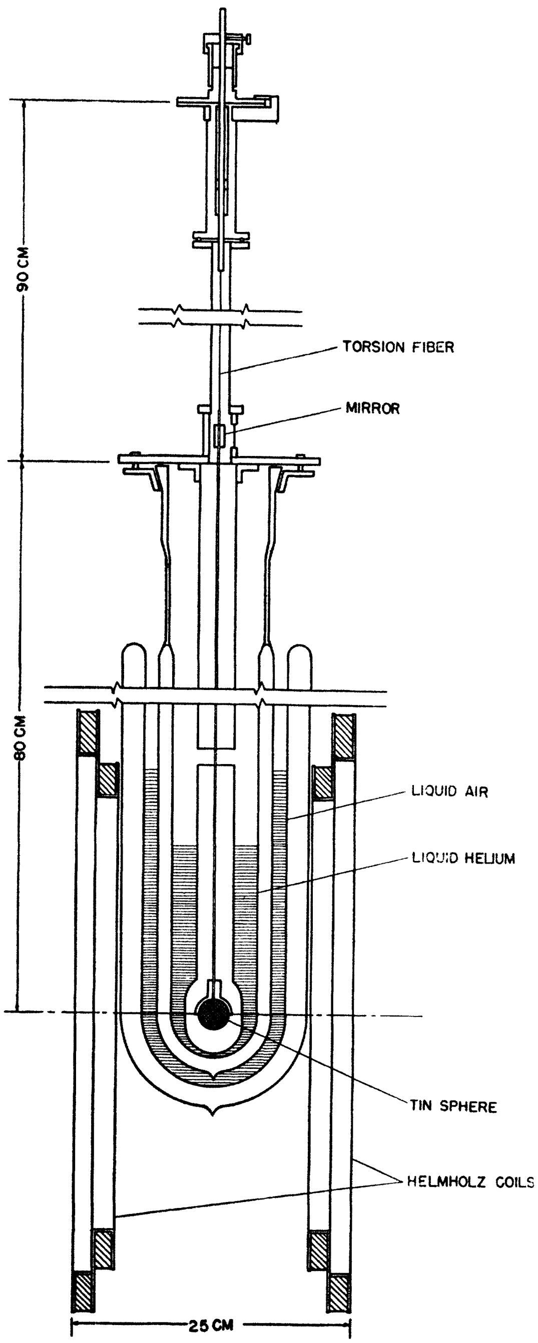

FIG. 1. Scheme of apparatus.

Figure 3 shows the damping coefficient at a temperature of 2.9°K. The observations were made in two series. The best least squares straight lines for the two series separately are designated as 1 and 2. Between series 1 and series 2 the suspension was relieved long enough to check the compensation of the magnetic field. The first reading after the sphere was again suspended led to the point marked A. This point was omitted in computing the lines, since it is obviously associated with changes in the suspension. The heavy line represents both series with a total of 24 observations. Its slope is less than 10^{-5} times the slope in Fig. 2.

Figure 4 shows the square of the period as a function of the square of the magnetic field at 4.2°K. The departures of the points from the least squares straight line represent about 0.01 sec. in period. In the superconducting state no such dependence of period on field strength was observed. The effect of the magnetic field on the period, below the transition temperature, could be associated with a small frozen-in moment.

IV. ANALYSIS AND CONCLUSIONS

One can immediately draw from these results the qualitative conclusion that the London currents in the superconductor appear to slide through the metal without any hindrance. To examine the question quantitatively, however, one must examine the currents to be expected, and the corresponding forces, in both the normal and the superconducting states.

Since the sphere oscillates through only a small angle, the field as seen from the sphere can be approximated closely by a constant field and a small alternating field at right angles to the first. The eddy current produced by the alternating field can be computed in the usual manner, and the force due to its interaction with the constant field gives the desired torque. The alternating field, and hence the torque, can be expressed in terms of the angle of rotation of the sphere. The computed torque is

$$L=-\frac{2\pi B_0{}^2R^3}{\mu_0}\left\{1-\frac{3}{x}\frac{\sinh x-\sin x}{\cosh x-\cos x}\right\}\theta -\frac{2\pi B_0{}^2R^3}{\mu_0\omega}\left\{\frac{3}{x}\frac{\sinh x+\sin x}{\cosh x-\cos x}-\frac{6}{x^2}\right\}\dot{\theta}, \quad (1)$$

where $x=(2\omega\mu_0R^2/\tau)^{1/2}$, R is the radius of the sphere, τ is its specific resistivity, ω is the angular frequency and μ_0 is necessary because of the use of rationalized MKS units.

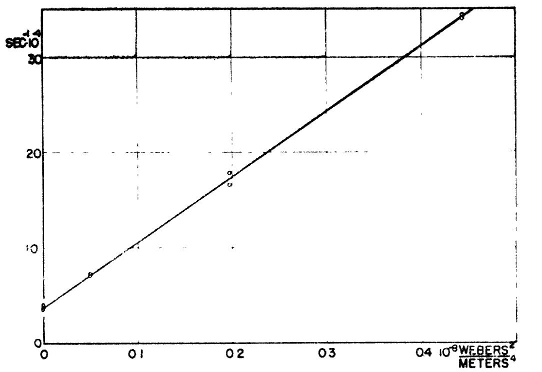

FIG. 2. Damping constant as a function of the magnetic induction with the tin sphere in the normal state at 4.2°K.

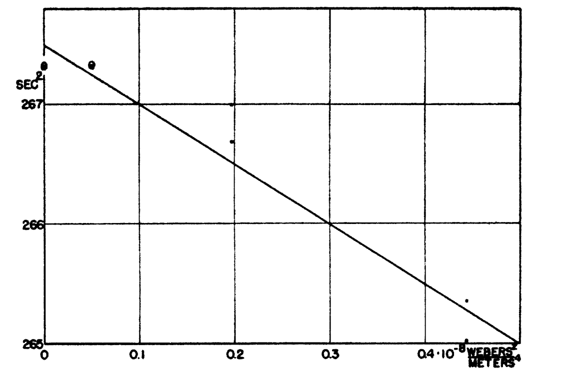

FIG. 4. Square of the period against square of the magnetic induction with the tin sphere in the normal state at 4.2°K.

The second term in Eq. (1), because of its proportionality to $\dot{\theta}$, represents a damping torque. The proportionality to B_0^2 is in accordance with the results plotted in Fig. 2. The magnitude of τ necessary to give the observed slope in Fig. 2 is a reasonable value for tin at 4.2°K.

The first term in Eq. (1) represents a restoring force which is very small until τ becomes small. However, with the τ for tin at 4.2°, x is about 2.6, so that an observable restoring force is to be expected. This is the interpretation of the period dependence on the magnetic field shown in Fig. 4.

The behavior of the sphere in the normal state is completely described by the ordinary eddy current theory in the range of conductivity for which it can be tested. One is then tempted to try to explain the results for the superconducting state along the same lines, by assuming a very small resistivity. As is shown in Fig. 5 the theoretical damping torque reaches a maximum at resistivities only a few times smaller than that of tin at 4.2°. The observed drop of the damping torque could then be explained as due to an increase in x by a factor somewhat over 10^5, corresponding to a decrease in τ by a factor of some 10^{10}. Such a change of resistance was, of course, the original interpretation of superconductivity. But Fig. 5 and Eq. (1) show that such a decrease in conductivity would also lead to a large restoring force and a decrease in period such as is not observed.

One might, however, question whether the restoring force is properly computed, since it is based on the interaction between the eddy currents and the constant field which is assumed to penetrate the sphere. If the conductivity of the sphere is really as high as must be assumed to account for the absence of damping, will the field penetrate the sphere in the time of the experiment or will it be excluded by the eddy currents?

Experimentally the dependence of the period on the magnetic field has been shown to increase with the increase in conductivity as far as its value at 4.2°K.

Theoretically if one considers the penetration of the field as a skin effect at a very low frequency but a very high conductivity, the skin depth for the small transverse field will be much less, in any case, than the penetration of the constant field which is maintained for over an hour at a time. Hence, while the calculated value of the restoring torque may not be quantitatively correct, there should at least be a significant decrease in period such as was not observed.

If the London theory is applied to the problem the damping torque and the restoring torque associated

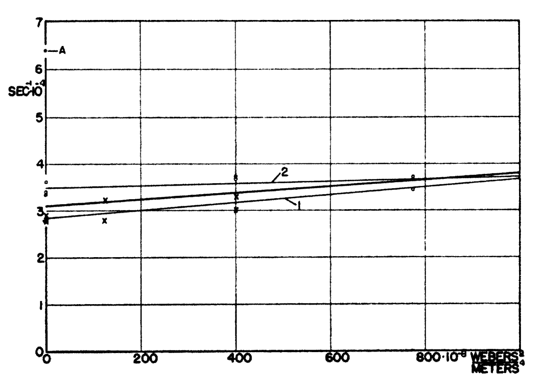

FIG. 3. Damping constant as a function of the magnetic induction with the tin sphere in the superconducting state at 2.9°K.

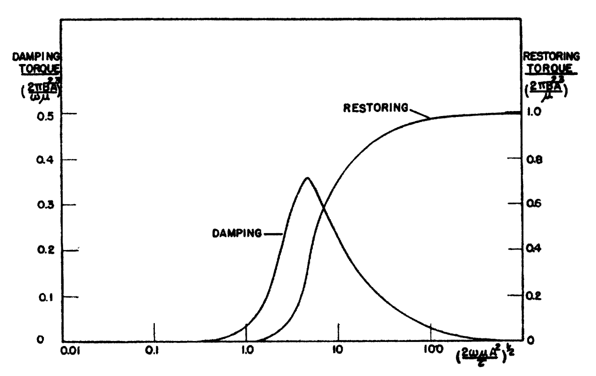

FIG. 5. Damping and restoring torques as a function of the specific resistivity as computed by ordinary methods for quasi stationary eddy currents.

with the magnetic field should be only those due to the small normal currents existing in the presence of the Meissner currents. A crude estimate suggests that these are far below the limits of accuracy of this experiment.

The results, then, support the London view of superconductivity, but a quantitative measurement of the constant λ requires an accuracy far exceeding that of this work.

PHYSICAL REVIEW VOLUME 79, NUMBER 6 SEPTEMBER 15, 1950

Electron Conductivity and Mean Free Paths

FRED P. ADLER
California Institute of Technology, Pasadena, California

AND

HENRY MARGENAU
Yale University, New Haven, Connecticut

(Received May 29, 1950)

Mean free path and electron density are calculated from measurements on the complex conductivity of a discharge. Formulas based on two alternative assumptions are tested against experimental facts; one is constancy of mean free path (independence of electron velocity), the other is constancy of mean free time.

IN a recent paper,[1] devoted to the measurement of complex electron conductivities in a positive column of a glow discharge, attention has been called to the opportunities which such experiments hold for gaining information about kinetic processes taking place in the conducting gas. It was shown that electron densities can be computed from the imaginary as well as the real part of the conductivity when the mean free path is given independently, and that the densities agree tolerably well. The purpose of the present note is to indicate how the mean free path itself can be determined from such measurements, and what light this sheds on other questions of interest in the kinetic theory of discharges.

The formula[2] used in the analysis of the experiments under discussion was based on the assumption of a constant mean free path of the electrons: λ is not a function of electron velocities v and hence is independent of the temperature, T. This is not generally true. An alternative and equally simple supposition is constancy of the mean free *time* τ, which is λ/v. What these hypotheses mean fundamentally is seen from the general relation

$$Nq\lambda=1, \qquad (1)$$

in which N is the number density of molecules intercepting the electrons (in this instance Hg atoms) and q the collision cross section, a function of v. It is clear that constancy of λ implies a constant q, whereas constancy of τ requires q to be proportional to v^{-1}. We shall discuss these two alternatives in sequence.

[1] F. P. Adler, J. App. Phys. **20**, 1125 (1949).
[2] H. Margenau, Phys. Rev. **69**, 508 (1946).

If $\lambda=$ const., the complex conductivity is given by

$$\sigma=\frac{4}{3}\frac{e^2\lambda n}{(2\pi mkT)^{\frac{1}{2}}}[K_2(x_1)-ix_1^{\frac{1}{2}}K_{\frac{3}{2}}(x_1)]\equiv\sigma_r-i\sigma_i, \qquad (2)$$

as was shown in reference 2. Here $x_1=(m\omega^2\lambda^2/2kT)$, and ω is the radian frequency of the microwaves. This formula is exact provided the distribution function for the electrons is Maxwellian, and T is the electron temperature. In reference 1, T was measured by probe methods. The ratio σ_i/σ_r is a function of x_1 alone, and it is plotted in reference 2. The experimental ratio leads therefore at once to knowledge of x_1 and, if T is known, to λ. Furthermore if the premise, $\lambda=$ const., is true, the values thus determined should not vary with T.

The situation is somewhat different if $\tau=$ const., and the analysis of reference 2 must be modified. Indeed the conductivity formula is then much simpler.[3] The equations lead to a distribution function which is strictly Maxwellian and corresponds to an electron temperature

$$T''=T+[M\gamma\tau^2/6k(1+\omega^2\tau^2)].$$

The complex conductivity[4] takes the form

$$\sigma=[ne^2\tau/m(1+\omega^2\tau^2)](1-i\omega\tau). \qquad (3)$$

Here the ratio σ_i/σ_r, which is $\omega\tau$, leads directly to the determination of τ. If the assumption $\tau=$ const. is true, the ratio test will thus reveal it. Both τ and λ are,

[3] Its derivation is given by J. H. Cahn, Phys. Rev. **75**, 838 (1949) and by E. Everhart and S. C. Brown, Phys. Rev. **76**, 839 (1949).
[4] This is very similar to the Lorentz formula for the conductivity of electrons. It results if the friction constant, g, in the equation of motion is equated to m/τ.