In conclusion, we want to express our thanks to R. Landshoff, E. Teller, S. Ulam, and J. Wheeler for valuable discussions.

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[5] See, for example, S. Rosseland, *The Pulsation Theory of Variable Stars* (Oxford University Press, London, 1949), in particular, Figs. 5, 6 and 8.

Pressure Broadening of Spectral Lines

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This note presents a brief summary of some theoretical work on the development of the statistical theory of pressure broadening in terms of the basic "Fourier integral" approach. A more complete report will appear at a later date.

We use here the simplest version of the Fourier integral theory, which assumes the non-degeneracy of both initial and final energy levels. Under this condition, essentially the only effect of interaction between a radiating atom and its neighbors is the modulation of the unperturbed line frequency, ω_0, with a "frequency perturbation" $\omega_p(t)$ equal to $1/\hbar$ times the instantaneous relative shift of the two energy levels. We introduce the "phase perturbation" $\varphi(t)=\int^t\omega_p(t)dt$ and set the zero of our frequency scale at ω_0. The Fourier integral formulation for the spectral intensity, $I(\omega)$, then reads[1]

$$I(\omega)=\langle|A(\omega)|^2\rangle_{\text{Av}},\tag{1}$$

$$A(\omega)=(2\pi T)^{-\frac{1}{2}}\int_{-T/2}^{+T/2}\exp\{i[\varphi(t)-\omega t]\}dt.\tag{2}$$

In these relations, the symbol $\langle\;\rangle_{\text{Av}}$ denotes an average over all types of collision, T is an arbitrarily large time interval, and the factor $(2\pi T)^{-\frac{1}{2}}$ takes care of the normalization of $I(\omega)$.

The derivation[2] of the statistical intensity distribution from (1) and (2) proceeds as follows. We take ω large enough so that, on the time scale in which $\omega_p(t)$ and its derivatives undergo appreciable relative variations, $e^{i\omega t}$ is a rapidly oscillating function. In this case the whole integrand of (2) oscillates rapidly except at points t_i such that $d\varphi/dt_i\equiv\omega_p(t_i)=\omega$. Hence, only the neighborhoods of these "coincidence" points contribute significantly to the integral. Developing $\varphi(t)$ in a Taylor series about each t_i and dropping terms of order higher than $(t-t_i)^2$, we have

$$\begin{aligned}A(\omega)&\cong(2\pi T)^{-\frac{1}{2}}\Sigma_i\exp\{i[\varphi(t_i)-\omega t_i]\}\\&\quad\times\int_{-\infty}^{+\infty}\exp[(i/2)(d\omega_p/dt_i)(t-t_i)^2]dt\\&=\Sigma_i\exp\{\varphi(t_i)-\omega t_i\pm i\pi/4]\}(T|d\omega_p/dt_i|)^{-\frac{1}{2}},\end{aligned}\tag{3}$$

the plus (minus) sign in the exponent prevailing when $d\omega_p/dt_i$ is positive (negative). Inserting (3) into (1) and ignoring phase correlations between different coincidence points, we obtain

$$I(\omega)=(1/T)\langle\Sigma_i|d\omega_p/dt_i|^{-1}\rangle_{\text{Av}}\tag{4}$$

which represents the statistical distribution in its most general form. Namely, the right-hand side of (4) is equal to the *occurrence distribution* of $\omega_p(t)$, i.e., that fraction of the time interval T for which $\omega_p(t)$ is contained in a unit frequency range at ω.

In our detailed calculations, aimed at determining the accuracy and domain of applicability of the above derivation of the statistical distribution, we have treated the case in which the interaction of a radiating atom with its neighbors consists of a succession of binary collisions (low density of perturbing atoms). Each of these encounters is assumed to provide a frequency modulation of the form

$$\omega_p(t)=C/r^n=C/[R_\alpha^2+v^2(t-t_\alpha)^2]^{n/2},\tag{5}$$

where C and n are interaction constants and v the relative velocity of colliding atoms. The averaging operation of (1) consists in summing over all values of the impact parameter, R_α, and time of closest approach, t_α, for each collision.

On introducing (5) into (2) and evaluating the integrals by the method of steepest descents,[3] we obtain

$$A(\omega)=\Sigma_\alpha\exp\{i[\varphi(t_\alpha)-\omega t_\alpha]\}A_\alpha(\omega)\tag{6}$$

where $A_\alpha(\omega)$, the contribution of the αth collision, is given by an asymptotic series of the form

$$\begin{aligned}A_\alpha(\omega)\cong&2(T|d\omega_p/dt_i|)^{-\frac{1}{2}}\cos[\varphi(t_i)-\varphi(t_\alpha)-\omega(t_i-t_\alpha)-\pi/4]\\&+\text{terms in }d^2\omega_p/dt_i^2,\quad d^3\omega_p/dt_i^3\cdots.\end{aligned}\tag{7}$$

In this expression, $t_i=t_\alpha+[(C/\omega)^{2/n}-R_\alpha^2]^{\frac{1}{2}}$. It is found that, for large ω—more precisely,[4] $\omega^{1-1/n}C^{1/n}/v\gg1$—Eq. (7) constitutes a satisfactory approximation for all $R_\alpha<(C/\omega)^{1/n}$ exclusive of the immediate neighborhood of the point $R_\alpha=(C/\omega)^{1/n}$ (at which $t_i=t_\alpha$). In this region we use an alternate approach. Expanding (5) binominally in powers of $v^2(t-t_\alpha)^2/R_\alpha^2$, we find it possible to express $A_\alpha(\omega)$ as a series of terms involving Airy's integral[5]

$$\int_{-\infty}^{+\infty}\exp[i(t^3\pm xt)]dt.$$

As $(C/\omega)^{1/n}-R_\alpha$ increases positively, the series in question passes over smoothly into that given by (7); as $(C/\omega)^{1/n}-R_\alpha$ increases negatively (no point of coincidence in the collision), $A_\alpha(\omega)$ drops rapidly to zero.

We now insert (6) into (1) and average over the t_α and R_α. The main effect of the average over t_α is to eliminate any coherence between the contributions of different collisions. In averaging over the R_α, we simply group the collisions according to the values of their impact parameters and integrate. We thus obtain

$$I(\omega)=\int_0^\infty|A_\alpha(\omega)|^2\cdot T\cdot2\pi NvR_\alpha dR_\alpha,\tag{8}$$

where N is equal to the density of perturbing atoms.

Detailed calculations following the procedure outlined above yield for $I(\omega)$ the expression

$$\begin{aligned}I(\omega)=\frac{4\pi NC^{3/n}}{n\omega^{1+3/n}}\{1-(n/36)(1+1/2n)(1-1/n^2)\\\times[\omega^{1-1/n}C^{1/n}/v]^{-2}\},\end{aligned}\tag{9}$$

which constitutes the first two terms of an asymptotic series in increasing negative powers of $\omega^{1-1/n}C^{1/n}/v$.

The factor in front of the curly bracket of (9) is the well-known statistical distribution of Kuhn and Margenau[6] for the case of binary collisions. From (9) it is obvious that this distribution is valid when $\omega\gg v^{n/(n-1)}/C^{1/(n-1)}$ (i.e., in the wing of the line). Now, Spitzer[7] has shown that the impact theory of pressure broadening is valid when $\omega\ll v^{n/(n-1)}/C^{1/(n-1)}$ (i.e., in the vicinity of the unperturbed line). Finally, in the case of binary collisions, the ratio of the statistical to the impact distribution[8] turns out to be $\sim(\omega^{1-1/n}C^{1/n}/v)^{(n-3)/(n-1)}$ which is equal to unity for $\omega=v^{n/(n-1)}C^{1/(n-1)}$. These results exhibit the transition between the two theories as well as the domains of applicability of each.

[1] See, for example, H. M. Foley, Phys. Rev. **69**, 616 (1946), p. 619, second column, first equation (where $P(t)$ is equal to our $\omega_p(t)$). This as well as the present formulation requires the additional specification that ω_0 is large compared to ω; such a restriction excludes certain cases of microwave broadening in which one is interested in values of ω comparable to ω_0.
[2] The method was previously employed in a quantum-mechanical treatment of pressure broadening by A. Jablonski, Phys. Rev. **68**, 78 (1945).
[3] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, New York, 1945), second edition, pp. 235, 236.
[4] It is instructive to write $\omega^{1-1/n}C^{1/n}/v$ in the form $\omega\tau_c$, where $\tau_c=(C/\omega)^{1/n}/v$ may be considered as the "time of collision."
[5] Reference 3, pp. 188–190.
[6] H. Kuhn, Phil. Mag. **18**, 987 (1934); H. Margenau, Phys. Rev. **48**, 755 (1935).
[7] L. Spitzer, Jr., Phys. Rev. **58**, 348 (1940).
[8] Reference 1, Eqs. (16), (17) and intervening text. For the case of binary collisions the denominator of (16) reduces to ω^2; the quantity γ in (17) is our C multiplied by an unimportant numerical factor.