On the Evaluation of the Fermi β-Distribution Function

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IT was recently announced[1] that an accurate computation is in progress of the Coulomb correction factor involved in Fermi's theory of β-decay. The need for this computation apparently occurs because the literature does not contain a satisfactory analytic expression for the function,

$$f(Z,\eta)=\eta^{2+2S}e^{\pi y}|\,\Gamma(1+S+iy)\,|^2 \qquad (1)$$

in the notation of reference 1.

Evaluation of Eq. (1) is difficult only because it depends upon the magnitude of the complex gamma-function. It is the purpose of this letter to point out that the complex gamma-function can be evaluated by means of the asymptotic formula for $\Gamma(z)$ for z large, *even though the value of* $|z|$ *may only be of order unity*. None of the approximate formulas quoted in reference 1 appear to utilize this property of $\Gamma(z)$.

The asymptotic expansion in question is given by the equation[2]

$$\ln\Gamma(z)=\tfrac{1}{2}\ln 2\pi+(z-\tfrac{1}{2})\ln z-z+1/12z+J_1(z), \qquad (2)$$

where, for the conditions of this problem

$$|J_1(z)|<1/360(1-\alpha^2)(1+\alpha^2/\eta^2)^{\frac{1}{2}}<1/200.$$

Calculations based on Eq. (2) with $J_1(z)=0$ will thus provide an accuracy for $f(Z,\eta)$ of better than one percent, and a somewhat better accuracy still for the shape of the curve (about three-tenths percent for the worst case of reference 1 with $Z=90$). On the basis of this procedure the following formula is obtained

$$f(Z,\eta)\sim 2\pi\eta^{2x}\exp(-2x+2y\tan^{-1}x/y)(1+\alpha^2/\eta^2)^{x-\frac{1}{2}} \times\exp\{1+x/6(1+\alpha^2/\eta^2)\}, \qquad (3)$$

where $x=(1-\alpha^2)^{\frac{1}{2}}$, $y=\alpha(1+\eta^2)^{\frac{1}{2}}/\eta$, $\alpha=Z/137$.

If greater accuracy is desired another term may be retained in the expansion of $\Gamma(z)$. In this case the neglected remainder, J_2, amounts to less than three-tenths percent in the resulting expression for $f(Z,\eta)$. The accuracy provided by Eq. (3) is probably more than adequate for use with available experimental data.

[1] I. Feister, Phys. Rev. **78**, 375 (1950).
[2] T. M. MacRobert, *Functions of a Complex Variable* (Macmillan Publishing Company, New York, 1917), p. 149.

Physical Theory of Ferromagnetic Domains

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A GENERALLY excellent review bearing the title of this note has recently been published by Kittel.[1] This review, however, fails to mention at least one rather fundamental contribution to the subject, and, possibly as a result of this oversight, claims priority for the observation by Williams and Shockley[2] that Barkhausen noises may be associated with minor fluctuations of a boundary between two ferromagnetic domains rather than with transfer of a whole domain from one direction of magnetization to another.

Such stepwise advance of a domain boundary, in the case of cobalt, was long ago described in detail by Elmore[3] and the significance of this observation in the calculations of domain size was clearly recognized in the last previous American review of this subject by Brown.[4] This review is one of the most important papers omitted from Kittel's bibliography.

The other gaps in the historical background are much less important and are believed to be due in part to the gradual shift in emphasis, during the early history of domain investigations, from crystal anisotropy to strain anisotropy as the most usual determinant for domain or sub-domain boundary fixation. Some of the pertinent stages in this history are mentioned in an accompanying note entitled "Ferromagnetic Block."

The value of Kittel's Appendix B, on crystal lattice sums of dipole arrays, would have been greater if it had mentioned the conditional convergence of the lattice sums presented, and had given some indication of work before[5] and since[6] the single 1930 paper[7] cited. The relative unimportance of dipole magnetic fields in usual materials is now obvious, as Kittel points out in this appendix, but they may yet amount to something in sensibly strain-free single crystals where local magnetic fields of very small magnitude, in comparison with the molecular field, have such striking effects in locating domain boundaries.

[1] C. Kittel, Rev. Mod. Phys. **21**, 541 (1949).
[2] H. J. Williams and W. Shockley, Phys. Rev. **75**, 178 (1949).
[3] W. C. Elmore, Phys. Rev. **53**, 757 (1938).
[4] W. F. Brown, Jr., J. App. Phys. **11**, 160 (1940).
[5] N. S. Akulov, Zeits. f. Physik **52**, 389 (1928).
[6] T. Hayasi, Zeits. f. Physik **72**, 177 (1931); **91**, 818 (1934); F. C. Powell, Proc. Camb. Phil. Soc. **27**, 561 (1931); L. W. McKeehan, Phys. Rev. **43**, 1022 (1933); J. M. Luttinger and L. Tisza, Phys. Rev. **70**, 954 (1946) corrected in Phys. Rev. **72**, 257 (1947).
[7] R. Becker, Zeits. f. Physik **62**, 253 (1930).

Nuclear Size Resonances

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RECENT data on slow neutron scattering by nuclei[1] have been examined to look for a possible regular variation of scattering amplitude with nuclear size, or with mass number, A. The bound scattering amplitude (=scattering length) plotted *vs.* mass number in Fig. 1 gives some indication of such a regularity, but is by no means conclusive.

If the neutron wave function is influenced in a regular manner by the nuclear size, the scattering amplitude plotted against the mass number curve should show resonances similar to those which appear when this quantity is plotted against neutron energy for a single element. The data indicate one such resonance in the vicinity of $A=55$. The scattering amplitude of Fe^{56} is high; that of Ni^{58} is very high. Mn^{55} and Ti^{48} have negative amplitudes. (The signs for V^{51} and Cr^{52} are unknown.)

Another resonance is indicated near $A=8$. Li^7 has a negative scattering amplitude; Be^9 has a large and positive amplitude. The scattering amplitudes show a regularly decreasing trend from $A=9$ to $A=50$ with the notable exception of Cl. The hard sphere scattering amplitude, however, would be everywhere increasing with A.

If the neutron-nucleus system is close to resonance, the slope of the wave function at the edge of the nucleus is near zero. Small changes in nuclear parameters, or changes in spin orientation will therefore cause large changes in scattering amplitude. Hence if the nuclear spin is different from zero, or if more than one isotope is present, the possibility of destructive interference exists, so that small coherent cross sections are not unexpected in this region. Thus the extremely small coherent cross section of V^{51} can be explained on the assumption that the system is near resonance for this element. Also Ti^{48} and Co^{59} have unusually small coherent cross sections, and that of Mn^{55} is rather small. (Fe^{56}, Ni^{58}, and Ni^{60}, being single isotopes of zero spin, do not have small coherent

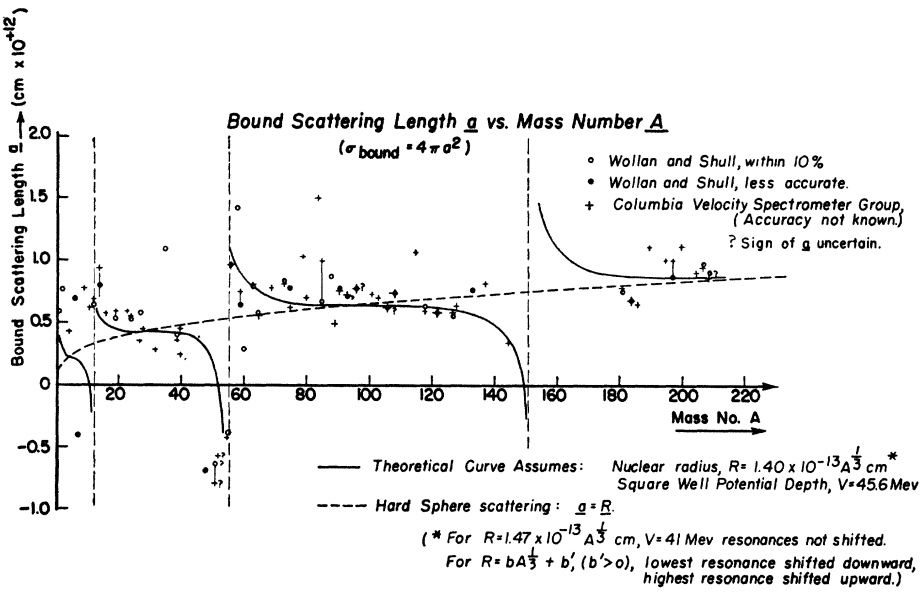

FIG. 1. Scattering of slow neutrons by nuclei.

cross sections.) Most other elements show coherent cross sections nearly as large as the bound cross sections.

Several models would predict qualitatively such resonance behavior. (a) A simple square well (without absorption) would give the plotted curve by adjusting the well depth to yield a resonance at $A=55+$, and using $R=1.40\times10^{-13}A^{\frac{1}{3}}$ cm. One obtains in this way a depth V=45 Mev. In this model, one assumes that the neutron moves in the average field of all the other nucleons, and that the collisions with the fluctuating part of the nucleon potential can be neglected to a first approximation. This is just the free particle approximation which underlies the shell structure model.

With this model, two other resonances are predicted, one at $A\cong12$, the other at $A\cong151$. For light nuclei, however, the formula for nuclear radius breaks down. Then $R/A^{\frac{1}{3}}>1.40\times10^{-13}$ cm, which depresses the $A=12$ resonance, so that the observed resonance at $A=8$ is reasonable. Unfortunately, no scattering amplitudes are known near $A=151$ except Nd^{145}, which is small. The most pronounced maximum in thermal neutron absorption cross sections occurs at about $A=155$, however. Otherwise the absorption cross sections do not vary very regularly, since they show arge isotope dependence (large for neutron deficient isotopes) and are more strongly influenced by low lying sharp resonance levels.

(b) The excitation of longitudinal nuclear waves (liquid droplet model) would predict resonances for values of the nuclear radius such that $(\frac{1}{2}n+\frac{1}{4})\lambda=R$, where $\lambda=$wave-length of the nuclear compression wave, and n is an integer. This is the same condition as that with the square well model, where the compression wavelength is replaced by the neutron wave-length inside the well, and could give resonances at the same places by adjustment of a single parameter. This demands an "inverse nuclear compressibility" of about 7 Mev$\times A^{\frac{1}{3}}/r_0^2(r_0\equiv R/A^{\frac{1}{3}})$, much smaller than Feenberg's estimate[2] of 50 to 100 Mev$\times A^{\frac{1}{3}}/r_0^2$. Only radially symmetric vibrations would be expected to be excited by thermal neutrons.

(c) An energy level varying in a regular way among the elements (decreasing with increasing A) would predict such a resonance behavior. It would need to be either very broad (~Mev) or very slowly varying with A. Such an energy level has no known theoretical justification.

[1] Wollan and Shull, via K. Way, mostly unpublished; Havens and Taylor Nucleonics 6-2, 74 (1950).
[2] E. Feenberg, Rev. Mod. Phys. 19, 239 (1947).