where

$$r_0^{-2}=\frac{64\pi^2 me^2}{h^3}\left(\frac{1}{2}\frac{\gamma^2}{\xi_\infty^{1/3}}+\gamma+\tfrac{1}{2}\xi_\infty^{1/3}\right)$$
$$=\frac{4(3n_\infty/\pi)^{2/3}}{a_0(3n_\infty/\pi)^{1/3}-(\nu/2\pi^2)}, \tag{7}$$

a_0 is the Bohr radius.

For $\nu=0$ this expression agrees with that obtained by Mott[3] who considered the screened Coulomb field in a metal ignoring exchange terms.

The potential is given by

$$V=-e^2\exp(-r/r_0)/r \tag{8}$$

and the charge distribution is

$$n(r)-n(\infty)=(1/4\pi r_0^2)\exp(-r/r_0)/r. \tag{9}$$

Sachs and Mayer[4] found that for a potential of the form (8) to have no bound state we must have

$$r_0<0.842a_0.$$

This criterion for our case leads to

$$4(0.842)^2(3n_\infty a_0^3/\pi)^{2/3}>(3n_\infty a_0^3/\pi)^{1/3}-(\nu/2\pi^2). \tag{10}$$

The inequality (10) is always satisfied if

$$\nu>1.7. \tag{11}$$

Equation (11) appears to be generally valid so that our model yields the result that a proton in a metal will have no bound state.

An analysis identical to that given above shows that a hydrogen-like atom with charge greater than unity will become singly ionized but not doubly ionized.

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The Virial Theorem and the Variation Principle

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IF, instead of using the correct wave function the best wave function of a certain form is used, the question arises as to whether or not the virial theorem is satisfied. The purpose of this note is to point out that the virial theorem is valid when the wave function is obtained by a variation of the expectation value of the Hamiltonian with respect to a change in scale and also under a slight but important generalization of this variation. Thus, suppose that

$$\psi'(x_i)\equiv\psi'(x_1, y_1, z_1; x_2, y_2, z_2; \cdots x_n, y_n, z_n) \tag{1}$$

is an arbitrary wave function for n particles. If, using the wave function

$$A(\lambda)\psi'(\lambda x_i)\equiv A(\lambda)\psi'(\lambda x_1, \lambda y_1, \lambda z_1; \cdots \lambda x_n, \lambda y_n, \lambda z_n) \tag{2}$$

the expectation value of the Hamiltonian is formed, then λ is determined from

$$\frac{\partial}{\partial\lambda}A^2\int\psi'^*H\psi' d^{3n}x=0; \tag{3}$$

A is a normalization factor.

Let

$$A(\lambda_m)\psi'(\lambda_m x_i)=\psi(x_i), \tag{4}$$

where λ_m satisfies (3).

Then we know that

$$\left[\frac{\partial}{\partial\lambda}\int N^2\psi^*(\lambda x_i)H\psi(\lambda x_i)d^{3n}x\right]_{\lambda=1}=0, \tag{5}$$

where N is determined from

$$N^2\int|\psi(\lambda x_i)|^2 d^{3n}x=1$$

or

$$N^2=\lambda^{3n}, \tag{6}$$

since $\psi(x_i)$ is normalized.

If

$$H=-\Sigma_i\frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial x_i^2}+V(x_i).$$

Equation (5) now reads

$$\left[\frac{\partial}{\partial\lambda}\left(\lambda^2\bar{T}+\int\psi^*(\xi_i)V(\xi_i/\lambda)\psi(\xi_i)d^{3n}\xi\right)\right]_{\lambda=1}=0,$$

$$\bar{T}\equiv-\int\psi^*(x_i)\Sigma_i\frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial x_i^2}\psi(x_i)d^{3n}x.$$

But

$$\left[\frac{\partial}{\partial\lambda}V(\xi_i/\lambda)\right]_{\lambda=1}=-\Sigma_i\xi_i\frac{\partial V(\xi_i)}{\partial\xi_i}. \tag{7}$$

Therefore

$$2\bar{T}-\Sigma_i\left\langle x_i\frac{\partial V(x_i)}{\partial x_i}\right\rangle_{\text{Av}}, \tag{8}$$

which is the virial theorem.

The above class of trial functions for which the virial theorem holds can be extended to include functions of the form $q(x_i)$, $f(x_i)$ where $q(x_i)$ is a homogeneous function of the x_i and only the scale of the argument of f need be varied. This includes cases like $r^l\exp(-\alpha\lambda r)$.

Now ψ may be a very poor trial function. Consequently the virial theorem cannot be used as a test of the "goodness" of the function chosen. On the other hand, since the above demonstration includes a great many cases which one meets in practice, it can be concluded that the virial theorem is applicable in most of the approximations commonly used.

The existence of a virial theorem depends only on the existence of an energy variation principle. Thus the above argument is applicable in the case of the statistical models, e.g., Thomas-Fermi, Thomas-Fermi-Dirac, the equations of which can be derived from a variation principle[1] in which the charge density is varied. Any trial charge density which has been fitted by a variation in scale will again satisfy a virial theorem.

The author would like to thank Professor C. Zener for very helpful discussions of this subject. This work arose through a conversation with Professor J. C. Slater.

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Superconductivity of Vanadium

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IT is well known that the superconductive properties of most samples of transition metals of the fourth and fifth groups of the periodic table are characterized by inordinately large values of the magnetic field required for the destruction of superconductivity. These values do not correspond to systems exhibiting a reversible "Meissner effect" and hence are of no value for thermodynamic calculations of the differences in specific heat of the normal and superconductive phases and of the heats of transition. Illustrations of this situation are to be found in the recent work on niobium[1] and uranium.[2] On the other hand, some samples of niobium and tantalum have been found to exhibit at least partial reversibility,[3] and the thermodynamic calculations have led to most interesting consequences.

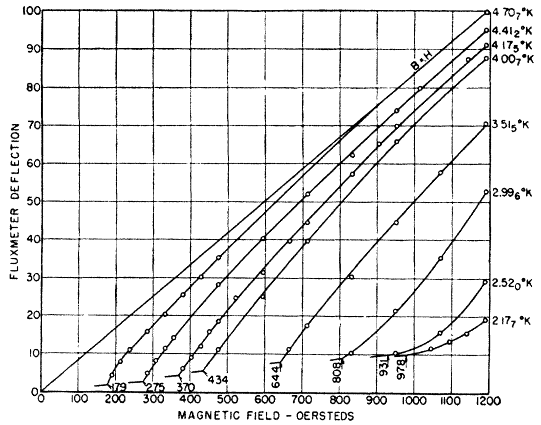

FIG. 1. Magnetic induction of superconductive vanadium.

It appeared to us that for the clarification of the nature of the properties of the hard superconductors the study of the superconductivity of vanadium would be most fruitful. In the first place, the temperature range in which this metal exhibits superconductivity is very convenient from an experimental viewpoint. Secondly, vanadium has a relatively low melting point, a factor which facilitates metallurgical treatment of the metal.

The superconductivity of vanadium, as determined by resistance measurements, was first reported by Meissner and Westerhoff.[4] The recent data of Webber *et al.*[5] gave an initial slope of 4100 oersteds per degree for a particular sample of vanadium.

Through the kindness of Dr. B. W. Gonser of the Battelle Memorial Institute a sample of pure vanadium was made available to us. The magnetic properties of the superconductive material were determined by a method which, in principle, was the same as that used by Keeley and Mendelssohn.[6] A coil of No. 40 copper wire was wrapped around the specimen which was a cylinder 0.070 in. in diameter and 1 in. long. The residual resistivity relative to the ice-point resistivity of a strip rolled from this cylinder was 0.051.

Isothermal measurements were made at each of eight temperatures. These consisted in the determination of a quantity proportional to the integral of the voltage induced in the coil when the field was reduced from a given value to zero. The data are indicated in Fig. 1. For each temperature, the fluxmeter deflection is a linear function of the field until a critical value is reached. These critical values have been assumed to represent the threshold fields at the various temperatures, and have been plotted in Fig. 2. The initial slope of this curve is 400 oersteds per degree; in general the curve is quite similar to that for tantalum on the basis of which Daunt and Mendelssohn have made thermodynamic calculations. From Fig. 1 it is seen that the "Meissner effect" is most pronounced near the critical temperature and that the reversibility becomes much poorer at lower temperatures.

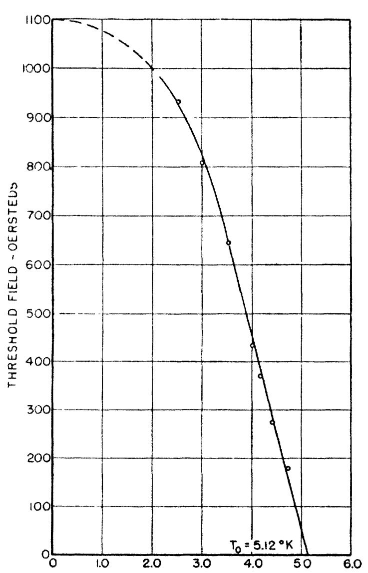

FIG. 2. Threshold fields for superconductive vanadium.

Work is in progress on the factors responsible for the irreversibility associated with the transition and hence the data herein reported are given provisionally.

A detailed account of the experimental methods used and of the effect of metallurgical factors upon the electrical and magnetic properties of superconductive vanadium will be reported in the near future.

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Cosmic Rays as the Source of General Galactic Radio Emission

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THE galactic radio emission is not a thermal free-free radiation of interstellar gas, as was first believed. The electronic temperature would have to be of the order of 100,000° in contradiction to all spectroscopic evidence which gives values around 10,000°. Stars could be considered as sources only under very artificial assumptions. The observed intensities, which must come from the outermost layers of stellar atmospheres, could not be blackbody radiation[1] and might be understood only in terms of coherent plasma oscillations of extended regions. The formation and maintenance of these oscillations is hardly possible in stellar atmospheres.[2]

It will now be shown that the general cosmic radiation of our star system is a high frequency source of sufficient power. In interstellar space, at least inside the interstellar clouds which occupy about 5 percent of space, the mean density of kinetic energy ought to be of the same order as the magnetic-field energy; therefore, fields of around 10^{-6} gauss are to be expected. An energetic electron with energy $W \gg m_0c^2$, which is circulating in this field, is radiating electromagnetic energy into a very narrow cone whose angular aperture is m_0c^2/W in the direction of motion. Therefore, an observer at rest receives very short pulses corresponding to a frequency which is very much higher than the classical Larmor frequency, ν_0. The mean spectral intensity distribution of this radiation will then be[3]

$$P(\nu) \approx (e^2/\pi R)(\nu/\nu_0)^{\frac{1}{3}}$$

for $\nu_0 \ll \nu < \nu_c$, where R is the radius of the electron's circular orbit and $\nu_c = \frac{2}{3}\nu_0(W/m_0c^2)^3$. If n_e is the number of electrons per cm^3 with energy W, the emissivity of high frequency radiation will be

$$\epsilon_\nu \Delta\nu = n_e P(\nu)\Delta\nu = (e^3H/\pi W)n_e(\nu/\nu_0)^{\frac{1}{3}}\Delta\nu \text{ ergs/cm}^3\text{/sec.}$$

This increases steadily with frequency until $\nu = \nu_c$ and then decreases rapidly. The observed distribution[1,4] within the frequency range of 10 to 3000 Mc seems rather to be $\propto \nu^{-0.3}$. We therefore expect to be already in a region with $\nu \geq \nu_c$. Also the