our theorem on the relation between T_c and the available energy, we know that the real T_c for the nearest neighbor region is

$$\left.\begin{aligned} T_{c3}&=\lambda(4\lambda-2\epsilon), \\ \text{and}\quad \theta/T_{c3}&=3(1+\epsilon/2\lambda)/[1-\epsilon/2\lambda]. \end{aligned}\right\} \tag{44}$$

This is plotted as the dashed curve (2) of Fig. 2.

It will be seen that $\theta/T_c=5$ is still barely allowed by our theory, but that the ϵ/λ ratio is definitely close to $\frac{1}{2}$ for MnO. The two possible ϵ/λ ratios for all compounds, under our theory, are included in Table I.[6] It is a nice confirmation of our theory that (a) MnO requires a high next nearest neighbor interaction simply because of its θ/T_c value, while Shull's data confirm that next nearest neighbors are furnishing the antiferromagnetic alignment; (b) the theory of superexchange predicts that the next nearest neighbor interaction should increase along the series MnO–MnS–MnSe, as is observed.[7] Shull has shown that MnSe has the "next nearest neighbor" arrangement of MnO, and thus the value $\epsilon/\lambda=1$ is to be preferred.

The reason for the large values of ϵ/λ occurring in the theory must be sought for in a high superexchange[7, 8] combined with a large separation of the magnetic ions in the antiferromagnetic crystals, leading to low direct exchange integrals. An examination of Kramers' theory indicates that if the superexchange is due to transitions of p-electrons from the negative to the positive ion (that is, to a partial covalent character of the bonds) the directionality of the superexchange must be that of a p-wave function; i.e., directly through the negative ion to the next nearest neighbor rather than the nearest neighbor. Also, since the other compounds should be expected to be less ionic than MnO, it is probable that their ϵ/λ values are the higher ones, $\cong 1$, rather than the lower ones, as would be more satisfying from a naive picture.

I should like to express my thanks to Drs. C. Kittel, G. H. Wannier, and C. G. Shull for their helpful interest in this work.

[6] It was of interest to carry out the logical extension of the simple Néel two-sublattice theory for high values of the next nearest neighbor interaction: i.e., to divide each sublattice into two sublattices. The result is very like Fig. 2 except that $\theta/T_c>3$ is not allowed in this case. For $\epsilon>\lambda/2$, the sublattices become antiferromagnetic, as in the f.c.c. structure. Thus simply on internal evidence alone the Néel theory cannot explain the θ/T_c ratio in MnO.

[7] P. W. Anderson, Phys. Rev. **79**, 350 (1950).

[8] H. A. Kramers, Physica **1**, 182 (1934).

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μ-Pair Theories and the π-Meson

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The μ-pair theory suggests the interpretation of the π-meson as a pair of μ's bound together by a nucleon pair field. Only a non-relativistic description, involving a cut-off in the momentum spaces, is attempted. Various π-meson types, such as are well known from the conventional Yukawa theories, can be constructed, depending on the type of coupling adopted in the interaction Hamiltonian of the pair theory. It appears, however, that only the pseudoscalar coupling, which leads to a pseudoscalar π-meson, is consistent with the experimental data; e.g., on the nuclear scattering of μ's, which indicate that the interaction of μ's with nucleons, at least in such processes, is rather weak. Then, also, the creation of μ-pairs in high energy nuclear collisions is expected to be an infrequent event, compared to the π-creation. Nonetheless, the μ-pair production should furnish a crucial experimental test. Another process predicted is the dissociation of a fast π-meson, passing through matter, into a pair. The existence of heavier mesons, involving more than two μ's, seems likely.

I. INTRODUCTION

As an alternative to Yukawa's theory, the pair theory of nuclear interactions was much discussed some years ago.[1] The great advantage of this kind of theory is that it accounts for the saturation character of the nuclear forces without *ad hoc* assumptions.[2] The present paper is concerned with another implication of the theory. According to μ-pair theories, μ-mesons may be bound together to form composite particles and, in particular, π-mesons may be interpreted as μ-pairs. Thus, a unified picture of both meson specimens seems possible. Heavier mesons may also be foreseen.

It is true that the μ-pair theory has been somewhat discredited lately by the lack of experimental evidence for a strong nuclear interaction of μ-mesons. Even if only pairs of μ's are supposed to interact with nucleons, one expects at least a strong nuclear scattering of

[1] For the literature up to 1944, see W. Pauli and N. Hu, Rev. Mod. Phys. **17**, 267 (1945).

[2] Wigner, Critchfield, and Teller, Phys. Rev. **56**, 530 (1939); G. Wentzel, Helv. Phys. Acta **15**, 111 (1942); A. Houriet, Helv. Phys. Acta **16**, 529 (1943).

μ's,[3] indeed much stronger than has been observed.[4] One way to avoid this difficulty is to assume a particular (pseudoscalar) type of coupling which leads to small matrix-elements for the scattering processes although other matrix-elements are large; for example, those determining the binding energies of compound mesons.

Postponing the further discussion of such questions, we first present the μ-pair theory in a form slightly different from the conventional one. If singly charged "pions"[5] are to be interpreted as μ-pairs, we have to associate a singly charged "muon" with its (hypothetical) neutral counterpart, the "nuon." Consider, then, the four elementary particles, each of spin $\frac{1}{2}$: proton (P), neutron (N), positive muon (μ), and nuon (ν), together with their antiparticles ($\bar{P}$, $\bar{N}$, $\bar{\mu}$, $\bar{\nu}$, to be described according to Dirac's hole theory), coupled by the interaction Hamiltonian

$$\eta\int d^3x(N^*AP)(\mu^*A\nu)+\text{conj.} \tag{1}$$

(the symbols N, P, μ, ν stand for the corresponding spinor field operators, and A is one of the five matrix operators known from β-decay theory). Typical elementary processes described by (1) are:

$$P+\nu\leftrightarrow N+\mu, \tag{1a}$$

i.e., the scattering of nuons or muons by nucleons, with exchange of charge;

$$P\leftrightarrow N+\mu+\bar{\nu}, \tag{1b}$$

i.e., meson pair creation or annihilation by a nucleon;

$$P+\bar{N}\leftrightarrow\mu+\bar{\nu}, \tag{1c}$$

e.g., the annihilation of a nucleon-antinucleon pair into a meson pair. Process (1c) also is the basis for our pion model: A muon and an antinuon combined have a negative self-energy owing to their ability to transform into a proton-antineutron pair and back; this self-energy will be interpreted as the binding energy of the pion. The presence, in our model, of virtual nucleon-antinucleon pairs causing the binding makes for a certain (limited) resemblance to the model proposed by Fermi and Yang.[6]

As to the divergences characteristic of all quantized field theories, since the infinities are even of higher order in pair theories than in Yukawa theories, we shall resort to a primitive cut-off in momentum space because so far no more systematic approach has been worked out. Such a non-relativistic theory will lead necessarily to ambiguities in the interpretation of the theoretical results. For instance, if one calculates the total electric current (space integral of the current density) carried by a pion of momentum $\mathbf{p}$, one finds an expression of the expected form $e\mathbf{p}/m$, but the mass m is not equal to the mass deduced from the energy of the particle at rest, not even in the limit of vanishing $\mathbf{p}$. This kind of difficulty is well known from the Lorentz electron model where the electromagnetic mass calculated from the reaction force differs from the electric rest energy (by a factor 4/3), and is typical for all "extended source" theories. A future theory involving a "universal length" may possibly eliminate such ambiguities. For the time being the least objectionable procedure will be to consider the rest system as a natural frame of reference; in other words, we shall tentatively accept the mass and other quantities, as they are calculated in the rest frame, as the correct ones and shall rely on Lorentz transformations to provide the values for a moving particle.

Even so, there remains the arbitrariness inherent in the cut-off procedure. We shall employ two weight functions $g(p)$ and $G(P)$ for the meson and nucleon momentum, respectively, which may, for instance, be chosen as step-functions:

$$g(p)=\begin{cases}1 \text{ for } p<k\\ 0 \text{ for } p>k,\end{cases}\qquad G(P)=\begin{cases}1 \text{ for } P<K\\ 0 \text{ for } P>K.\end{cases} \tag{2}$$

We then have essentially three constants disposable to adapt the theory to the observational data; namely, the coupling constant η in (1), and the two cut-off momenta k, K in (2). Of course, there are several ways to make the theory even more flexible, but it seems that the present experimental knowledge does not require such further complications.

Actually, the parameter k is determined by the range of the nuclear forces, or by the nucleon density in heavy nuclei. Indeed, the two-nucleon potentials, as derived from pair theories, exhibit, for short distances r, a very strong r-dependence (e.g., as r^{-5}) so long as the cut-off is ineffective, and it is the cut-off radius ($\sim 10^{-13}$ cm), or the corresponding cut-off momentum ($\sim 10^{13}$ cm^{-1}), which actually determines the range of the forces.[7] As in this argument the motion of the nucleons is ignored (static approximation), only the cut-off of the mesonic momenta is relevant. We conclude that k, in (2), is to be chosen of the order of magnitude of the mesonic masses. Later we shall see that, similarly, K must be assigned a value close to the nucleonic masses, which seems most adequate in the framework of a non-

[3] J. W. Weinberg, Phys. Rev. **59**, 776 (1941); J. M. Jauch. Helv. Phys. Acta **15**, 221 (1942).

[4] Cf. B. Rossi, Rev. Mod. Phys. **20**, 537 (1948), Section 11.

[5] We adopt the nomenclature introduced by E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).

[6] See reference 5. According to the Fermi-Yang theory, a positive pion results from a proton and an antineutron bound together by a strong attractive close distance interaction which is supposed to lead to a binding energy of about 93 percent of the rest energy of the two free nucleons. In our model, because of the smaller masses of the main constituents ($\mu+\bar{\nu}$), the binding energy required is only about 34 percent, which may seem more natural.

[7] A more complete analysis is found in the two last papers quoted in reference 2, where rigorous static solutions (nucleons at rest) are discussed, not only for the two-nucleon problem but also for many nucleons arranged in lattice form. The saturation of the nuclear forces sets in for lattice constants smaller than the cut-off radius.

relativistic theory. Incidentally, it should be noted that the above argument is not invalidated by the fact that, besides the nuclear forces transmitted by the meson-pair field, our theory predicts additional forces of the type familiar from Yukawa theories, transmitted by pions or bound meson pairs.

In pair theories it has been customary to introduce the cut-off in such a way that it amounts to an averaging of every individual field operator in the interaction term over a small region in x-space.[2] Following this procedure we supply the matrix element of the interaction (1) with four form factors of type (2):

$$\eta\delta(\mathbf{P}_0-\mathbf{P}+\mathbf{p}-\mathbf{p}_0)G(P_0)G(P)g(p)g(p_0) \times\sum_\alpha (\mathbf{P}_0\Lambda_0|A_\alpha|\mathbf{P}\Lambda)(\mathbf{p}\lambda|A_\alpha|\mathbf{p}_0\lambda_0). \quad (3)$$

Here, $\mathbf{P}_0\Lambda_0$, $\mathbf{P}\Lambda$, $\mathbf{p}\lambda$, $\mathbf{p}_0\lambda_0$ characterize the momentum and spin states of the neutron, proton, muon, and nuon, respectively. The suffix α has been added to distinguish the components of vectors, tensors, or pseudovectors. Even linear combinations of the five coupling types may be admitted. It should be noted that in application of (3) to antiparticles (holes), $\mathbf{P}_{(0)}$ or $\mathbf{p}_{(0)}$ designate the momentum of the vacant state rather than the particle momentum which is $-\mathbf{P}_{(0)}$ or $-\mathbf{p}_{(0)}$.

II. THE PION MODEL

For the description of the pion at rest we introduce the following probability amplitudes: $f(\mathbf{p}\lambda\lambda_0)$ for the presence of a muon with momentum $\mathbf{p}$ and spin λ and of an antinuon with momentum $-\mathbf{p}$ and spin λ_0 (i.e. the vacant state is to be labeled $+\mathbf{p}\lambda_0$); $F(\mathbf{P}\Lambda\Lambda_0)$ for the presence of a proton with momentum $\mathbf{P}$ and spin Λ and of an antineutron with momentum $-\mathbf{P}$ and spin Λ_0. The interaction (3) couples these probability amplitudes according to the following Schrödinger equation:

$$\begin{aligned} 0&=[-\epsilon+(m^2+p^2)^{\frac{1}{2}}+(m_0{}^2+p^2)^{\frac{1}{2}}]f(\mathbf{p}\lambda\lambda_0) \\ &\quad+\eta g^2(p)\sum_\alpha(\mathbf{p}\lambda|A_\alpha|\mathbf{p}\lambda_0) \\ &\quad\times\int d^3PG^2(P)\sum_{\Lambda\Lambda_0}(\mathbf{P}\Lambda_0|A_\alpha|\mathbf{P}\Lambda)F(\mathbf{P}\Lambda\Lambda_0), \\ 0&=[-\epsilon+(M^2+P^2)^{\frac{1}{2}}+(M_0{}^2+P^2)^{\frac{1}{2}}]F(\mathbf{P}\Lambda\Lambda_0) \\ &\quad+\eta G^2(P)\sum_\alpha(\mathbf{P}\Lambda|A_\alpha|\mathbf{P}\Lambda_0) \\ &\quad\times\int d^3pg^2(p)\sum_{\lambda\lambda_0}(\mathbf{p}\lambda_0|A_\alpha|\mathbf{p}\lambda)f(\mathbf{p}\lambda\lambda_0). \end{aligned} \quad (4)$$

These equations are not complete because they ignore the existence of virtual intermediate states involving more than one meson and/or nucleon pair. Some of the virtual transitions passing through such states give rise to vacuum and mass renormalization terms only, but there are others which cannot be claimed to be physically meaningless or numerically unimportant. Consider, as an example, the following process involving the virtual creation of both a meson pair and a nucleon pair:

$$(\mu+\bar{\nu})\to(\mu+\bar{\nu})+(\bar{P}+N)+(\mu'+\bar{\nu}')\to(\mu'+\bar{\nu}').$$

Compared with the process treated in (4), *viz.*,

$$(\mu+\bar{\nu})\to(P+\bar{N})\to(\mu'+\bar{\nu}'),$$

its contribution to the energy is certainly not negligible. However, if one extends the Schrödinger equation accordingly, it turns out that its solutions, for the purposes of our qualitative discussions and rough estimates, are practically equivalent to those of the abbreviated Eq. (4). (The important items are: the structure of the eigenfunction components f, $F(5)$, the general character of the eigenvalue condition (9), and the normalization (11a), neither of which is tangibly altered.) Therefore, studying a more comprehensive Schrödinger equation would mean an unnecessary complication, and the numerical improvement achieved would even remain doubtful at any stage of approximation.

Returning to Eqs. (4), we want to simplify the calculations further by assuming the masses of charged and neutral mesons to be (very nearly) equal:[8]

$$m=m_0, \quad \text{and also} \quad M=M_0.$$

For the free particle energies we write

$$(m^2+p^2)^{\frac{1}{2}}=e(p), \quad (M^2+P^2)^{\frac{1}{2}}=E(P).$$

From the structure of the integral Eqs. (4) it is seen that a solution corresponding to a bound state must have the form:

$$\begin{aligned} f(\mathbf{p}\lambda\lambda_0)&=\frac{g^2(p)}{2e(p)-\epsilon}\sum_\alpha(\mathbf{p}\lambda|A_\alpha|\mathbf{p}\lambda_0)u_\alpha \\ F(\mathbf{P}\Lambda\Lambda_0)&=\frac{G^2(P)}{2E(P)-\epsilon}\sum_\alpha(\mathbf{P}\Lambda|A_\alpha|\mathbf{P}\Lambda_0)U_\alpha. \end{aligned} \quad (5)$$

Inserting this into (4), one obtains the following equations for the coefficients u_α, U_α:

$$u_\alpha+\eta\sum_\beta C_{\alpha\beta}U_\beta=0, \quad U_\alpha+\eta\sum_\beta c_{\alpha\beta}u_\beta=0, \quad (6)$$

where

$$\begin{aligned} c_{\alpha\beta}&=\int\frac{d^3pg^4(p)}{2e(p)-\epsilon}\sum_{\lambda\lambda_0}(\mathbf{p}\lambda_0|A_\alpha|\mathbf{p}\lambda)(\mathbf{p}\lambda|A_\beta|\mathbf{p}\lambda_0) \\ C_{\alpha\beta}&=\int\frac{d^3PG^4(P)}{2E(P)-\epsilon}\sum_{\Lambda\Lambda_0}(\mathbf{P}\Lambda_0|A_\alpha|\mathbf{P}\Lambda)(\mathbf{P}\Lambda|A_\beta|\mathbf{P}\Lambda_0). \end{aligned} \quad (7)$$

[8] Actually, a lower bound for m_0 is set by the requirement that the process $P+\bar{\mu}\to N+\bar{\nu}$, for a negative muon captured in the Coulomb field of any light nucleus, be energetically forbidden. Otherwise this capture process would be much too rapid to permit a β-decay of the negative muon. See V. F. Weisskopf, Phys. Rev. **72**, 155 (1947): S. Noma, Prog. Theor. Phys. **2**, 159 (1947).

The determinant Δ of Eqs. (6) is a function of the energy parameter ϵ, and the condition

$$\Delta(\epsilon)=0, \quad 0<\epsilon<2m \tag{8}$$

will determine the stationary states (negative eigenvalues of ϵ must be excluded for obvious physical reasons).

As an example let us consider the vector coupling case:

$$A_\alpha=(\boldsymbol{\alpha}, i).$$

The summations in (7) where λ, $\Lambda(\lambda_0, \Lambda_0)$ refer to positive (negative) energy states, can be carried out by the projection operator technique, and, taking spherical symmetry of $g(p)$ and $G(P)$ for granted, one obtains:

$$c_{\alpha\beta}=c_\alpha\delta_{\alpha\beta}, \quad C_{\alpha\beta}=C_\alpha\delta_{\alpha\beta},$$

$$c_1=c_2=c_3=2\int\frac{d^3pg^4(p)}{2e(p)-\epsilon}\left[1-\frac{1}{3}\frac{p^2}{e^2(p)}\right],$$

$$c_4=0,$$

$$C_1=C_2=C_3=2\int\frac{d^3PG^4(P)}{2E(P)-\epsilon}\left[1-\frac{1}{3}\frac{P^2}{E^2(P)}\right],$$

$$C_4=0.$$

Inserting into (6):

$$u_K+\eta C_1U_K=0, \quad U_K+\eta c_1u_K=0 \quad (K=1, 2, 3),$$
$$u_4=0, \quad U_4=0.$$

The eigenvalue condition becomes:

$$c_1(\epsilon)C_1(\epsilon)=\eta^{-2}.$$

Since c_1C_1, for $\epsilon<2m$, increases monotonically with increasing ϵ, there is one bound state (of threefold degeneracy) provided that η is chosen such that

$$c_1(0)C_1(0)<\eta^{-2}<c_1(2m)C_1(2m).$$

In particular, one can determine η such that ϵ corresponds to the observed mass of the pion. This composite particle would be a vector meson because u_1, u_2, u_3 transform like vector components under rotations (note that the vectors $\mathbf{u}$ and $\mathbf{U}$ are parallel).

The results for other coupling types are summarized in Table I. The first column lists the five "pure" coupling types (A_α), the second the transformation properties of the u_α or U_α of the resulting bound states. The next two columns refer to the eigenvalue condition which in every case has the general form

$$c(\epsilon)C(\epsilon)=\eta^{-2}, \tag{9}$$

$$\left.\begin{aligned} c(\epsilon)&=2\int\frac{d^3pg^4(p)}{2e(p)-\epsilon}\left[a+b\frac{p^2}{e^2(p)}\right],\\ C(\epsilon)&=2\int\frac{d^3PG^4(P)}{2E(P)-\epsilon}\left[a+b\frac{P^2}{E^2(P)}\right],\end{aligned}\right\} \tag{10}$$

TABLE I. Coupling conditions for composite particles.

Coupling type A	Type of pion	Coefficients in (10) a	b	Pion-nucleon interaction Coupling type	Kemmer[a]
Scalar	Scalar	0	1	Scalar	$H_1{}^a(f_a=0)$
Vector	Vector	1	$-\frac{1}{3}$	Vector	$H_1{}^b(f_b=0)$
Tensor	Vector	1	$-\frac{2}{3}$	Tensor	$H_1{}^b(g_b=0)$
	Axial vector	0	$\frac{1}{3}$	Tensor	$H_1{}^c(f_c=0)$
Pseudovector	Axial vector	0	$\frac{2}{3}$	Pseudovector	$H_1{}^c(g_c=0)$
	Pseudoscalar	1	-1	Pseudovector	$H_1{}^d(f_d=0)$
Pseudoscalar	Pseudoscalar	1	0	Pseudoscalar	$H_1{}^d(g_d=0)$

[a] N. Kemmer, reference 9 (see text for explanation).

with numerical coefficients a and b which are collected in Table I. The last two columns will be explained later.

Both in the tensor and the pseudovector coupling cases, two types of bound states are possible, one of which may, however, be non-existent (if not both). In the tensor case, for instance, the vector pion alone survives if η falls below a certain critical value, whereas the axial vector pion could only exist together with a vector pion of smaller mass ϵ (possibly $\epsilon<0$ which is to be excluded at all events).

If the interaction is a mixture of the pure coupling types, the tensors $c_{\alpha\beta}$, $C_{\alpha\beta}$ (7) may have off-diagonal components, for instance, if A_α and A_β refer to the vector states resulting from the vector and tensor coupling respectively; this coupling gives rise to two modified vector states. The same applies to the two pseudoscalar states listed in the table, whereas the axial vector states happen to remain uncoupled (because of $m=m_0$, $M=M_0$).

For each bound state, once its energy ϵ is known, the Schrödinger-function can be calculated from (6) and (5), except for the normalization constant, which shall be determined by

$$\int d^3p\sum_{\lambda\lambda_0}|f(\mathbf{p}\lambda\lambda_0)|^2+\int d^3P\sum_{\Lambda\Lambda_0}|F(\mathbf{P}\Lambda\Lambda_0)|^2=1. \tag{11}$$

With (5), this reduces to a normalization condition for the "vectors" u and U which has the general form

$$\sum_{\alpha\beta}(d_{\alpha\beta}u_\alpha{}^*u_\beta+D_{\alpha\beta}U_\alpha{}^*U_\beta)=1. \tag{11a}$$

III. PION CREATION

The fundamental process of the Yukawa theory, namely

$$P\leftrightarrow N+\pi, \tag{12}$$

can now be interpreted as a $(\mu+\bar{\nu})$ pair creation, according to (1b), but with the pair particles bound to each other. We can immediately write down the matrix-element for this process in the rest system of the pion for which frame we have above calculated the eigenfunction. Disregarding the F-part which gives only higher order contributions, and supplementing the

f-part with the factor $\delta(\mathbf{p}-\mathbf{p}_0)$ to account for the translational state, we obtain from (3), (5), and (7):

$$H_\pi=\eta G^2(P)\sum_\alpha (\mathbf{P}\Lambda_0|A_\alpha|\mathbf{P}\Lambda)$$

$$\times\int d^3p g^2(p)\sum_{\lambda\lambda_0} f^*(\mathbf{p}\lambda\lambda_0)(\mathbf{p}\lambda|A_\alpha|\mathbf{p}\lambda_0)$$

$$=\eta G^2(P)\sum_\alpha (\mathbf{P}\Lambda_0|A_\alpha|\mathbf{P}\Lambda)(\sum_\beta c_{\alpha\beta}u_\beta)^*,$$

and using (6):

$$H_\pi=-G^2(P)\sum_\alpha (\mathbf{P}\Lambda_0|A_\alpha|\mathbf{P}\Lambda)U_\alpha^*. \tag{13}$$

In H_π, we have omitted the factor $\delta(\mathbf{P}_0-\mathbf{P})$ which only expresses the trivial fact that the nucleon momentum remains unchanged in the rest system of the emitted particle.

Choosing for A one of the pure coupling types, and for U the corresponding scalar, vector, axial vector, or pseudoscalar, according to Table I, one sees immediately that (13) agrees (except for the cut-off factor G^2) with the interaction Hamiltonians currently used in "Yukawa theories." The last two columns of Table I indicate the various interaction types, first in general terms, and then, for the sake of clearer identification, in Kemmer's notation.[9] Of course, the comparison can only be made for the rest system of the pion, while, as explained in the introduction, we assume H_π to transform properly under Lorentz-transformations.

As to the magnitude of the matrix-element (13), the determining factor is the magnitude of U which is given by the normalization condition (11a), combined with (6). Since the coefficients d, D, c, C are strongly dependent on the choice of the cut-off functions g and G, there is no difficulty in adapting the theory to the experimental knowledge; e.g., to the measured cross sections for the production of pions in nucleon-nucleon collisions or by photo-effect.[10] As an example, consider the case of pseudoscalar coupling where the matrix-element for the creation of a pion with momentum $\boldsymbol{\pi}$, in a non-relativistic approximation becomes

$$H_\pi=-(\mathbf{P}_0\Lambda_0|\beta\gamma_5|\mathbf{P}\Lambda)U^*=-((\Lambda_0|\boldsymbol{\sigma}|\Lambda)\cdot\boldsymbol{\pi})U^*/2M.$$

Using the theoretical computations of Feshbach and Lax[11] for the photo-creation cross section, a value $|U|\sim M^{-\frac{1}{2}}$ or slightly larger (in rational units: $\hbar=c=1$) is seen to fit the experimental data,[10] and the same value seems to give the correct order of magnitude for the pion creation by 350-Mev protons.[12] On the other hand, from (11a) and (6), such a $|U|$ value is most naturally obtained by cutting off the nucleon momenta P at $K\sim M$ (certainly $K>M/3$), assuming $k\sim m$ as explained in the introduction.

The pseudoscalar theory is exceptional in that it requires a particularly large $|U|$ value (small K) to compensate the small factor $\pi/2M$ in H_π; in all other theories, $|U|$ should be made a good deal smaller, thus requiring a weaker cut-off ($K>M$). The pseudoscalar theory, however, has been reported to give by far the best agreement with the observed angular and energy distribution of photo-mesons,[13] and emphasis on this theory is also indicated by the analysis of the μ-scattering problem (Section V).

With the known values of the three parameters k, K, and η, or at least their orders of magnitude, further properties of the pion model can be deduced. A comment may be welcome as to the relative weights of the meson-pair and nucleon-pair components (f and F parts) in the pion eigenfunction (see Eq. (11)). Very roughly, one finds for the weight ratio:

$$d|u|^2/D|U|^2=dC/Dc\sim K/k\sim M/m\sim 10.$$

Thus, the pion is predominantly a meson pair, with a comparatively small nucleon pair admixture.

IV. DISINTEGRATION PROCESSES

Since the pion is supposed to be an extended structure, the question will be raised whether it is sufficiently stable against dissociation into two mesons ($\mu+\bar{\nu}$) when passing through matter with high velocity $\mathbf{v}$. In order to examine the effect of the Coulomb field of an atomic nucleus (charge Ze) we transform into the rest frame of the pion where the effective potential is

$$V(\mathbf{r},t)=\frac{Ze}{2\pi^2}(1+\boldsymbol{\alpha}\cdot\mathbf{v})\int d^3g\frac{\exp[i\mathbf{g}\cdot(\mathbf{r}+\mathbf{v}t)]}{g^2-(\mathbf{v}\cdot\mathbf{g})^2}$$

and apply the Born-Dirac approximation. The F-part of the pion eigenfunction is again ignored so that V acts on the charged muon only. Then, the matrix-element for the dissociation of the pion (at rest) into a muon of momentum $\mathbf{p}$ and spin λ, and an antinuon of momentum $-\mathbf{p}_0$ and spin λ_0, becomes

$$\frac{Ze}{2\pi^2}\sum_{\lambda'}\frac{(\mathbf{p}\lambda|1+\boldsymbol{\alpha}\cdot\mathbf{v}|\mathbf{p}_0\lambda')f(\mathbf{p}_0\lambda'\lambda_0)}{|\mathbf{p}-\mathbf{p}_0|^2-(\mathbf{v}\cdot(\mathbf{p}-\mathbf{p}_0))^2}e^{i\omega t}, \tag{14}$$

where

$$\omega=[e(p)+\mathbf{v}\cdot\mathbf{p}]+[e(p_0)-\mathbf{v}\cdot\mathbf{p}_0]-\epsilon.$$

The quantity $\omega(1-v^2)^{-\frac{1}{2}}$ is the energy change in the rest frame of the atomic nucleus. Since $\omega=0$ for the real process, it follows that

$$v|\mathbf{p}-\mathbf{p}_0|\geq\mathbf{v}\cdot(\mathbf{p}_0-\mathbf{p})>2m-\epsilon\quad(\approx 70\text{ Mev})$$

[9] N. Kemmer, Proc. Roy. Soc. **A166**, 127 (1938). See in particular his Eqs. (63a–d), p. 143.

[10] C. Richman, H. Wilcox, Phys. Rev. **78**, 85 (1950); M. Weissbluth, Phys. Rev. **78**, 86 (1950); Peterson, McMillan, and White, Phys. Rev. **78**, 84 (1950).

[11] H. Feshbach and M. Lax, Phys. Rev. **76**, 134 (1949).

[12] T. B. Taylor and G. F. Chew, Phys. Rev. **78**, 86 (1950).

[13] K. A. Brueckner, Phys. Rev. **78**, 84 (1950).

which sets a lower bound to $|\mathbf{p}-\mathbf{p}_0|$, i.e., the total momentum of the two ejected particles, and therefore also to the denominator in (14). Then, remembering that $\int d^3p_0|f(\mathbf{p}_0\lambda'\lambda_0)|^2<1$, according to (11), even if the integration is extended over the entire $\mathbf{p}_0$ space, it is easy to establish an upper limit to the total cross section for dissociation, by comparing it with the cross section for scattering of a muon by the same Coulomb field: Even at energies far above the threshold, the dissociation will be certainly less probable than the wide angle scattering of a 100-Mev muon (cross section $\ll Z^2\times 10^{-29}$ cm^2). The same argument can be used for non-electric (nuclear) interactions. We expect, therefore, that the dissociation $\pi\rightarrow\mu+\bar{\nu}$ will be hard to ascertain experimentally, the more so because of the difficulty of distinguishing it from a scattering process or a $\pi-\mu$-decay.

Another process which might, according to our model, destroy the pion too rapidly is the β-disintegration. Actually, the pion is known to decay after a mean lifetime (at rest) of about 2×10^{-8} sec.,[14] into a muon and a neutral particle, presumably a neutrino, and it is concluded that its mean lifetime against β-decay is at least about 10^{-7} sec. In Fermi's β-theory, the direct transition

$$P\rightarrow N+p+\bar{n} \qquad (15a)$$

(p=positron, n=neutrino) is supposed to be an elementary process characterized by the interaction term

$$\eta_\beta\int d^3x(N^*BP)(p^*Bn)+\text{conj.}, \qquad (15)$$

similar to (1), where η_β is empirically determined by the lifetimes of β-active nuclei. Then, the matrix-element for the process

$$P+\bar{N}\rightarrow p+\bar{n}$$

is also known, and it is easy to calculate the probability of the decay process

$$\pi\rightarrow p+\bar{n}$$

due to the F-part ($P+\bar{N}$ admixture) of the pion eigenfunction. Indeed, the matrix-element for this process is

$$\eta_\beta\sum_\beta(\mathbf{p}\lambda|B_\beta|\mathbf{p}\lambda_0)\int d^3P\sum_{\Lambda\Lambda_0}(\mathbf{P}\Lambda_0|B_\beta|\mathbf{P}\Lambda)F(\mathbf{P}\Lambda\Lambda_0). \qquad (16)$$

One possibility is that the coupling types in (1) and (15) are the same: $B=A$. In this case, (16) can be written, with the help of (5), (6), (7) (and assuming (2)):

$$-(\eta_\beta/\eta)\sum_\beta(\mathbf{p}\lambda|A_\beta|\mathbf{p}\lambda_0)u_\beta.$$

It turns out, however, that this case ($B=A$) must be excluded because the resulting pion lifetime would be too short ($<10^{-9}$ sec.). For the same reason, the following combinations of coupling types are to be excluded:

A, B or B, A=vector, tensor;
A, B or B, A=pseudovector, pseudoscalar.

All other combinations are unobjectionable, giving zero for the matrix element (16). For instance, favoring for A the pseudoscalar, we may consider for B the scalar, vector, or tensor coupling.

An alternative β-theory may be based on the assumption that, instead of (15a),

$$\mu\rightarrow\nu+p+\bar{n} \qquad (17a)$$

is the elementary process, so that the nuclear β-decay becomes a two-step process involving (1b):

$$P\rightarrow N+\mu+\bar{\nu}\rightarrow N+p+\bar{n},$$

whereas the pion would disintegrate immediately by $\mu+\bar{\nu}\rightarrow p+\bar{n}$ (f-part). Using B now to denote the coupling type in (17a), the conclusion regarding the objectionable or admissible A, B combinations is the same as above.

The well-known decay of the muon into (at least) three light particles, with a mean lifetime of 2×10^{-6} sec., may also cause a decay of a pion into an antinuon and the three light particles, but this process is, of course, much too slow to compete with the $\pi-\mu$-decay.

As to the latter process which is beyond the scope of this paper, the following comment may be welcome. If the $\bar{\mu}$-capture by light nuclei is interpreted[15] as the transition $P+\bar{\mu}\rightarrow N+\bar{n}$, e.g., described by the interaction term

$$\eta'\int d^3x(N^*A'P)(\mu^*A'n)+\text{conj.}$$

(similar to (1) but with the nuon replaced by the neutrino), this automatically entails the decay of the pion, through its F-part, into a muon and antineutrino ($P+\bar{N}\rightarrow\mu+\bar{n}$). But, again, $A'=A$ and other combinations must be excluded because otherwise the $\pi-\mu$-decay would be too fast, as compared with the measured pion lifetime.[14]

V. NUCLEAR SCATTERING AND PAIR CREATION OF MESONS

Weinberg and Jauch[3] have derived rigorous solutions for the scattering of muons by infinitely heavy nucleons according to the pair theory, assuming scalar or vector coupling. For other coupling types such rigorous solutions are not available because the interaction in the limit of nucleons at rest either involves the nucleon spin (tensor, pseudovector), or vanishes altogether (pseudoscalar). Moreover, the interaction (1) differs from those studied by Weinberg and Jauch in that the elementary process (1a) involves a charge exchange (the proton charge is transferred to the meson) which again prohibits a rigorous (static) solution. We therefore content ourselves with an approximation similar

[14] J. R. Richardson, Phys. Rev. **74**, 1720 (1948); E. A. Martinelli and W. K. H. Panofsky, Phys. Rev. **77**, 465 (1950).

[15] B. Pontecorvo, Phys. Rev. **72**, 246 (1947), see also reference 8.

to the one used in studying the pion (Section II): only states with one nucleon and one meson will be considered, i.e., higher states involving additional pairs will be disregarded. The nucleon velocity need not be small.

Let us consider the process $P+\bar{\mu}\leftrightarrow N+\bar{\nu}$. Introducing the probability amplitude $\varphi(\mathbf{p}\Lambda\lambda)$ for the presence of a proton with momentum $\mathbf{p}$ and spin Λ and of an antimuon with momentum $-\mathbf{p}$ and spin λ, and similarly $\varphi'(\mathbf{p}'\Lambda'\lambda')$ for the presence of a neutron and an antinuon, we obtain a Schrödinger equation of the form

$$0=[-\epsilon^0+E(p)+e(p)]\varphi(\mathbf{p}\Lambda\lambda)+\eta G(p)g(p)\int d^3p'G(p')g(p')\times\sum_{\lambda'\Lambda'}\sum_{\alpha}(\mathbf{p}\Lambda|A_\alpha|\mathbf{p}'\Lambda')(\mathbf{p}'\lambda'|A_\alpha|\mathbf{p}\lambda)\varphi'(\mathbf{p}'\Lambda'\lambda') \quad (18)$$

and a similar equation, with primed and unprimed quantities interchanged. It is convenient to use the relation

$$\sum_\alpha(\mathbf{p}\Lambda|A_\alpha|\mathbf{p}'\Lambda')(\mathbf{p}'\lambda'|A_\alpha|\mathbf{p}\lambda)=\sum_{K=1}^{5}g_K\sum_\alpha(\mathbf{p}\Lambda|A_\alpha{}^K|\mathbf{p}\lambda)(\mathbf{p}'\lambda'|A_\alpha{}^K|\mathbf{p}'\Lambda')$$

where $A^K(K=1\cdots5)$ stands for any of the pure coupling matrices; the numerical coefficients g_K may be taken from a paper by Fierz.[16] A solution of (18), corresponding to an initial $(P+\bar{\mu})$ state $[\epsilon^0=E(p^0)+e(p^0)]$ may be written as

$$\left.\begin{aligned}\varphi(\mathbf{p}\Lambda\lambda)&=\delta(\mathbf{p}-\mathbf{p}^0)\delta_{\Lambda\Lambda^0}\delta_{\lambda\lambda^0}+\frac{G(p)g(p)}{E(p)+e(p)-\epsilon^0}\sum_{K\alpha}(\mathbf{p}\Lambda|A_\alpha{}^K|\mathbf{p}\lambda)u_\alpha{}^K,\\ \varphi'(\mathbf{p}'\Lambda'\lambda')&=\frac{G(p')g(p')}{E(p')+e(p')-\epsilon^0}\times\sum_{K\alpha}(\mathbf{p}'\Lambda'|A_\alpha{}^K|\mathbf{p}'\lambda')v_\alpha{}^K,\end{aligned}\right\} \quad (19)$$

$$\left.\begin{aligned}&u_\alpha{}^K+\eta g_K\sum_{l\beta}c_{\alpha\beta}{}^{Kl}v_\beta{}^l=0,\\ &v_\alpha{}^K+\eta g_K\sum_{l\beta}c_{\alpha\beta}{}^{Kl}u_\beta{}^l=-\eta G(p^0)g(p^0)g_K(\mathbf{p}^0\lambda^0|A_\alpha{}^K|\mathbf{p}^0\Lambda^0),\end{aligned}\right\} \quad (20)$$

$$c_{\alpha\beta}{}^{Kl}=\int\frac{d^3pG^2(p)g^2(p)}{E(p)+e(p)-\epsilon^0}\times\sum_{\lambda\Lambda}(\mathbf{p}\lambda|A_\alpha{}^K|\mathbf{p}\Lambda)(\mathbf{p}\Lambda|A_\beta{}^l|\mathbf{p}\lambda). \quad (21)$$

[16] M. Fierz, Zeits. f. Physik **104**, 553 (1937), Eq. (1.4).

In order that the singularity at $p=p_0$ provide for outgoing waves in ordinary space, ϵ^0 should be given a small, positive imaginary part. φ', of course, represents a scattering with charge exchange, whereas the second term in φ describes the ordinary scattering, the proton and negative muon each retaining their charges.

This method of approximation, when applied to the similar problems treated by Weinberg and Jauch, gives very close agreement with their rigorous results, and there is no reason to doubt its reliability for estimating the cross sections, even in the pseudoscalar coupling case. Agreement with the Born approximation is obtained for $|\eta c_{\alpha\beta}{}^{Kl}|\ll1$. In the first-order Born approximation ($u_\alpha{}^K\rightarrow0$) only the scattering with charge exchange remains.

If we wish to retain, for the coupling parameter η and the cut-off momenta k, K, such values as were found compatible with the pion data (Sections II and III), the theory predicts, in general, scattering cross sections roughly a hundred times larger than those observed. As was mentioned in the introduction, this difficulty may be avoided by choosing for A the pseudoscalar $\beta\gamma_5$. To prove this, a Born approximation is sufficient. Indeed, if one expresses the solution of Eqs. (20) as expansions in powers of η, it is easily seen that, owing to certain small matrix elements of $\beta\gamma_5$, the condition

$$\left|\eta\frac{k}{M}\int\frac{d^3pG^2(p)g^2(p)}{E(p)+e(p)-\epsilon^0}\right|\ll1$$

suffices to ensure a rapid convergence of the Born expansion, and this condition is well satisfied for $k\sim m$, $K\sim M$, and η determined by (9), (10). In other words, with respect to the scattering, the coupling (1) may be considered as weak although this is not so for the stationary state problem of the pion (where the coupling strength may be called "intermediate"). The first order approximation leads to the following value for the total scattering cross section:

$$(16\pi^5/3)\eta^2[G(p^0)g(p^0)p^0]^4[E(p^0)+e(p^0)]^{-2}. \quad (22)$$

As mentioned, this value pertains to the scattering with charge exchange, the ordinary scattering having a much smaller probability. Inserting numerical values, one finds that the cross section (22) is quite small except for $p^0\approx k$ where it reaches a peak value of some 10^{-28} cm^2. The experimental data are hardly complete enough to disprove this result.

A phenomenon which should furnish an easier and more decisive observational test is the μ-pair creation in nuclear collisions. Indeed, our theory predicts an immediate correlation between the probabilities for the creation of a $\mu+\bar{\nu}$ pair (free) according to (1b), and for the creation of a pion ($\mu+\bar{\nu}$ bound) according to (12), and this quite independent of the mechanism of excitation. This is a simple consequence of the fact that, in the center of mass system of the mesons, the two matrix-elements (3) and (13), for $A=\beta\gamma_5$, have the ratio

$$H_{\mu+\nu}/H_\pi=\eta g^2(p)(\mathbf{p}\lambda|\beta\gamma_5|\mathbf{p}\lambda_0)/U^* \tag{23}$$

which is of the order $|\eta U^{-1}|=|(cC)^{-\frac{1}{2}}U^{-1}|$ (cf. (9), (10)) if $p(=p_0)<k$. The ratio of the cross sections is essentially the square of this constant times the ratio of the meson phase volumina. This latter factor is, of course, strongly energy dependent and makes the pair creation quite insignificant near the threshold. At higher energies the factor $g^2(p)$ in (23) cuts down the momentum space available to the pair particles and causes the μ/π ratio to reach a saturation value of the order $(4\pi/3)k^3|\eta U^{-1}|^2$, i.e., between one and ten percent. The smallness of this numerical value may explain why as yet the experiments have failed to reveal with certainty the creation of muons in high energy nuclear collisions.[17] For 350-Mev protons, the above saturation value should be almost reached, in other words, we expect such protons to produce a few muons per hundred pions. Since most of these muons will be more energetic than those resulting from $\pi-\mu$-decays, it should not be too difficult to test this prediction.

VI. CONCLUDING REMARKS

Of the problems which remain to be examined, those concerning the nuclear forces and the magnetic moments of proton and neutron are the most conspicuous. Here we must consider the contributions of both the pair field and the pion field; they are additive in a first approximation only. A further and more serious complication ensues from the apparent existence of a neutral pion ("neutretto") which one would like to interpret as a bound neutral pair ($\mu+\bar{\mu}$ and/or $\nu+\bar{\nu}$) and which calls for an obvious generalization of the interaction (1). For the problems studied in Sections II to V this is of minor importance (at least the orders of magnitude will be unaffected), but evidently in a problem like the charge dependence of the nuclear forces the effects of neutral pair and pion fields cannot be disregarded.[18] The same complication will encumber the problem of heavier mesons with more than two elementary constituents.[19]

The most objectionable element in our theoretical analysis is the use of the cut-off method which has been taken seriously to the extent that the momentum bounds k and K are postulated to have the same values in all individual applications. Whether this makes sense when viewed in the light of a future more perfect theory is, of course, quite doubtful, and it may be useful to keep some of the possibilities in mind which were discarded above. On the other hand, while no better method is available, it seems encouraging that even when imposing the rather rigid cut-off rules we can obtain a consistent picture of a considerable variety of phenomena which so far is nowhere obviously in conflict with experience.

[17] See for instance, O. Piccioni, Phys. Rev. **77**, 1 (1950); P. H. Fowler, Phil. Mag. **41**, 169 (1950).

[18] See for instance N. Kemmer, Phys. Rev. **52**, 906 (1937), who discusses the charge dependence of forces transmitted by charged and neutral pair fields in terms of isotopic spin operators. In a similar investigations by S. Noma, Prog. Theor. Phys. **3**, 54 (1948), the nuon and the antinuon are assumed to be identical particles, like a Majorana neutrino. The forces due to neutral pions are too well known to call for comments.

[19] To mention one example: Consider the simultaneous creation of 2 pairs: $(\mu+\bar{\nu})+(\nu+\bar{\nu})$; since presumably each two of the four particles attract one another, they may form a particle of "mesonic weight" 4. (The force between μ and ν arises from their ability to exchange a $(P+\bar{N})$ or $(\bar{P}+N)$ pair. Such force might even lead to a bound state $(\mu+\nu)$—different from $\pi=(\mu+\bar{\nu})$ discussed —; but the pair $(\mu+\nu)$ could not be created alone, according to (1).) Most of the compound mesons would be very short-lived due to γ-decay, but metastable states with longer lifetimes may exist.