Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

A Method of Approximation for Cooperative Phenomena

RYOICHI KIKUCHI
Department of Physics, University of Tokyo, Tokyo, Japan
May 4, 1950

A NEW method for approximating the entropy of an order-disorder system having an assigned energy has been developed. By a suitable choice of variables it gives the Bethe-Takagi approximation[1,2] in one case, and the "variation method" introduced by Kramers and Wannier[3] in another. The new method is more intuitive than the latter and is more effective than the quasi-chemical method by Yin-Yuan Li.[4] It can be improved and extended successively.

Two-dimensional square net.—The probability of appearance of a configuration of a cell (a bond, a spin) given on the left column of each part of Fig. 1 is denoted by z_ν, y_ν, or x_ν on the corresponding second column. $\gamma_\nu(\beta_\nu, \alpha_\nu)$ denotes the number of configurations with the same probability z_ν, y_ν, or x_ν. Modifying the method of Takagi[2] for approximating the number of ways of constructing a crystal, the entropy of the system composed of M spins each located at a square net point can be approximated as:

$$S=kM[2\sum_{\nu=1}^{3}\beta_\nu y_\nu \ln y_\nu-\sum_{\nu=1}^{2}\alpha_\nu x_\nu \ln x_\nu-\sum_{\nu=1}^{6}\gamma_\nu z_\nu \ln z_\nu].$$

The energy E is rigorously expressed as

$$E=\tfrac{1}{2}4M\sum_{\nu=1}^{3}\epsilon_\nu\beta_\nu y_\nu,$$

where ϵ_ν is the energy of the bond. Minimizing the free energy $F=E-TS$, we get results identical with those obtained by the "variation method" by Kramers and Wannier.[3]

As a by-product it was found that the Kramers-Wannier result[3] for the ordered phase can be simplified as follows: Their Eq. (91.a) can be factorized as

$$\{k^2z^2+(k^4+1)z+k^6\}\{k^4z^2-k^2(k^4-3)z+1\}=0.$$

The true solution is contained in the second factor, which gives

$$\lambda=\{k^8+8k^4-11+(k^4-5)^{\frac{3}{2}}(k^4-1)^{\frac{1}{2}}\}/(2k^6)$$

in place of their Eq. (91.b).

Taking y_ν and x_ν as variables, we can approximate the entropy as

$$S=kM[3\sum_{\nu=1}^{2}\alpha_\nu x_\nu \ln x_\nu-2\sum_{\nu=1}^{3}\beta_\nu y_\nu \ln y_\nu]$$

which gives a result identical with Takagi's[2] and Bethe's.[1]

Simple cubic lattice.—Taking z_ν, y_ν, and x_ν as variables, the entropy of a system with M spins, each located at a simple cubic lattice point, can be approximated as

$$S=kM[9\sum_{\nu=1}^{3}\beta_\nu y_\nu \ln y_\nu-7\sum_{\nu=1}^{2}\alpha_\nu x_\nu \ln x_\nu-3\sum_{\nu=1}^{6}\gamma_\nu z_\nu \ln z_\nu].$$

Minimizing the free energy, we get the following results:

(a). The transition is of the second order and the transition temperature T_t is

$$kT_t/\epsilon=4.6097.$$

(b). The specific heat c_{t+} and c_{t-} on the higher and the lower temperature side respectively at T_t is

$$c_{t+}/k=0.389 \quad\text{and}\quad c_{t-}/k=2.902.$$

(c). Extrapolated, the specific heat for the disordered phase becomes infinite at

$$kT/\epsilon=4.2221.$$

Therefore it might not be in error to infer that the correct transition point T_c would be in the range

$$4.2221<kT_c/\epsilon<4.6097.$$

When we take the probabilities of configurations of a *cubic cell* as variables, we shall have a better approximation, which will be identical with the extension of the "variation method" to the simple cubic case tried by ter Haar and Martin.[5]

Face-centered cubic lattice (Ising model).—Taking as variables the probabilities of configurations of a tetrahedron, we reach the following results:

(a). The transition is of the second order and

$$kT_t/\epsilon=10.0260.$$

(b). $c_{t+}/\epsilon=0.336$ and $c_{t-}/\epsilon=2.789$.

(c). By the same inference as above we would obtain the following range for the correct transition point, T_c,

$$9.2388<kT_c/\epsilon<10.0260.$$

A more detailed account will be published elsewhere.

Cell	Prob.	γ_ν
⊕—⊕ / ⊕—⊕	Z_1	1
⊕—⊕ / ⊕—⊖ etc	Z_2	4
⊕—⊖ / ⊕—⊖ etc	Z_3	4
⊖—⊕ / ⊕—⊖ etc	Z_4	2
⊕—⊖ / ⊖—⊖ etc	Z_5	4
⊖—⊖ / ⊖—⊖	Z_6	1

(a)

Bond	Prob.	β_ν	ε_ν
⊕—⊕	y_1	1	$-\varepsilon$
⊕—⊖ / ⊖—⊕	y_2	2	$+\varepsilon$
⊖—⊖	y_3	1	$-\varepsilon$

(b)

Spin	Prob.	α_ν
⊕	x_1	1
⊖	x_2	1

(c)

FIG. 1. Configuration scheme.

[1] H. A. Bethe, Proc. Roy. Soc. **A150**, 552 (1935).
[2] Y. Takagi, Proc. Phys. Math. Soc. Japan **23**, 44 (1941).
[3] H. A. Kramers and G. H. Wannier, Phys. Rev. **60**, 252 and 263 (1941).
[4] Yin-Yuan Li, J. Chem. Phys. **17**, 447 (1949); Phys. Rev. **76**, 972 (1949).
[5] D. ter Haar and B. Martin, Phys. Rev. **77**, 721 (1950).

Cosmic Rays Underground

C. M. GARELLI AND G. WATAGHIN
Instituto di Fisica della Università di Torino, Turin, Italy
June 5, 1950

THE variation of cosmic-ray intensity underground has been studied recently by several authors.[1,2] In this note are reported results of calculations of the absorption of π- and μ-mesons underground, performed taking into account the process of creation of π-mesons by direct electromagnetic interaction of charged incident mesons with nuclei. The cross section for this process was recently calculated by several authors.[3,4] The supplementary energy losses become more important than pair production and bremsstrahlung for energies >100 Bev. Therefore these losses must be taken into account for mesons reaching great depths.

Indeed from the Sneddon and Touschek formulas[3] follows:

$$-dE/dx \cong 3.9\times 10^{-7}E(\tfrac{7}{12}\ln 2E-\tfrac{1}{3}) \tag{1}$$

from which, for energies ~ 100 or 1000 Bev, follows:

$$-dE/dx \cong 3.10^{-6}E \tag{2}$$

where E is the energy, x is the depth in g/cm^2. We shall assume that, in spite of some approximations made in the deduction of these formulas, (1) gives the right order of magnitude.

For depths $\geqslant 100$ m H_2O underground, the cosmic-ray intensity is supposed to be due only to the components deriving from charged mesons of sufficiently high energy. Although slow π-mesons show a strong nuclear interaction and are absorbed with a cross section of the order of the geometrical cross section, there exists no experimental evidence for values of the cross section of π-mesons having energies >100 Bev. Therefore, we shall examine both extreme cases: (1) negligible nuclear absorption for high energy π-mesons; (2) strong nuclear absorption.

Starting from known experimental and theoretical data on the spectrum of π- and μ-mesons at sea level, calculations were performed taking into account the energy losses by ionization, decay, bremsstrahlung, electron pair production and π-meson production. In the first hypothesis (negligible nuclear absorption of energetic π-mesons) the resulting curve is given in Fig. 1 (curve I) and is in satisfactory agreement with the observations (experimental points are marked by ● and ⊙). A similar calculation, assuming a strong nuclear absorption of energetic π-mesons in the matter, gives curve II (Fig. 1), and seems not to be in agreement with the experimental points at great depths (data of Wilson[5] and Nishina and Miyazaki[6,7]).

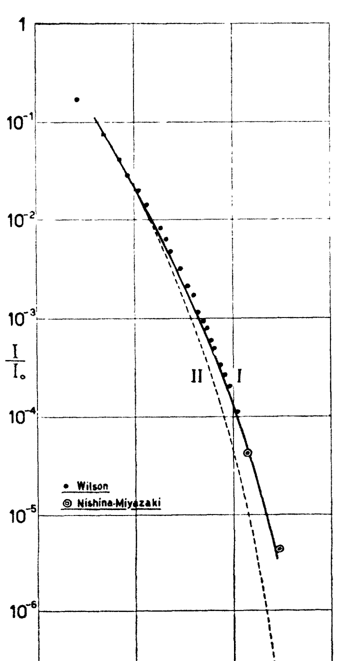

FIG. 1. Curves I and II represent calculated values of relative intensity of cosmic rays as functions of depth. Points ● and ⊙ indicate measured values.

If further experiments confirm this finding one should consider it as an argument showing that the cross section for energetic π-mesons decreases strongly with the energy. This decrease of the cross section signifies a corresponding decrease of the probability of all types of absorption processes for π-mesons. The problem will be discussed elsewhere. Detailed calculations will be published shortly.

[1] K. Greisen, Phys. Rev. **73**, 521 (1948); **76**, 1718 (1949).
[2] S. Hayakawa and S. Tomonaga, Prog. Theor. Phys. **4**, 287 (1949); Phys. Rev. **75**, 1958 (1949).
[3] I. N. Sneddon and B. F. Touschek, Proc. Roy. Soc. (A) **199**, 352 (1949).
[4] E. Strick and D. ter Haar, Phys. Rev. **78**, 68 (1950).
[5] W. C. Wilson, Phys. Rev. **53**, 337 (1938).
[6] Nishina, Sekido, Miyazaki, and Masuda, Phys. Rev. **59**, 401 (1941).
[7] Y. Miyazaki, Phys. Rev. **76**, 1733 (1949).

Energy Levels in C^{12} from $Be^9(\alpha, n)C^{12}$

WILLIAM H. GUIER* AND JAMES H. ROBERTS
Northwestern University, Evanston, Illinois
June 19, 1950

THE neutron energy spectrum from the $Be^9(\alpha, n)C^{12}$ reaction using a thin Po alpha-source and thin Be target was measured by the proton recoil method in Kodak NTA nuclear emulsions. The thin Po source was prepared by the Canadian Radium and Uranium Corporation by plating polonium onto a nickel backing. The beryllium target was prepared by H. Bradner at the Radiation Laboratory of the University of California and was 0.25 mg/cm^2 thick.

Both the target and source had an effective area of about 4 cm^2 and were mounted 2.2 cm apart. Photographic plates were mounted about 10 cm from the target in the forward, backward, and 90° positions. To date, 308 acceptable tracks have been measured in the forward direction.

After processing the plates, the tracks were measured with a Leitz Ortholux microscope with an oil immersion objective. Tracks were considered acceptable if they made an angle of 12° or less with the average direction of the incident neutrons. The energies of the recoil protons were determined from their ranges by using the Ilford range energy curves.[1] The neutron energies were found from the proton energies by $E_n = E_p \sec^2\theta$, where θ is the angle between the incident neutron direction and the recoil proton. The intensities must be corrected for the probability of leaving the emulsion[2] and for the $n-p$ scattering cross section. The final distribution is shown in Fig. 1.

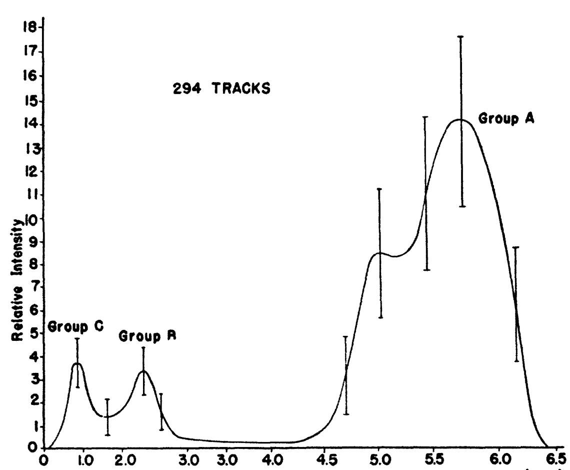

FIG. 1. Neutron spectrum of the reaction $Be^9(\alpha, n)C^{12}$.