inside neutral lines give no extra divergence. This also leads to the finiteness of all neutron magnetic moment graphs, pointed out by Matthews as essential for renormalization to go through.)

By use of the above technique we easily get the following identities:

$$\Lambda_\mu{}^a(p,p) = -(1/2\pi)\partial\Sigma_P{}^*(p)/\partial p_\mu,$$
$$\Lambda_\mu{}^b(p,p) = -(1/2\pi i)\partial\Sigma_M{}^*(p)/\partial p_\mu,$$
$$\Theta_{\mu\nu}(p,p,0) = \tfrac{1}{2}\partial\Lambda_\mu{}^b(p,p)/\partial p_\mu,$$

so that

$$L^a = -B/2\pi, \quad L^b = R = -C/2\pi i,$$

while M^b is finite. The procedure above covers reducible as well as irreducible graphs.

To have a complete proof of renormalization one still must take account of scattering of mesons by mesons (M parts) and also of considerable overlaps between vertex and C parts. It has been verified, however, that the three-field mixture introduces no great complexities besides those associated with mesons in the electromagnetic field. Work on the complete problem is in progress.

After the above work was completed, the author received a communication from Mr. F. J. Dyson, proving similar identities for interaction of mesons with electromagnetic field, from general arguments of gauge invariance.

Acknowledgments are due Dr. P. T. Matthews, for suggesting the investigation and numerous discussions, and Dr. N. Kemmer for continual help and encouragement.

[1] P. T. Matthews, Phys. Rev. (to be published).
[2] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[3] J. C. Ward, Phys. Rev. **77**, 293 (1949); **78**, 182 (1950).

Isotope Effect and Lattice Properties in Superconductivity*

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IN a recent communication,[1] Serin, Reynolds, and Nesbitt have suggested that the isotopic dependence of the superconducting transition temperature in mercury can be expressed by the approximate relation $M^{\frac{1}{2}}T_c$ = constant, where M is the average atomic mass and T_c is the transition temperature. From this they have inferred that the ratio of the Debye temperature to the transition temperature is a constant for each of the different isotopes.

It should be pointed out, however, that the data represent a relatively small spread in M and T_c and consequently in trying to fit a relation of the form $M^\alpha T_c$ = constant, one finds that the product is approximately constant, to the degree noted by Serin, Reynolds, and Nesbitt, for a comparatively large range of α's. We have examined all of the available data on the isotopes of mercury for the purpose of determining the best value of the exponent. These data are given in Table I and are plotted for mercury in logarithmic form in Fig. 1. A least-squares fit of the best straight line to the points of Fig. 1 yields a slope of 0.378 $\pm$0.021. A line of slope $\frac{1}{2}$ is shown for purposes of comparison.

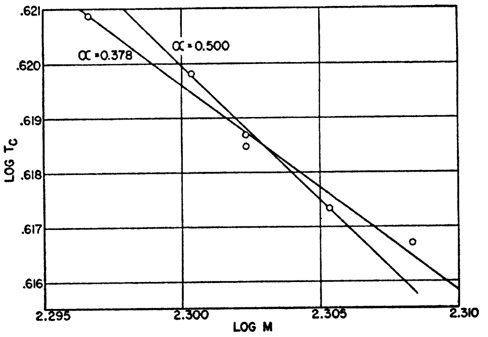

FIG. 1. Plot of $\log T_c$ *vs.* $\log M$ for mercury.

TABLE I. Isotope effect on the transition temperature.

Element	M	T_c
Hg^{198}	198.0	4.177[a]
Hg^{199}	199.7	4.167[b]
Nat. Hg	200.6	4.154[b]
Nat. Hg	200.6	4.156[a]
Hg^{202}	202.0	4.147[b]
Hg^{204}	203.4	4.137[b]
Sn^{124}	123.1	3.662[c]
Nat. Sn	118.7	3.715[d]

[a] E. Maxwell, Phys. Rev. **78**, 477 (1950).
[b] Serin, Reynolds, and Nesbitt, Phys. Rev. **78**, 813 (1950).
[c] E. Maxwell, Phys. Rev. **79**, 173 (1950).
[d] E. Laurmann and D. Shoenberg, Proc. Roy. Soc. **A198**, 560 (1949).

TABLE II. Possible theoretical relations of the form $M^\alpha T_c$ = constant.

Physical quantity assumed constant	Relation between M and T_c
Thermal energy of the lattice[a]	$M^{3/8}T_c$ = const.
Ratio of thermal energy to zero-point energy[a]	$M^{1/2}T_c$ = const.
Mean square linear momentum[a]	$M^{5/8}T_c$ = const.
Mean square thermal velocity[a]	$M^{1/8}T_c$ = const.
Ideal part of normal conductivity just before transition[b]	$M^{2/5}T_c$ = const.

[a] From the Debye theory.
[b] A. Sommerfeld and H. Bethe, *Handbuch der Physik* (Springer, Berlin, 1933), Vol. 24, part 2, p. 529, Eq. (37.16).

Recent results[2] on the isotope effect in Sn^{124} tend to reinforce this conclusion. One finds a value of 0.394 for the exponent using the data for tin in Table I.

We may ask whether the equation, $M^{0.378}T_c$ = constant, corresponds to the invariance of any significant physical quantity.[3] In Table II we list the relation between M and T_c which would hold if various quantities have critical values (independent of the particular isotope) at the transition temperature. From a purely empirical point of view the total thermal energy of the lattice is indicated as a first choice and the ideal part of the normal conductivity as a second. Evidently more experimental data are needed.

* Supported in part by the ONR.
[1] Serin, Reynolds, and Nesbitt, Phys. Rev. **78**, 813 (1950).
[2] E. Maxwell, Phys. Rev. **79**, 173 (1950).
[3] This constant is not the same for all elements, of course.

Ionization Chamber Bursts at Very High Altitudes

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AN investigation is being conducted to study the atmospheric absorption and the latitude dependence of the radiation which produces high energy nuclear disintegrations in the upper part of the atmosphere. The balloon-borne instruments which are being used throughout the projected series of measurements, contain a cylindrical electron-collection ionization chamber 20 cm long and 6.4 cm in diameter filled with pure argon at a pressure of 5 atmospheres. The ionization pulses are amplified and fed into a radio transmitter keying circuit which is biased to respond to pulses exceeding a preset amplitude. Each chamber contains

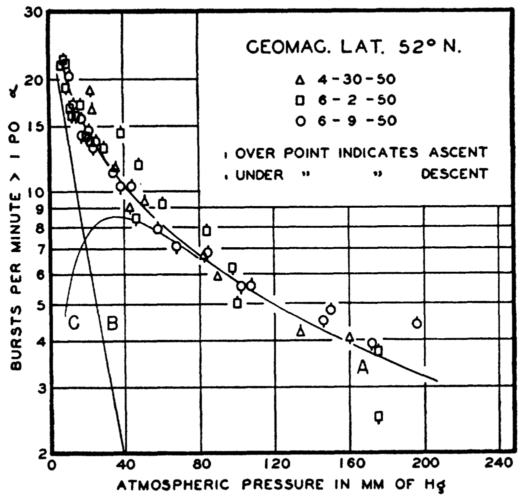

FIG. 1. *A*. Counting rate *vs.* atmospheric pressure for bursts exceeding 1 Po α. *B*. Calculated heavy-nucleus contribution to the total counting rate. *C*. Difference between *A* and *B*. -- bursts produced by protons, neutrons and α-particle.

an electrically controlled Po α-particle source which is used both for calibration and for checking the over-all stability of the instrument during ground tests. The usual radio-sonde methods are employed for relaying temperature and pressure data to the ground station.

In the first experiments a series of instruments biased to respond to pulses ≥ 1.0 Po α have been sent aloft to altitudes up to 103,000 ft. at geomagnetic latitude 52°N. The counting rates recorded in three flights are plotted as a function of atmospheric pressure in Fig. 1. Points obtained during ascent and descent are distinguished by separate symbols. In all the flights the statistical standard deviations in the counting rates vary from 20 percent for points in the neighborhood of 200 mm Hg to 10 percent for the points at the highest altitudes reached. The curve *A* is drawn so that it agrees with most of the points within the standard deviation.

The counts recorded in these measurements result predominantly from two sources: (1) nuclear evaporations occurring in the chamber walls, and (2) heavy nuclei (charge ≥ 8) of the primary radiation.

From photographic emulsion studies Bradt and Peters[1] have determined the mean free paths of heavy nuclei of various charges, and have estimated the intensity of primary nuclei of charge $Z>6$ to be 1.45×10^{-3}/cm^2/sec./steradian above the atmosphere. Assuming this value to be correct, the present chamber would be expected to give 41 heavy-nucleus counts/min. at the top of the atmosphere. If it is assumed further that the effective mean free path for collisions in which particles of $Z>8$ are broken down into lighter fragments which cannot be counted by the ionization chamber is 30 g/cm^2, we obtain curve *B* (Fig. 1) as the heavy nucleus contribution to the ionization chamber counting rate. This calculation takes into account the variation in path length with zenith angle, and is based upon an isotropic angular distribution at the top of the atmosphere.

The difference between the calculated heavy-nucleus counting rate and the total rate measured by the instrument is indicated by Fig. 1, curve *C*. Within the uncertainties involved in the calculation, this curve represents the altitude variation of total flux of protons, neutrons and α-particle, these being presumably the principal source of nuclear disintegrations in the chamber walls. The approximations used in arriving at the heavy-nucleus contribution are of such a nature that values given by curve *B* may be somewhat too high; however, it is doubtful that the over-all error exceeds 50 percent at any point on the calculated curve.

The most interesting feature of curve *C* is the maximum which appears at a depth of approximately 36 mm Hg (49 g/cm^2). The occurrence of such a maximum is a characteristic feature of a component of the cosmic radiation which becomes multiplied by a cascade process in penetrating the atmosphere. The fact that the peak occurs at a depth somewhat less than the geometrical mean free path for proton collisions ($\sim$70 g/cm^2) is to be expected from the wide zenith-angle distribution of the primary rays.

Similar measurements will be obtained at geomagnetic latitude 69°N during a forthcoming National Geographic Society expedition.

[1] H. L. Bradt and B. Peters, Phys. Rev. **77**, 54 (1950).

On the Origin of the Cosmic Radiation

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RECENT measurements[1,2] of the proton component of the cosmic radiation show that the differential energy spectrum, as shown in Fig. 1, appears to indicate that there is a maximum in the low energy end with a gradually increasing slope toward high energies. It is the purpose of this note to show that this fact could be a consequence of the circumstance that the majority of the protons originate from the nucleus of our galaxy.

A description of the mechanism of acceleration of protons in the interstellar spaces is given in Fermi's theory of the origin of the cosmic radiation.[3] According to this theory, low energy protons are accelerated by collisions with the wandering magnetic clouds in the interstellar spaces provided that their initial energies exceed a certain threshold value. Taking into account the average energy loss between collisions, the energy of a proton of initial energy w_0 (including rest energy) after N collisions is given by the formula:

$$w=(w_0-\epsilon/B^2)\exp(B^2N)+\epsilon/B^2 \tag{1}$$

where ϵ is the average energy loss between collisions and B is the average velocity of the wandering magnetic clouds in units of the velocity of light. Because we are working in such a high energy range it is sufficient, for present purposes, to take ϵ constant. The value of B is taken from Fermi's paper to be 10^{-4}.

Now assume that protons originate from some source at a distance r from the earth, and that some of them reach the earth after many collisions. We can therefore apply the principle of random flight to deduce the energy distribution which reads:

$$P(r,y)dy=\frac{s}{(\pi R^2\ln y)^{\frac{3}{2}}}\cdot\exp\left(-\frac{r^2}{R^2\ln y}\right)\cdot\frac{dy}{y^{1+s}}, \tag{2}$$

where

$$y=\frac{w-\epsilon/B^2}{w_0-\epsilon/B^2},\quad R^2=\frac{4L^2}{3B^2},\quad s=\frac{\sigma}{M}\left(\frac{D\rho}{B^2}\right).$$

Here L is the average linear displacement between collisions, σ is the proton absorption cross section per nucleon, M is the proton mass, D is the average length of path between collisions, and ρ is the average density of the interstellar matter.

Equation (2) gives the expected spectrum of the proton component if protons originated from a "point source." The adjustment of Eq. (2) to fit the experimental results is facilitated by the following facts.

(a) The experimental energy spectrum around $E\approx 10$ Bev has the form $1/E^2$, and that around $E\approx 10^4$ Bev has the form $1/E^{2.7}$ (Hilberry's spectrum). This fact suggests that $s=1.5$ in our Eq. (2). Taking $\sigma=2.5\times 10^{-25}$ cm^2 from Fermi's paper, and the energy loss of a high energy proton in passing through the interstellar medium as about 8 Mev/g/cm^2, we find $(w-\epsilon/B^2)\approx$ the kinetic energy E of the proton.