We wish to thank Frank Wagner, Jr. for assistance in operating the spectrometer and Jack May for his aid in the computations. A more detailed report of this work will be submitted to this Iournal.

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## Temperature Dependence of the Energy Gap in Germanium from Conductivity and Hall Data\*

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PTICAL absorption experiments, measurements of the threshold of internal photoelectric effects, and consideration of crystal volume change with temperature all show that the energy gaps in silicon and germanium decrease with rising temperature.1 The same result has been obtained by analysis of the high temperature resistivity and Hall data for silicon.<sup>2</sup> We describe here the calculation of the temperature dependence of the energy gap in germanium from high temperature (500°K to 850°K) conductivity and Hall data.

The free electron concentration,  $n_e$ , and hole concentration,  $n_h$ , in a semiconductor in thermal equilibrium<sup>3</sup> are related by

$$n_e n_h = A^2 T^3 \exp(-E_G/kT), \tag{1}$$

where  $E_G$  is the energy gap between the full and conduction bands, and

$$A^{2} = 32h^{-6}(m_{e}m_{h})^{\frac{3}{2}}(\pi k)^{3}, \qquad (2)$$

where k is the Boltzmann constant, h the Planck constant, and  $m_e$  and  $m_h$  the effective electron and hole masses. If  $m_e \approx m_h \approx m_0$ , the free electron mass, then  $A = 4.84 \times 10^{15}$  cm<sup>-3</sup> deg.<sup>-3</sup>. If one writes

$$E_G = E_G^0 + (\partial E_G / \partial T)T, \qquad (3)$$

 $(n_e n_h/T^3)^{\frac{1}{2}} = A' \exp\{-E_G^0/(2kT)\},\$ (4)

$$A' = A \exp\{-\left(\frac{\partial E_G}{\partial T}\right)/(2k)\}.$$
(5)

The  $n_e$  and  $n_h$  values at various temperatures, high enough for impurity scattering to be negligible, were calculated from the measured conductivity  $(\sigma)$  and Hall coefficient (R) curves for several N-type germanium samples.<sup>4</sup> First, c, the ratio of electron to hole mobility, is calculated from

$$1 - \frac{1}{c} = \frac{-3\pi e R (\sigma/eb_e)^2 / 8 - N}{(\sigma/eb_e) - N},$$
(6)

where  $b_e$  is the electron mobility and N the electron density at exhaustion. In view of recent measurements<sup>5</sup> and calculations<sup>6</sup> pertaining to electron mobility in germanium, we used  $b_e = 17$  $\times 10^{6} T^{-\frac{1}{2}} \text{ cm}^{2}/\text{volt-sec.}$  Thus c is found to be approximately 1.5 for all samples. Then  $n_h$  is calculated from

$$n_h = \{(\sigma/eb_e) - N\} (1 + 1/c)^{-1}, \tag{7}$$

and  $n_e$  from  $n_h + N$ . When  $\ln\{(n_e n_h)^{\frac{1}{2}}/T^{\frac{1}{2}}\}$  is plotted vs. 1/T, a straight line is obtained for each sample (Fig. 1). The slope is



FIG. 1. Plot of  $(n_e n_h/T^3)^{\frac{1}{2}}$  vs. 1/T for two of the germanium samples investigated. The  $n_e$  and  $n_h$  values are calculated from measured conductivities and Hall curves.

essentially the same for each sample and its value determines  $E_{G^0}$ as about 0.73 ev. The average of the intercepts gives  $A' = 9.2 \times 10^{15}$ cm<sup>-3</sup> deg.<sup>-1/2</sup>. By use of Eq. (5), one obtains  $\partial E_G/\partial T = (-1.1 \pm 0.1)$  $\times 10^{-4}$  ev/°K if  $(m_e m_h/m_0^2)^{\frac{3}{4}} = 1$ . This value compares well with the value calculated by Fan1 by considering volume change and the thermal excitation of lattice vibrations, but is too low by a factor of 4 to agree with the optical measurements.7 Two possibilities should be considered in connection with this discrepancy: (1)  $\partial E_G / \partial T$  would have a larger negative value if  $m_e m_h$  is less than  $m_0^2$ ; and (2) the value of  $\partial E_G/\partial T$  may be a function of temperature. However, since the R and  $\sigma$  data indicate that  $\partial E_G/\partial T$  is constant between 500°K and 850°K, and the optical data indicate a constant value, four times larger, between 80°K and 300°K, it is not likely that one value changes to the other in the intervening temperature range; this point will be checked by extending the optical measurements to high temperatures.

\* Assisted by Signal Corps Contract.
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<sup>7</sup> M. Becker, private communication.

## Emission of Neutral Radiation in **Cosmic-Ray Stars**

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N a search for electron pairs associated with meson showers in photographic emulsions we have so far obtained one fairly definite case of an identified electron pair associated with a 43-pronged star. We also find numerous electron pairs not obviously associated with stars,<sup>1,2</sup> with a fairly wide distribution in energy around several hundred Mev.

Figure 1 shows a photo-micrograph of the event, found in Ilford G5 emulsion  $(300\mu)$  developed to make minimum ionization tracks particularly visible. The pair originates  $120\mu$  from the star center, inside a wide angle cone of about 15 shower particles. The electrons were identified by grain counting and multiple