Letters to the Editor

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A Method of Approximation for Cooperative Phenomena

RYOICHI KIKUCHI
Department of Physics, University of Tokyo, Tokyo, Japan
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A NEW method for approximating the entropy of an order-disorder system having an assigned energy has been developed. By a suitable choice of variables it gives the Bethe-Takagi approximation[1,2] in one case, and the "variation method" introduced by Kramers and Wannier[3] in another. The new method is more intuitive than the latter and is more effective than the quasi-chemical method by Yin-Yuan Li.[4] It can be improved and extended successively.

Two-dimensional square net.—The probability of appearance of a configuration of a cell (a bond, a spin) given on the left column of each part of Fig. 1 is denoted by z_ν, y_ν, or x_ν on the corresponding second column. $\gamma_\nu(\beta_\nu, \alpha_\nu)$ denotes the number of configurations with the same probability z_ν, y_ν, or x_ν. Modifying the method of Takagi[2] for approximating the number of ways of constructing a crystal, the entropy of the system composed of M spins each located at a square net point can be approximated as:

$$S=kM[2\sum_{\nu=1}^{3}\beta_\nu y_\nu \ln y_\nu - \sum_{\nu=1}^{2}\alpha_\nu x_\nu \ln x_\nu - \sum_{\nu=1}^{6}\gamma_\nu z_\nu \ln z_\nu].$$

The energy E is rigorously expressed as

$$E=\tfrac{1}{2}4M\sum_{\nu=1}^{3}\epsilon_\nu\beta_\nu y_\nu,$$

where ϵ_ν is the energy of the bond. Minimizing the free energy $F=E-TS$, we get results identical with those obtained by the "variation method" by Kramers and Wannier.[3]

As a by-product it was found that the Kramers-Wannier result[3] for the ordered phase can be simplified as follows: Their Eq. (91.a) can be factorized as

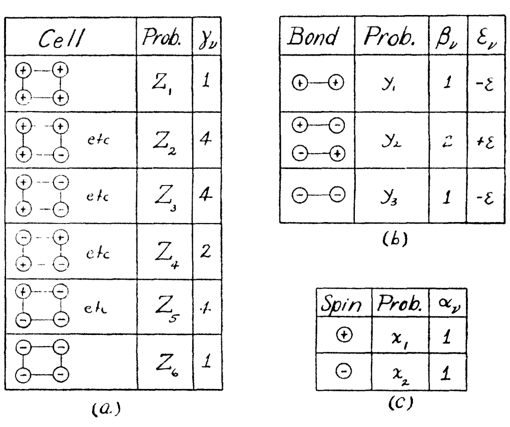

FIG. 1. Configuration scheme.

$$\{k^2z^2+(k^4+1)z+k^6\}\{k^4z^2-k^2(k^4-3)z+1\}=0.$$

The true solution is contained in the second factor, which gives

$$\lambda=\{k^8+8k^4-11+(k^4-5)^{\frac{1}{2}}(k^4-1)^{\frac{1}{2}}\}/(2k^6)$$

in place of their Eq. (91.b).

Taking y_ν and x_ν as variables, we can approximate the entropy as

$$S=kM[3\sum_{\nu=1}^{2}\alpha_\nu x_\nu \ln x_\nu - 2\sum_{\nu=1}^{3}\beta_\nu y_\nu \ln y_\nu]$$

which gives a result identical with Takagi's[2] and Bethe's.[1]

Simple cubic lattice.—Taking z_ν, y_ν, and x_ν as variables, the entropy of a system with M spins, each located at a simple cubic lattice point, can be approximated as

$$S=kM[9\sum_{\nu=1}^{3}\beta_\nu y_\nu \ln y_\nu - 7\sum_{\nu=1}^{2}\alpha_\nu x_\nu \ln x_\nu - 3\sum_{\nu=1}^{6}\gamma_\nu z_\nu \ln z_\nu].$$

Minimizing the free energy, we get the following results:

(a). The transition is of the second order and the transition temperature T_t is

$$kT_t/\epsilon=4.6097.$$

(b). The specific heat c_{t+} and c_{t-} on the higher and the lower temperature side respectively at T_t is

$$c_{t+}/k=0.389 \quad \text{and} \quad c_{t-}/k=2.902.$$

(c). Extrapolated, the specific heat for the disordered phase becomes infinite at

$$kT/\epsilon=4.2221.$$

Therefore it might not be in error to infer that the correct transition point T_c would be in the range

$$4.2221<kT_c/\epsilon<4.6097.$$

When we take the probabilities of configurations of a *cubic cell* as variables, we shall have a better approximation, which will be identical with the extension of the "variation method" to the simple cubic case tried by ter Haar and Martin.[5]

Face-centered cubic lattice (Ising model).—Taking as variables the probabilities of configurations of a tetrahedron, we reach the following results:

(a). The transition is of the second order and

$$kT_t/\epsilon=10.0260.$$

(b). $c_{t+}/\epsilon=0.336$ and $c_{t-}/\epsilon=2.789$.

(c). By the same inference as above we would obtain the following range for the correct transition point, T_c,

$$9.2388<kT_c/\epsilon<10.0260.$$

A more detailed account will be published elsewhere.

[1] H. A. Bethe, Proc. Roy. Soc. **A150**, 552 (1935).
[2] Y. Takagi, Proc. Phys. Math. Soc. Japan **23**, 44 (1941).
[3] H. A. Kramers and G. H. Wannier, Phys. Rev. **60**, 252 and 263 (1941).
[4] Yin-Yuan Li, J. Chem. Phys. **17**, 447 (1949); Phys. Rev. **76**, 972 (1949).
[5] D. ter Haar and B. Martin, Phys. Rev. **77**, 721 (1950).

Cosmic Rays Underground

C. M. GARELLI AND G. WATAGHIN
Instituto di Fisica della Università di Torino, Turin, Italy
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THE variation of cosmic-ray intensity underground has been studied recently by several authors.[1,2] In this note are reported results of calculations of the absorption of π- and μ-mesons underground, performed taking into account the process of creation of π-mesons by direct electromagnetic interaction of charged