

## A Precision Evaluation of $h/e$ by X-Rays

J. A. BEARDEN AND GUENTER SCHWARZ\*  
*Johns Hopkins University, Baltimore, Maryland*  
 (Received March 13, 1950)

Precision x-ray measurements of  $h/e$  have been made at potentials from 8000 to 19000 volts. A double crystal spectrometer with perfect calcite crystals was used as the monochromator. The window width was 0.7 volt. A unipotential cathode was developed and the use of this with the narrow monochromator window gave isochromats which broke very sharply at the high frequency limit. Isochromats were recorded with both tungsten and oxide cathodes which experimentally indicated the necessity for making the work function correction to the applied voltage. No evidence was found to support the previously reported effects due to surface films on the target or gas pressure in the x-ray tube. Targets of Au, W, Ta, Mo, Ni, and Cu were used. The pronounced structure near the high frequency limit appears to be much the same for all targets, differing only in some details and magnitude. The value found for  $h/e$  is  $1.37938 \pm 0.00008 \times 10^{-17}$  erg·sec·e.s.u.<sup>-1</sup> and is independent of the target or voltage used.

### I. INTRODUCTION

THE excellent studies on the natural constants made by Birge beginning with his review article<sup>1</sup> in 1929 have served not only to supply physicists with a self-consistent set of constants, but of equal importance has been the highlighting of a series of "discrepancies" which his work revealed. Notable among these have been the  $e$  and  $e/m$ , and more recently the  $h/e$ , discrepancies. The latter discrepancy stimulated the present experimental work on a new precision evaluation of  $h/e$  from the high frequency limit of the continuous x-ray spectrum.

For a direct measurement of  $h/e$  we have several methods available: (1) The measurement of the high frequency limit of the continuous x-ray spectrum; (2) The measurement of the excitation voltages of x-ray lines compared to the corresponding absorption edges; (3) The determination of the ionization and excitation potentials; (4) The photoelectric effect; (5) The measurement of the radiation constant. To date the first method stands out over all the others in attainable precision. However, it is possible that careful experiments and theoretical analysis of the absorption limits might make the second method comparable in precision to the first. In the present work, a few measurements<sup>2</sup> by the second method have been made, but the most conclusive results have been obtained through a series of measurements on the high frequency limit.

In devising a precision experiment on the high frequency limit, the spectral region in the neighborhood of 1.25A to 1.5A has been chosen for most of the present work which corresponds to a potential range of 8000 to 10,000 volts. The advantages of this spectral region are: (1) A double crystal spectrometer of high resolving power and a precisely divided circle can be employed in air; (2) A Geiger counter of high efficiency and low background can be used for measuring the low x-ray intensities observed near the high frequency limit;

(3) The potentials can be measured directly with high precision resistance standards; (4) An x-ray tube has been developed for this voltage range with unipotential indirectly heated cathodes; (5) Potentials applied to the x-ray tube and the current through the tube can be highly stabilized (one part in 30,000 to 50,000).

### II. EXPERIMENTAL ARRANGEMENTS

#### (1) The X-ray Tube

The main features of the x-ray tube are shown in Fig. 1. Use has been made of the method whereby gases from the Neoprene gaskets must diffuse a considerable distance between carefully machined surfaces before reaching the high vacuum. The base and body of the tube are made of copper and the flanges which hold the gaskets are of brass. The body and target are water-cooled. The body of the tube can be rotated about a vertical axis so that x-rays can emerge from the small window  $W$  at any desired angle with respect to the first crystal of the spectrometer. Two target assemblies have been used: (1) The axis on which the water-cooled target rotates, passes through its surface such that the angle of the emitted x-rays with respect to the target surface can be varied; (2) A four-sided target was designed such that different targets could be employed by rotating the target support by 90°.

The cathode structure was mounted on a glass insulator. In addition to the usual 5-volt, 4-ampere spiral tungsten filament cathode, we developed an indirectly heated oxide cathode. Details of the latter are shown in Fig. 2. Focusing of the electron beam is accomplished by properly locating the oxide surface in the cathode and by adjusting the cathode-target distance. The forming of the oxide cathode surface is a slow process, but once accomplished the life has been found to be 100 or more hours. After the cathode has been properly formed the x-ray tube may be opened to air for short intervals without seriously affecting the cathode emission.

The main difficulty encountered in operating the oxide cathode was in controlling the emission. This was

\* Now at Florida State University, Tallahassee, Florida.

<sup>1</sup> R. T. Birge, *Rev. Mod. Phys.* **1**, 1 (1929).

<sup>2</sup> J. A. Bearden and G. Schwarz, *Phys. Rev.* **59**, 934 (1941).

finally accomplished by heating the cathode (back surface) with electron bombardment and controlling the bombarding voltage with a voltage regulator actuated by the current flowing through the x-ray tube. When the regulator was used to control the bombarding current the heating inertia of the filament was too great and "hunting" occurred.

The x-ray tube is exhausted by a three-stage oil fractionating pump backed by a high speed mechanical fore pump. The pressure is measured near the base of the x-ray tube with an ionization gauge. The normal pressure in the x-ray tube when operating at a load of 500 watts was better than  $10^{-6}$  mm and was about  $2 \times 10^{-7}$  mm when the tube was not operating. When tungsten filament cathodes are used the evaporated tungsten gradually forms a visible film over the surface of the target. In the case of the oxide cathode, no visible film was observed.

### (2) The X-ray Power Supply

The essential requirement of the high voltage supply is stability and freedom from ripple. In the first measurements<sup>2</sup> a 500-cycle power source was used with a conventional capacity inductance filter. This supply was partially satisfactory but required continuous adjustment by the operator and even then changes of one or two volts frequently occurred during the measurement of the intensity at a selected voltage. A block diagram of the final high voltage power supply is shown in Fig. 3. Two high voltage systems of this type have been constructed. The one used for most of the present work was designed for a maximum of 12,000 volts and 50 ma, the second operates at the same current, but at voltages up to 100,000 volts. Tests on both supplies show excellent regulation. A 10 percent change in input a.c. voltage changes the 12,000-volt d.c. output by less than one volt. The 100,000-volt supply has an equally good percentage performance. The output impedance in each case is approximately 25 ohms.

The ripple voltage was determined by measuring the drop across a 100,000-ohm non-inductive resistance which was connected between ground and a large (0.4 mf) high voltage condenser which was connected to the high voltage. Both an oscilloscope and a peak-reading vacuum tube voltmeter were used to measure the ripple voltage. The measured ripple on the 12,000-volt supply was less than 70 millivolts r.m.s. and on the 100,000-volt supply it was less than 175 millivolts. In both cases the measurements were made at full operating loads.

When a tungsten filament was used as a source of electrons insulated heavy duty storage batteries were used as the source of power. The x-ray tube current was then controlled manually by a resistor which was in series with the batteries and filament. The voltage drop across the filament must be taken into account in determining the high voltage through

which the x-ray tube electrons are accelerated. Isochromats were obtained with the high voltage connected in turn to each end of the filament. It was found that the cut-off point of the isochromat in the two cases differed by almost exactly the voltage applied to the filament. This indicates that the  $IR$  drop through the filament leads and filament is confined almost entirely to the thermionic emitting region of the filament. The great majority of the electrons emitted do not produce x-rays within a volt of the high frequency limit and hence the isochromat observed does not show a sharp discontinuity at the quantum limit.

In order to eliminate the part of the indefiniteness of the isochromat which is due to the spread in energy of the electrons, a unipotential cathode of the oxide type described above was developed. The cut-off points of the isochromats obtained with this cathode were much more clearly defined and the structures observed above the limit were more pronounced than when the tungsten filament was employed. The oxide cathode operated very satisfactorily for voltages less than 10,000 and while we were able to get a few measurements with it at 20,000 volts, the operation was not comparable to the lower voltages.

### (3) Voltage Measurement

The principle used for measuring the high voltage applied to the x-ray tube is shown in Fig. 3. In a number of measurements we used a modification of the above method. The principal advantage of this method over that normally used is that possible errors in the Type K Leeds and Northrup potentiometer are greatly reduced by selecting a value for the high voltage such that the

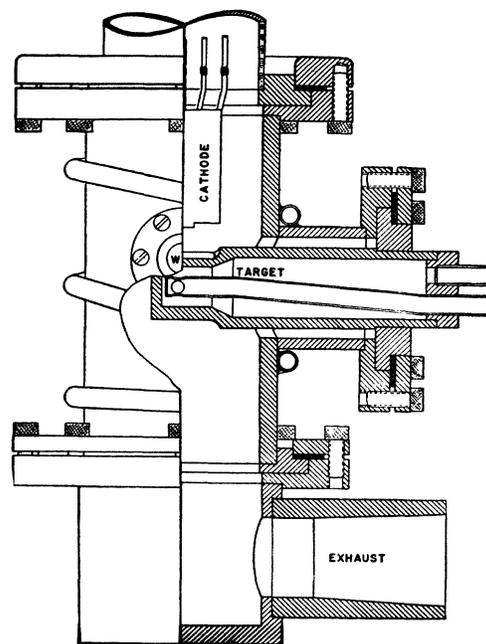


FIG. 1. X-ray tube.

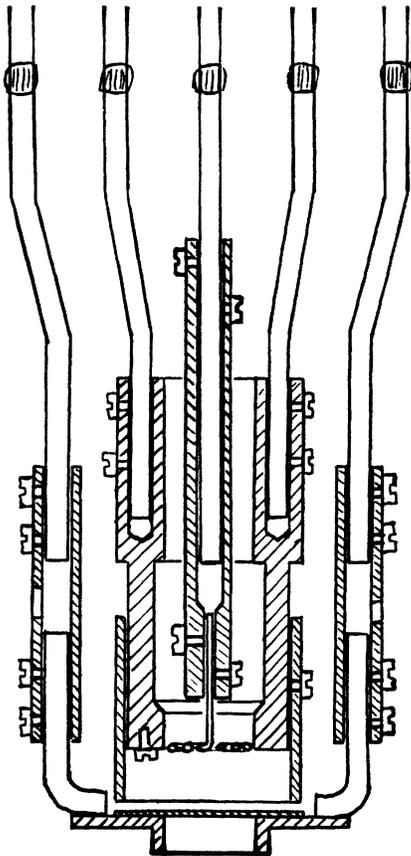


FIG. 2. Indirectly heated oxide cathode.

$IR$  drop across the 100-ohm standard in Fig. 3 is almost balanced by the potential of a second standard cell. The potential then measured by the potentiometer is only the difference between the  $IR$  drop across the 100-ohm standard and the potential of the standard cell. The potentiometer was carefully calibrated by the National Bureau of Standards for the particular ratios used in all experiments. As a further precaution the potentiometer was rechecked at the conclusion of the experiment.

Two one-megohm manganin wire standards made by Otto Wolf were made available to us for this work.<sup>3</sup> Each standard was made of 40 coils wound on tubes about 15 mm in diameter and 100 mm in length. A constant temperature oil bath was constructed which forced oil through and around each coil. The temperature of the oil was maintained within  $\pm 0.01^\circ\text{C}$ . The two megohm standards have been checked as shown in Table I, by the National Bureau of Standards and no changes have been observed which are greater than

<sup>3</sup> We are greatly indebted to Dr. J. L. Thomas for his splendid cooperation in loaning us standard resistances and for his special interest in calibrating these standards as well as calibrating the Type K potentiometer. We also wish to express our appreciation of Professor Wheeler Loomis and the Department of Physics of the University of Illinois for the loan of one of the Otto Wolf resistance standards.

normal for high resistance standards. The temperature coefficient of both standards is slightly negative at the temperature used. Three 100-ohm Leeds and Northrup precision resistance standards were periodically inter-compared and one was calibrated twice, as shown in Table I.

Through the cooperation of Dr. G. W. Vinal of the National Bureau of Standards we had available three of their standard cells of the saturated type. The calibration is shown in Table I. The cells were kept in the constant temperature oil bath and periodic inter-comparisons were made of their voltages. Over a period of eight years no change of more than  $10^{-5}$  volt has been observed in the voltage of any cell.

Each coil of the megohm standard dissipates 2.5 watts when operating at full voltage. Even though rapid circulation of oil was maintained in and around the coils, the insulated manganin wire was at some temperature above that of the oil. Two experiments were undertaken to measure the effect of this heating. In the first experiment the Bureau of Standards used the high powered resistance standards from their high voltage laboratory in order to test the resistance of the Wolf standard under low and high load conditions. The high voltage standards are capable of dissipating large amounts of power without appreciable heating. A bridge circuit was formed using these resistances together with the Wolf standard as the unknown. The latter was operated in the same circulating constant temperature oil bath as was used in the x-ray measurements. A balance was first obtained with 1000 volts on the bridge. Under this condition the coils of the Wolf standard were dissipating only 0.025 watt which should produce a negligible heating of the manganin wire. 10,000 volts were then applied to the bridge and the change in resistance observed was assumed to be due to the heating of the manganin wire. The results were somewhat erratic and the two Wolf standards gave slightly different results. The change in resistance appeared to be about 30 parts in  $10^6$  for standard 4564 and 15 parts in  $10^6$  for standard 1427. These values represented just about the limit of accuracy of the measurements.

The second method of measuring the change in resistance with load was made in our own laboratory in which use was made of the  $10^5$ -ohm subdivisions of the Wolf standards. A bridge was formed in which a  $10^5$ -ohm subdivision of one Wolf standard was compared to the entire  $10^6$ -ohm resistance of the other. A balance was first obtained with 100 volts on the bridge and the change in resistance measured when the voltage was increased to 1000 volts. Under the latter condition, a  $10^5$ -ohm section has the same load as in normal use with 10,000 volts on the 10 sections in series. Each  $10^5$ -ohm section was compared in turn and the sum of the corrections amounted to  $-11$  ohms in the total resistances, for standard 4564 and  $-4$  ohms for standard

1427. The latter measurements are accurate to  $\pm 3$  ohms. From the two experiments it seems safe to conclude that no change in resistance occurred greater than one part in  $10^5$  when the standards were used at full load.

The voltage divider measures the x-ray cathode to ground voltage. The x-ray target, however, is grounded through a milliammeter and over current relay and the resistance across which is developed the voltages for the operation of the current and voltage stabilizers. These resistances have been carefully measured and in addition the voltage drop has been measured with a precision vacuum tube voltmeter while isochromats were being recorded. However, in most of the isochromats the voltage correction was obtained by computing the voltage drop from the x-ray tube current and the resistance between cathode and ground.

#### (4) Monochrometer

The spectrometer used was a high precision double crystal instrument built by the Société Genevoise. The spectrometer was altered such that the first crystal is mounted on a horizontal slide midway between the focal spot of the x-ray tube and the second crystal. The second crystal is accordingly located on the axis of rotation of the high precision divided circle. The diffraction angles of the second crystal are read on the divided circle by four microscopes. The circle has been carefully calibrated by the four microscopes  $90^\circ$  method and the excellence of the circle is indicated by the fact that over the greater part of the circle corrections are less than 0.3 sec. In the wave-length region employed in the present measurements this corresponds to less than 0.1 volt in the position of the isochromats. The measured crystal angles at room temperature were corrected to  $18^\circ\text{C}$  by the relation

$$d\theta'' = 2.18(t^\circ - 18^\circ\text{C}) \tan\theta.$$

The sharpness of the isochromat cut-off in the experiments where the oxide cathode was employed is primarily dependent upon the resolving power of the monochrometer. The low x-ray intensity near the quantum limit has forced most previous experimenters to use monochrometers of very low resolving power. Geiger counters have been perfected for recording x-ray intensities to such an extent that theoretically perfect calcites can now be used in the (1+1) position and still give measurable intensities within one volt of the quantum limit. The resolving power may be computed from the relation  $\lambda/d\lambda = \lambda D/w$  where  $D$  is the dispersion and  $w$  is the full width at half-maximum in radians of the (1-1) rocking curve. The calcite crystals used were the same as those used in previous experiments reported many years ago. The crystals have been rechecked at 1.54A and the (1-1) width was 10.0 sec. or  $4.85 \times 10^{-5}$  radian and  $D$  is equal to 0.34 which gives 10,800 as the resolving power. The full energy width of the monochrometer "window" at half-maximum intensity is 0.7 volt. The effect of the window on the

sharpness of the limit of the isochromats is thus negligible.

The first 4 or 5 volts of an isochromat essentially determines the quantum threshold. Within this range, and under the conditions of the present experiment, the number of quanta diffracted by the calcite crystals is of the order of 10 to 15 quanta per second. In an ionization chamber the radioactive contamination and the cosmic-ray background would make the observation of this number of x-ray photons practically impossible. On the other hand, the background of a properly shielded Geiger counter can be reduced to less than 0.5 count per second. The principal difficulty with Geiger counters is their low absorption of x-rays and this has been overcome by the use of argon and krypton at high pressure. Counters filled with argon to 70 cm pressure and krypton counters filled to 30 cm pressure, plus quenching gases of methylene bromide or halogene mixtures have proved satisfactory. The argon counters absorb about 80 percent of the radiation of the 1.54A range and the krypton counters absorb about 50 percent of the radiation, at 0.6A. The efficiency of these counters makes it possible to record intensities very near the quantum limit. The body of the counters is of stainless steel with a mica window and metal-to-glass insulating seal. Eight inches of lead and iron were placed over the counters to reduce the cosmic-ray background. A scaling circuit arranged to measure the time for a fixed number of counts was used for recording the intensities.

### III. CORRECTIONS AND AUXILIARY CONSTANTS

#### (1) Work Function Correction

DuMond<sup>4</sup> and his co-workers have pointed out the necessity for correcting the measured voltage applied to the x-ray tube by adding the work function of the cathode which supplies the electrons. Ohlin,<sup>5</sup> on the

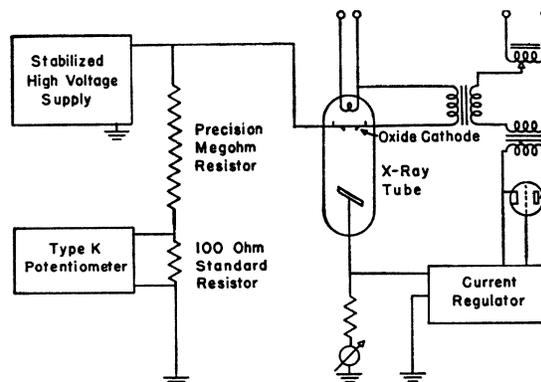


FIG. 3. Source of x-rays and measurement of applied voltage.

<sup>4</sup> J. W. M. DuMond and V. L. Bollman, *Phys. Rev.* **51**, 412 (1937). Panofsky, Green, and DuMond, *Phys. Rev.* **62**, 224 (1942).

<sup>5</sup> P. Ohlin, Inaugural Dissertation, Uppsala (1941); *Arkiv. f. Mat. Astr. o. Fys.* **27B**, No. 10; **29A**, No. 3; **29B**, No. 4; **31A**, No. 9; **33A**, No. 23.

TABLE I. Calibration of resistance and voltage standards by the National Bureau of Standards.

| Standard no.   | Resistances   |                | Resistance absolute ohms |
|----------------|---------------|----------------|--------------------------|
|                | Date          | Temp.          |                          |
| 1427           | 2/41          | 22.0°C         | 1,000,715                |
| 1427           | 2/41          | 23.8           | 1,000,726                |
| 1427           | 7/48          | 23.6           | 1,000,759                |
| 1427           | 5/49          | 26.0           | 1,000,736                |
| 4564           | 2/48          | 25.0           | 1,001,110                |
| 4564           | 7/48          | 24.6           | 1,001,111                |
| 4564           | 5/49          | 26.0           | 1,001,085                |
| 649482         | 5/47          | 25.0           | 100.0500                 |
| 649482         | 4/49          | 25.0           | 100.0507                 |
| Standard cells |               |                |                          |
| Cell no.       | Acidity       | Ab. volts 20°C | Ab. volts 28°C           |
| 968            | 0.04 <i>N</i> | 1.018614       | 1.018241                 |
| 970            | 0.04 <i>N</i> | 1.018612       | 1.018239                 |
| 987            | 0.04 <i>N</i> | 1.018601       | 1.018227                 |

other hand, has not corrected his measurements for this effect primarily because his resulting values of  $h/e$  at voltages from 2983.3 to 4651.6 volts are more consistent if the work function correction is not made. Furthermore, he has used a BaO-coated platinum cathode in which the work function should be materially different from that of tungsten and the value of  $h/e$  observed agrees with his other results if the work function correction is not made. Thus, from two types of measurements he concludes that the energy gained by the electrons in passing through the cathode barrier does not exhibit itself in the position of the high frequency limit of the continuous spectrum. DuMond and Cohen<sup>6</sup> have corrected Ohlin's values for the work function and the resulting values of  $h/e$  are spread from  $1.3798$  to  $1.3810 \times 10^{-17}$  which are much higher than any previous experimental results and also considerably higher than the least squares adjusted value<sup>6</sup> of  $h/e$ .

In spite of Ohlin's<sup>5</sup> experimental results (which are not confirmed by the present experiments) the writers are convinced that it is necessary to apply a correction for the energy gained by the electrons due to the heating of the cathode. A slightly different approach to the problem may help to understand the theoretical analysis<sup>4</sup> which indicates the necessity for making the correction. In Fig. 4, the potential,  $V$ , of the battery measured by the voltage divider is applied to the conduction levels (dashed lines) of the cathode and target. If we assume the cathode and target to be infinite in extent then the electric field  $F$  is equal to the potential  $V$  plus the contact potential divided by the separation of the plates,  $d$ . Electrons emerge from the cathode with practically zero velocity. The energy gained by the electrons in passing from just outside the cathode to the outside of the target surface is  $FXd$ . However, those electrons that reach the conduction band (indicated by dotted line) of the target without a collision gain an

<sup>6</sup> J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. 20, 82 (1948).

additional energy equal to the work function of the target  $W_t$ . The maximum energy an electron can attain then is  $E_{\max} = [(V + \text{contact potential})/d] \times d + W_t$ . The contact potential is the difference in work function of the cathode and target or  $E_{\max} = V + (W_c - W_t) + W_t = V + W_c$  which is exactly the result obtained by the more elaborate argument presented by DuMond.<sup>4</sup> The same result can be arrived at with a thermodynamical cyclic process. If we take as reference point the top of the conduction band of the target, one sees that the maximum energy gained by an electron in a complete cycle is the potential  $V$  plus the energy gained by the electron in escaping from a cathode, which again confirms the previous conclusions that the energy added to the electron by cathode heating becomes available for the excitation of the high frequency limit. In addition to the theoretical arguments, it will be shown later that the present experiments with tungsten and with oxide coated cathodes will give inconsistent values of  $h/e$  unless proper corrections are made for the work functions.

### (2) Isochromats with Target at High Temperature

In many experiments involving electrons and metal surfaces it has been noted that appreciable charges exist even on surfaces newly formed in vacuum. It is conceivable that such surface charges could affect the values of  $h/e$  computed from the high frequency limit. These surface effects should disappear at high temperature. A tungsten target with a long heat conduction path to the water-cooled target support was developed such that with normal input power the temperature of the target was raised to about 1600°K. Isochromats, Fig. 5, were taken with the hot target at 10,180 volts and compared to those taken at the same voltage with a water-cooled target. A shift of less than one volt in the location of the isochromat could have been detected and within this experimental error no shift was observed.

### (3) Effect of Surface Films on the Target

In the early experiments,<sup>2</sup> where the pumping speed was inferior to the present system, the target accumulated a visible film of carbon and tungsten in a few hours' operation similar to that noted by DuMond<sup>4</sup> and his co-workers. In order to test for the effect of this film, a piece of molybdenum, to which a small piece of iron was spotwelded, was used as a cover over the target during the initial evacuation and early operation of the x-ray tube. The voltage was then adjusted such that a critical point on the initial rise of the isochromat could be observed. The molybdenum cover was removed by a magnet and the intensity of this point quickly observed. Continued measurements were made on the isochromat until an opaque film formed on the target. The structure in the vicinity of the high frequency limit

changed as the film formed but the position of the high frequency limit remained unchanged.

Panofsky, Green, and DuMond<sup>4</sup> observed a displacement of 90 volts or 0.9 percent between the positions of the isochromats with and without a carbon layer. In view of the fact that DuMond and Cohen<sup>6</sup> suggest that Ohlin's<sup>5</sup> experimental results may have been influenced by target films, it is worth while to examine these observations in more detail.

DuMond<sup>7</sup> has pointed out that an error of  $10^3$  made in computing the retardation of primary electrons into targets completely invalidated his theory for the fine structure which is observed near the high frequency limit. The same error invalidates the assumed thickness of target films in Ohlin's experiments. Films of only a few molecules in thickness are required to retard the primary electrons and if the primary electron in either Panofsky, Green, and DuMond, or Ohlin's experiments were retarded by carbon layers, then pronounced diffuseness of the foot of the isochromats should have been observed. A large part of the retardation of high velocity electrons in solids is due to very distant electronic encounters. A reasonable value for the loss per encounter is about 10 electron volts. Therefore, it would require about 9 encounters on the average to lose the 90 volts observed by Panofsky, Green, and DuMond.<sup>4</sup> With as many as 9 collisions on the average a few electrons should still reach the underlying target metal with undiminished energy. Therefore the isochromats observed with the carbon layer should have started with a very low intensity at the same point as the isochromats without the carbon deposit and the intensity should have gradually increased in accordance with the distribution losses suffered by the impinging high speed electrons. The fact that no such smearing was observed suggests that the effect observed was due to some other experimental cause.

In the case of Ohlin's<sup>5</sup> experiments the films would need to be only a very few molecules thick and the primary electrons should experience retardations due to probably one encounter. The effect of this would be a displacement of an isochromat of low intensity leaving the main isochromat undisplaced and sharp. The sharpness of Ohlin's isochromats and those of the present experiments indicate that target films normally encountered do not shift the position of the observed isochromats.

#### (4) $h/e$ as a Function of Gas Pressure in the X-Ray Tube

Ohlin<sup>5</sup> has emphasized that the value of  $h/e$  increases as the vacuum in the x-ray tube is improved. In the wave-length range from 2.5Å to 4.1Å he obtained a value of  $h/e$  at pressures of the order of  $10^{-4}$  mm of mercury, which was about 0.1 percent smaller than

when the vacuum was maintained at better than  $10^{-5}$  mm of mercury.

In some of our earlier experiments<sup>2</sup> a glass body x-ray tube was used which was evacuated by a moderate speed pumping system. In these experiments a deliberate attempt was made to observe a change in the position of the high frequency limit when the pressure in the tube was such that (estimated to be about 2 to  $3 \times 10^{-4}$  mm) it was barely possible to prevent a gas discharge in the tube. We were unable to observe any shift in the position of the isochromat which could be attributed to the gas pressure in the tube. Instability in operation made it impossible to make precision measurements under these conditions but we believe any changes of the order of magnitude of those reported by Ohlin would have been observed.

It has been suggested that Ohlin's observations may have arisen through scattering by the residual gas in the slit in front of the Geiger counter. An estimate of the expected scattering is so small that the present writers believe this explanation unlikely. The effect is more likely due to a shift in the position of the focal spot of the x-ray tube or some other instrumental difficulty.

#### (5) Auxiliary Constants Used in Computing $h/e$

If  $V$  represents the total voltage (applied potential plus work function) through which an electron falls, and  $\nu$  the frequency of the quantum limit, then  $Ve = h\nu$ . Or, in terms of the wave-length  $h/e = v\lambda/c$ . Details of the proper procedure for precisely relating  $V$  and  $\lambda$  will be discussed in the section on computations of  $h/e$ . The velocity of light,  $c = (2.99776 \pm 0.00004) \times 10^{10}$  cm/sec.

The wave-length  $\lambda$  depends directly on the ruled grating measurement of x-ray wave-lengths. From the excellent agreement in the independent measurements of Becklin, Sodermann, and Bearden<sup>8</sup> the wave-lengths appeared to be known within one or two parts in  $10^5$ . More recent measurements by Tyren<sup>9</sup> however, differ from the average of the above by about 4 parts in  $10^5$ . Crystallographers have decided to give some weight to Tyren's work by lowering the previously accepted ratio of  $\lambda_g/\lambda_s$  by one part in  $10^5$ . However, Birge,<sup>10</sup> and more recently, DuMond and Cohen<sup>6</sup> have retained  $\lambda_g/\lambda_s = 1.00203$  and we have used this value in computing our present values of  $h/e$ .

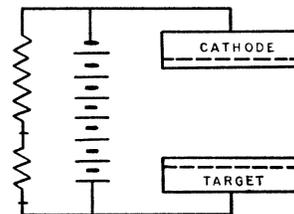


FIG. 4. Work function correction.

<sup>8</sup> J. A. Bearden, J. App. Phys. 12, 395 (1941).

<sup>9</sup> F. Tyren, Inaugural Dissertation, Uppsala (1940).

<sup>10</sup> R. T. Birge, Am. J. Phys. 13, 63 (1945).

<sup>7</sup> J. W. M. DuMond, Phys. Rev. 74, 1883 (1948).

The applied voltage must be corrected by the addition of the work function of the cathode. In the case of tungsten, we have used 4.52 volts. The work function of oxide cathodes<sup>11</sup> depends on the supporting metal and on the process used for activating the cathode. The values of the work function vary from about 0.8 to 1.5 volts. We have assumed an average value of 1.2 volts as applicable to the present measurements. It will be noted in Fig. 6, that if this correction were not made then the resulting isochromats taken with different cathodes would not agree. This correction introduces a greater uncertainty in our final results than all other factors combined. A new experiment is in progress in which we hope to eliminate the major part of this uncertainty.

#### ISOCHROMATS AND COMPUTATIONS OF $h/e$

##### (1) Recording of Isochromats

The x-ray intensity transmitted by the monochromator was recorded as a function of the voltage applied to the x-ray tube (isochromat method). The number of counts recorded for each point on the isochromat was selected such that a curve could be completed in from one to two hours. The statistical fluctuation of the points on a single curve made it impossible to compute a precise value of  $h/e$  from such an isochromat. A large number of isochromats were recorded under identical operating conditions. The intensities for each voltage setting were averaged and the average isochromat plotted as in Figs. 6-8. Most isochromats have been recorded at voltages lower and higher than those plotted. The number of runs averaged varied from 8 to 24. In this method of averaging it is essential that the x-ray tube current, the temperature of the crystals, standard cells, and resistances remain constant. These factors

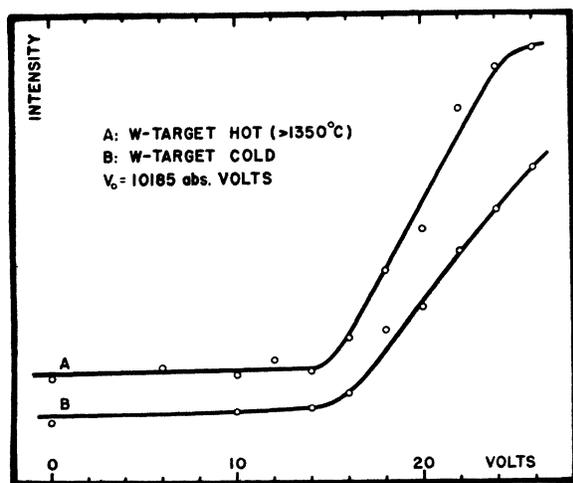


FIG. 5. Isochromats recorded at near room temperature and at 1350°C. The monochromator selected the same wave-length in each case.

<sup>11</sup> Jacobs, Hees, and Crossley, Proc. I.R.E. 36, 1109 (1948).

were so well controlled that curves taken many days apart could be averaged without difficulty.

In order to make it possible to compare isochromats taken with different targets and with different voltages, we have computed the voltage in each case which corresponds to a value of  $h/e$  equal to  $1.3792 \times 10^{-17}$ . This voltage is plotted as zero in Figs. 6-8, and the observed x-ray intensities of the isochromats have been plotted as functions of the incremental x-ray voltage.

##### (2) Fine Structure in the Isochromats

In our early experiments<sup>2</sup> in 1941-42 structure was observed in the isochromats similar to that shown in Figs. 6-8. Accumulation of surface film on the target usually caused the structure to vanish after a few hours operation, but as emphasized above the location of the high frequency limit was not altered. In the present work where high pumping speeds have been employed deposits and cathode evaporation have been so small that the structure remains pronounced after many hours' operation of the x-ray tube. Furthermore, in every case where we have repeated experiments after replacing cathode and targets, the details of the structure repeat within experimental error.

In all curves in Figs. 6-8, except the one for copper there appears a first peak in the isochromat at from 6 to 9 volts from the high frequency limit. A second peak occurs in the region of 16 to 18 volts and copper shows a peak at this point. Some curves show evidence of additional structure beyond this point, but it is so washed out as to make analysis difficult. Comparing the curves for tungsten at 8045 volts and at 19,600 volts (Fig. 6—curve A) one notes that the first peak occurs at 6 to 8 volts in both cases. Cathode difficulties prevented the curve A from being extended but rough details can be observed in the "washed out" filament cathode curve B. Nilsson and Ohlin<sup>12</sup> have recorded with a curved crystal spectrograph a very similar isochromat for tungsten at 3690 volts. It appears, therefore, that the structure is independent of the applied x-ray voltage. Nilsson and Ohlin<sup>12</sup> recorded similar isochromats for targets of Cr, Fe, Cu, Zn, Au, and Pb. In general, their results are in good agreement with those of Figs. 6-8. Panofsky, Green, and DuMond<sup>4</sup> failed to observe such fine structure in their isochromats both at 10 and 20 kv. They used a gold target and from Fig. 8, one notes that the minimum is not as pronounced in this case as for the other targets. Low resolving power of the monochromator could easily cause the structure in gold to be overlooked.

Considerable speculation on the possible source of the unexpected structure has occurred but no analysis has explained any of the details observed. Nijboer<sup>13</sup> pro-

<sup>12</sup> A. Nilsson and P. Ohlin, Arkiv. f. Mat. Astr. o. Fys. 33A, No. 23.

<sup>13</sup> B. R. A. Nijboer, Physica 12, 461 (1946).

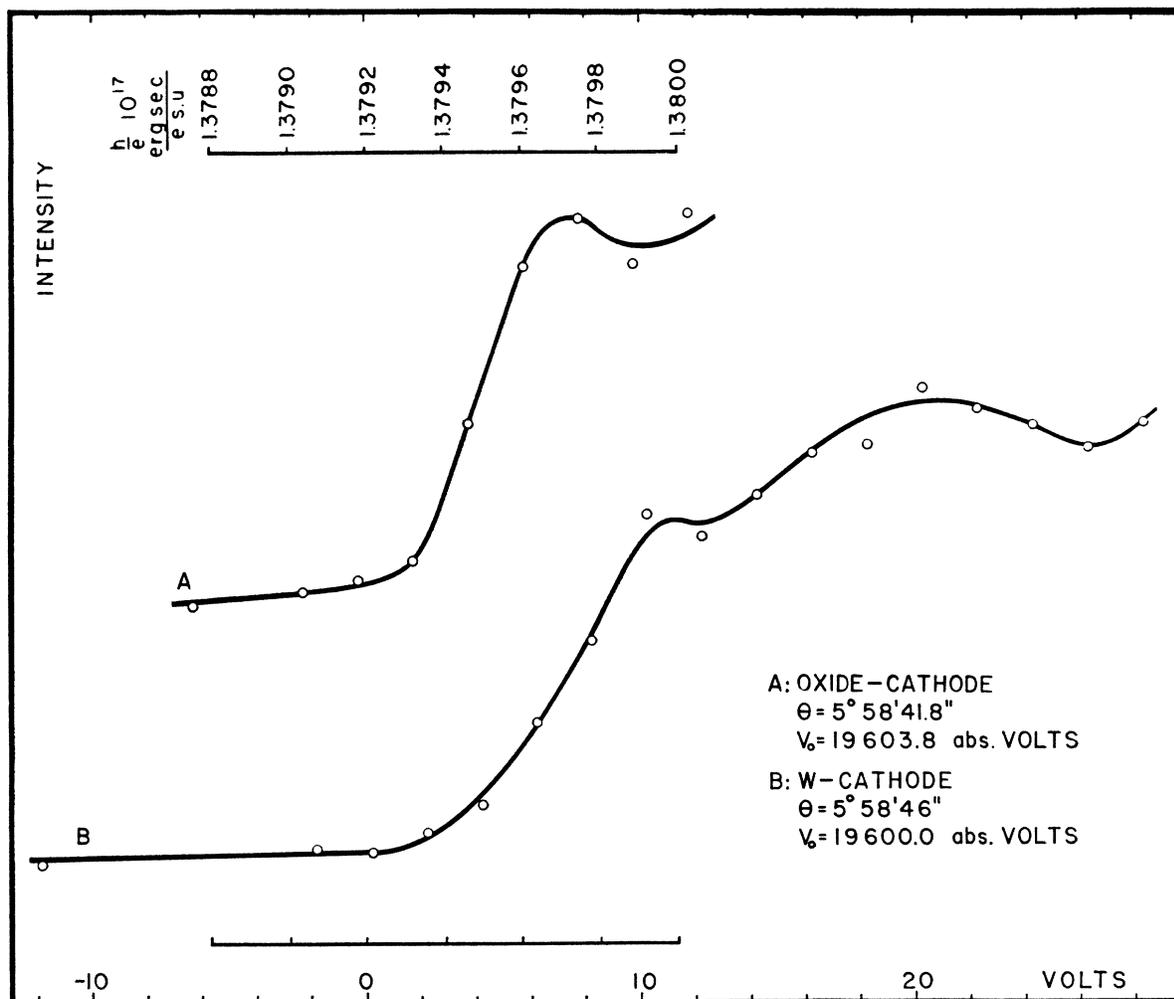


FIG. 6. The increased sharpness of the break in the isochromat due to the unipotential cathode is clearly shown in curve A. The structure at this voltage corresponds to that observed at lower voltages.

poses that the minima could be explained from an analogy with the fine structure observed in x-ray absorption edges. He assumes that in both cases the final state of the electron is one of the lowest unoccupied levels. Further, it is assumed that for small energy losses of the primary electron the energy loss curve for metals shows large fluctuations. Nijboer<sup>13</sup> was unable to make any quantitative calculations of the isochromat to be expected but concluded that it seemed possible to account qualitatively for the first minima in the observed isochromat.

An examination of the isochromats suggests that the curves in general (except copper) are a superposition of isochromats displaced about 10 and 20 volts from the high frequency limit. The energy lost by a high velocity electron in traversing a solid is primarily due to a large number of distant encounters in which the average energy loss is in the range from 7 to 12 volts. The high frequency limit would on this basis correspond to electrons with no energy loss and following this we would

have a second high frequency limit corresponding to electrons which had had one collision and hence had lost from 7 to 12 volts, and correspondingly the third high frequency limit would correspond to those electrons which had experienced two encounters and lost approximately 20 volts. The primary difficulty with such an explanation lies in the fact that theoretically the isochromats observed from thin targets should be rectangular in shape and therefore by superposition alone one could not explain the minima observed. Experimental measurements on thin targets with high resolving power have never been made near the high frequency limit. The fine structure observed with thick targets emphasizes the importance of performing such experiments.

### (3) Analysis and Computation of $h/e$ from the Isochromats

The "window" of the monochromator has a sharp peak at a particular wave-length, with the intensity

decaying asymptotically on either side. The energy of the electron beam is not perfectly homogeneous. The result of these two effects is to produce a curvature at the cut-off point of the isochromat. This may be made small as in the present experiments, but it cannot be entirely eliminated. DuMond has shown that the cut-off point is actually represented by the point of maximum bending of the isochromat or the maximum of the second derivative of the isochromat. He further shows theoretically that it is not permissible to locate the cut-off points by extending the straight line portion of the isochromat to the voltage axis. In the present experiments where the resolving power of the monochromator is approximately that given for perfect crystals, and where unipotential cathodes have been used, the resulting isochromats are so sharply defined that the cut-off points estimated by visual inspection and those obtained by the second derivative method agree within 0.2 or 0.3 of a volt.

The voltages at which certain values of  $h/e$  occur have been computed from the wave-length setting of the monochromator. These values have been indicated on the isochromats of Figs. 6-8. In addition to the iso-

chromats shown, a large number have been recorded from which a value of  $h/e$  has been computed. Table II summarizes the values of  $h/e$  as was obtained in our best experiments for several targets and voltages.

Many additional results have been obtained with a tungsten filament cathode which agree with those listed in the table but the error introduced by the lack of sharpness in the isochromat cut-off was so much greater than in the results with unipotential cathodes that we listed only one tungsten filament result. However the accuracy was always sufficient to show the necessity for making the work function correction.

#### (4) Discussion of Errors

Each value of  $h/e$  in Table II is an independent measurement, as far as observational errors are concerned. The possible error due to the use of different targets, change in the work function of differently activated oxide cathodes, adjustment of spectrometer, observational errors in the voltage and current measurement, uncertainty in the location of the high frequency limit from the isochromats, are all contained in the probable error calculated from the independent results

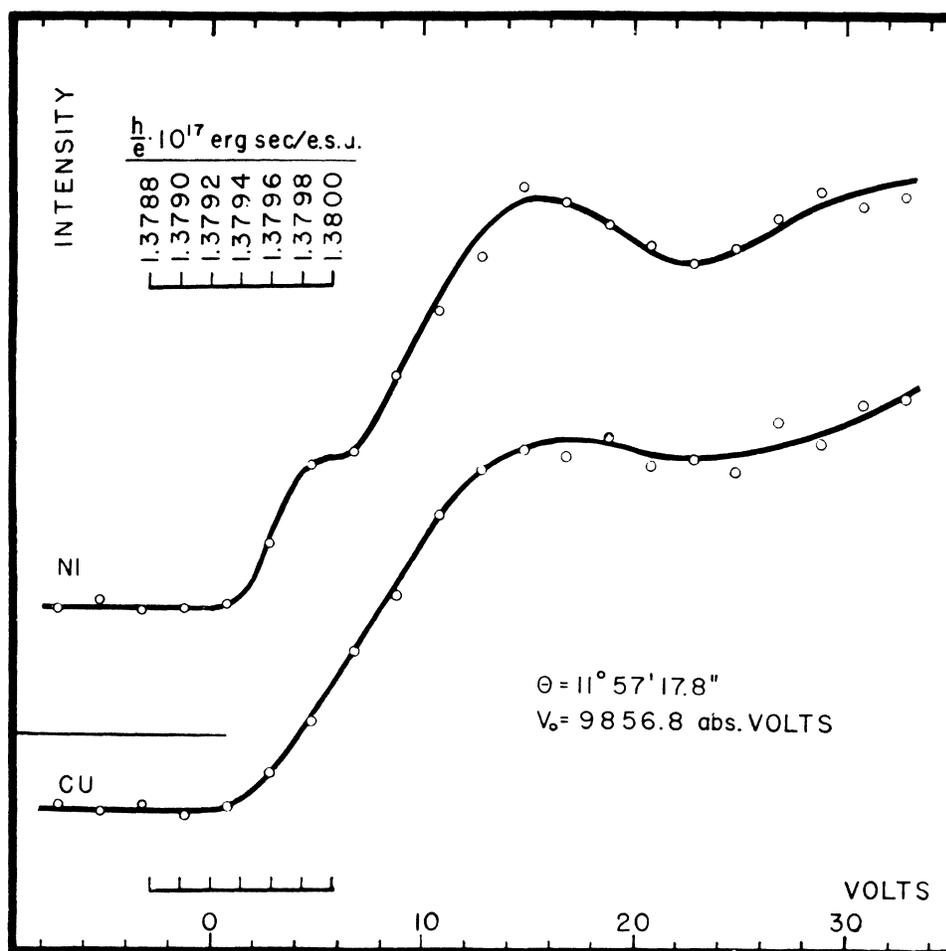


FIG. 7. Isochromats for copper and nickel target at the same voltage.

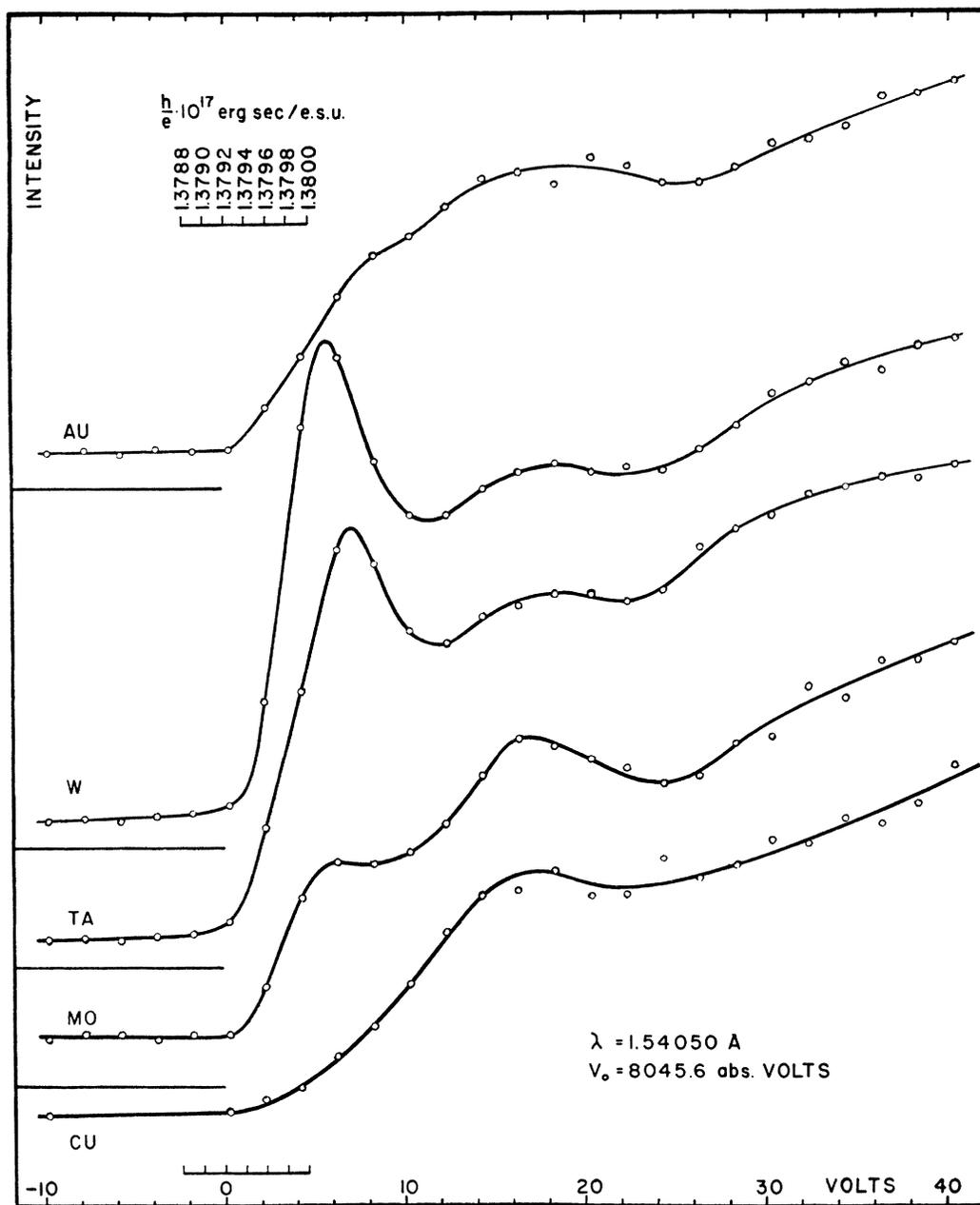


Fig. 8. Isochromats for targets of gold, tungsten, tantalum, molybdenum, and copper at the wave-length of the copper  $K\alpha_1$  line.

of Table II. The probable error as calculated from Table II due to all of these factors is hence  $3 \times 10^{-5}$ . The estimated probable errors common to all measurements and their values are as follows:

- |   |                    |
|---|--------------------|
| (1) x-ray monochromator                           | $1 \times 10^{-5}$ |
| (2) uncertainty in ruled grating wave-length      | $2 \times 10^{-5}$ |
| (3) resistance of voltage divider                 | $4 \times 10^{-6}$ |
| (4) standard cells, including temperature effects | $2 \times 10^{-6}$ |
| (5) Leeds-Northrup Type K-2 potentiometer         | $4 \times 10^{-6}$ |
| (6) work function of oxide cathodes               | $5 \times 10^{-6}$ |

The combination of these probable errors in the usual way gives  $6 \times 10^{-5}$  or

$$h/e = 1.3793_8 \pm 0.00008 \times 10^{-17} \text{ erg} \cdot \text{sec} \cdot \text{e.s.u.}^{-1}.$$

It is well known that the work function of an oxide cathode is quite dependent on the backing metal and on the processing required to activate the surface. In the present use of a cathode in a high potential x-ray tube positive ion bombardment may induce additional uncertainty in the effective value of the work function.

The sharpness of the first peak in the structure for the tungsten target suggests that one could eliminate the cathode correction by making measurements at two voltages and then compute  $h/e$  from the difference in the voltages and the frequency separation of the peaks. Such an experiment appears entirely feasible and the precision attainable should be such that the remaining uncertainty in  $h/e$  by the x-ray methods would be primarily due to the probable error in the ruled grating x-ray wave-lengths.

### (5) Discussion of the Evaluations of $h/e$

It appears that the values of  $h/e$  reported by Ohlin<sup>5</sup> should definitely be corrected for the work function of the cathode used. This correction increases the value of  $h/e$  as reported by him from 1.3787 to  $1.3803 \times 10^{-17}$  erg·sec·e.s.u.<sup>-1</sup>. Panofsky, Green, and DuMond obtained the value  $1.3786 \pm 0.0004 \times 10^{-17}$  erg·sec·e.s.u.<sup>-1</sup> and the present measurements give  $1.37938 \pm 0.00008 \times 10^{-17}$  erg·sec·e.s.u.<sup>-1</sup>. DuMond and Cohen<sup>6</sup> computed the value  $1.37926 \pm 0.00009 \times 10^{-17}$  erg·sec·e.s.u.<sup>-1</sup>. The last two values agree within the limits of the probable errors and for the first time since the  $h/e$  "discrepancy" was highlighted we have agreement between the computed and experimental value. In fact if the present experimental value were used as input data for the least squares adjustment the agreement

TABLE II. Values of  $h/e$ .

| Target | Approximate voltage | Cathode | $h/e \times 10^{17}$<br>erg·sec·e.s.u. <sup>-1</sup> |
|--------|---------------------|---------|--|
| Cu     | 8,050               | oxide   | 1.3795   |
| Cu     | 9,860               | oxide   | 1.3793   |
| Ni     | 9,860               | oxide   | 1.3793   |
| Mo     | 8,050               | oxide   | 1.3794   |
| Ta     | 8,050               | oxide   | 1.3794   |
| W      | 8,050               | oxide   | 1.3793   |
| W      | 19,600              | oxide   | 1.3793   |
| W      | 19,600              | W. fil  | 1.3795   |
| Au     | 8,050               | oxide   | 1.3794   |
|        |                     |         | Average 1.37938 <sup>a</sup>                         |

<sup>a</sup> In computing  $h/e$  the absolute volt must be converted to the electrostatic system by the approximate factor of 300. We are tremendously embarrassed by the fact that all of our previous reports on the measurement of  $h/e$  have been computed with this approximate factor rather than the correct value 299.776. Thus, we have spent much effort in attempting to find an explanation of the low "experimental results" observed whereas in fact all of our results have been in excellent agreement with the value of  $h/e$  computed by least squares analysis. The search for the cause of the "discrepancy" has resulted in the attainment of a precision in the final value that otherwise would not have been the case.

between computed and experimental would be even closer.

We wish to acknowledge the assistance given by the Bureau of Ordnance USN and our appreciation to Dr. Thomas and Dr. Vinal of the National Bureau of Standards for their cooperation and interest in calibrating the electrical standards, and to Mr. E. H. Byerly for his assistance in developing the electronic equipment which contributed markedly to the success of the present experiments.