The Production of Mesons by Photons*

K. A. BRUECKNER
Radiation Laboratory, University of California, Berkeley, California
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The general features of the process of photo-meson production are discussed on a semiclassical basis and also on the basis of perturbation theory in the weak coupling approximation applied to scalar, pseudoscalar, vector, and pseudovector meson theory. These calculations are carried out with the Feynman-Dyson methods to give covariant expressions for the cross sections. The corrections of next higher order in the meson-nucleon coupling are calculated for pseudoscalar mesons and are shown to give large contributions. Comparison of the conclusions of the theory with the experimental results seems to indicate that the meson is of spin zero and is closely bound to the nucleons as is characteristic of pseudoscalar theory.

I. INTRODUCTION

THE production of mesons by photons has recently been accomplished experimentally,[1] and extensive experimental work is now being done on the problem.[2] It has been found that the present experimental techniques permit the study of the production of mesons by photons with greater accuracy and detail than is possible for production by nucleon-nucleon collision. The theoretical analysis of this information is also considerably simplified by the nature of the interaction. It will be shown that the results depend very markedly on the nature of the coupling of the mesons to the electromagnetic field. Since processes involving this coupling can be handled readily for non-relativistic energies by treating the interactions as weak, the usual methods of perturbation theory can be applied with some confidence to this aspect of the production process. The uncertainties concerning the nature of the coupling of mesons to nucleons, which, as is well known, lead to incorrect predictions of scattering phenomena, do not strongly affect this process. In fact, the characteristic differences between the behavior of the photon-ejected spin zero and spin one mesons will be shown to be due almost entirely to the nature of the meson coupling to the electromagnetic field.

The theory of the production of photo-mesons has been studied by a number of people,[3] the most complete work being the recent contribution of Feshbach and Lax. The methods used have been those of perturbation theory in the weak coupling approximation. Effects of the recoil of the nucleons have been neglected. The results obtained differ markedly for the various theories. It is not immediately apparent how the characteristic differences are related to the detailed features of the theories. It is therefore of interest to attempt to understand by classical arguments, without reference to perturbation theory, some details of the process. It has further been thought worth while to carry out the calculations using the new covariant formalism, eliminating unnecessary approximations and exhibiting the simplicity of the methods. Finally, the higher order corrections in the meson-nucleon interaction have been calculated for the pseudoscalar theory, using the new subtraction techniques, and found to give finite and unambiguous results.

II. GENERAL DESCRIPTION

A. Ratio of Cross Section for Production of Negative and Positive Mesons

One of the most striking of the early experimental observations on the production of mesons by photons was the excess of negative over positive mesons.[1] At present this is still the best established experimental fact. It was pointed out by Brueckner and Goldberger[4] that very simple classical arguments could give an explanation of this result. These arguments simply pointed out that there is an essential asymmetry in the production of negative and positive mesons. When the former are produced from neutrons, the charge-carrying nucleon is the final proton with large recoil velocity. When positive mesons are produced, the proton is initially at rest and does not interact through its charge. Using the interaction

$$e\mathbf{v}\cdot\mathbf{A}/(1-v/c\cos\theta) \tag{1}$$

which differs from the non-relativistic expression $e\mathbf{v}\cdot\mathbf{A}$ because of retardation effects in the interaction of the charge with the electromagnetic field, one obtains for the ratio of the interactions leading to the production of positive and negative mesons

$$\frac{I(\text{positives})}{I(\text{negatives})}=\frac{e\mathbf{v}\cdot\mathbf{A}/(1-v/c\cos\theta)(\text{meson})}{-e\dfrac{\mathbf{v}\cdot\mathbf{A}}{1-v/c\cos\theta}(\text{meson})+e\dfrac{\mathbf{v}\cdot\mathbf{A}}{1-v/c\cos\theta}(\text{recoil proton})}. \tag{2}$$

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[1] McMillan, Peterson, and White, Science **110**, 579 (1949).

[2] Cook, Steinberger, McMillan, Peterson, and White (private communications).

[3] W. Heitler, Proc. Roy. Soc. **166**, 529 (1938); M. Kobayashi and T. Okayama, Proc. Phys. Math. Soc. Japan **21**, 1 (1939); H. S. W. Massey and H. C. Corben, Proc. Camb. Soc. **35**, 84 (1939) and **35**, 463 (1939); L. Nordheim and G. Nordheim, Phys. Rev. **54**, 254 (1938); H. Feshbach and M. Lax, Phys. Rev. **76**, 134 (1949); L. Foldy, Phys. Rev. **76**, 372 (1949).

[4] K. Brueckner and M. Goldberger, Phys. Rev. **76**, 1725 (1949).

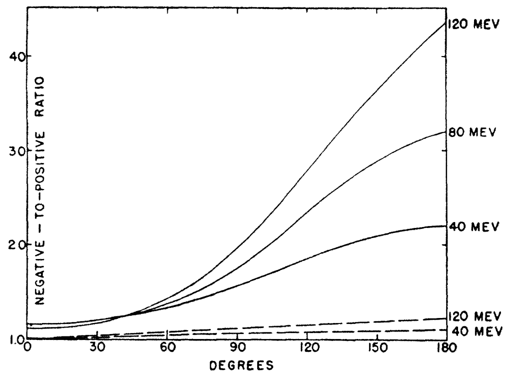

FIG. 1. The ratio of the cross sections for the production of negative and positive mesons. The solid curves are calculated with the assumption that the neutron does not interact with the electromagnetic field (Eq. (3)), the dotted curves with the assumption that the neutron interacts with the anomalous moment observed in a static field (Eq. (5)). The energies indicated in this and the following figures are the meson energies, which are related to the photon energy as shown in Fig. 6.

The ratio of the cross sections then is the square of the ratio of the interactions. Using over-all energy and momentum conservation, this ratio can be written

$$\frac{\sigma(\text{positives})}{\sigma(\text{negatives})} = [1-(q_0/Mc^2)(1-v/c\cos\theta)]^2, \quad (3)$$

where q_0 = meson energy including rest energy, M = nucleon mass, v = meson velocity, θ = angle between meson and photon velocity vectors. The dependence on meson energy and angle of this function is given explicitly in Fig. 1 (solid curves).

In this very simple argument the effects of the magnetic moments of the particles have been ignored. However, in Section V a generalization of the argument given here shows, for theories in which the electric dipole interactions are predominant, that the negative-to-positive ratio is changed by only a few percent. If, as can happen in some theories, the electric dipole moment is suppressed, and the magnetic moment interactions predominate, the ratio is given by

$$\sigma(+)/\sigma(-) = [1-(\gamma_p-\gamma_n/\gamma_p+\gamma_n)q_0/Mc^2(1-v/c\cos\theta)]^2, \quad (4)$$

where γ_n and γ_p are the magnitudes of the magnetic moments of the neutron and proton. It is interesting to observe that, if the magnetic moment of the nucleon is ignored, the ratio would be the same as that given by Eq. (3). If the numerical values of the static magnetic moments are substituted in this expression, we obtain

$$\sigma(+)/\sigma(-) = [1-0.20q_0/Mc^2(1-v/c\cos\theta)]^2. \quad (5)$$

This function is given graphically in Fig. 1 (dashed curves). It is apparent that, if nucleon moment interactions are important, the introduction of a magnetic moment for the neutron removes much of the asymmetry in the process leading to the production of positive and negative mesons and leads to a plus-to-minus ratio which is nearly unity.

We remark here that we have assumed that, with photons at energies of about the meson rest energy, the nucleons interact with the anomalous moments observed in a static field. This will be true only if the circulating currents which give rise to the anomalous moments are confined to a region small compared with the wavelength of the radiation. If this is not true, the anomalous moments will show energy dependence and the values of the static moments cannot be used.

We have ignored the interaction of the meson magnetic moments. These could contribute only if the meson were a vector particle, i.e., with spin one. It is apparent that such interactions are symmetrical for the production of either negative or positive mesons. Therefore strong meson-moment interactions would give a negative-to-positive ratio close to one.

One can conclude that a verification of the results of Eq. (3) would indicate that the meson does not interact strongly through a magnetic moment and that the neutron anomalous magnetic moment does not play an important part in the process. We have seen that these conclusions do not depend on the nature of the coupling of the mesons to the nucleons.

B. Angular Distribution

If the photon is absorbed by the ejected meson at photon energies for example of 200 to 300 Mev, $\beta = v/c$

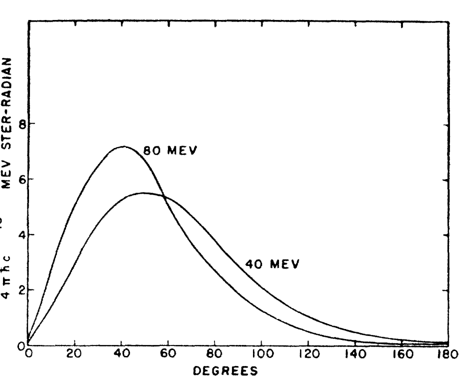

FIG. 2. Angular distribution of positive scalar mesons from dK/K photon spectrum.

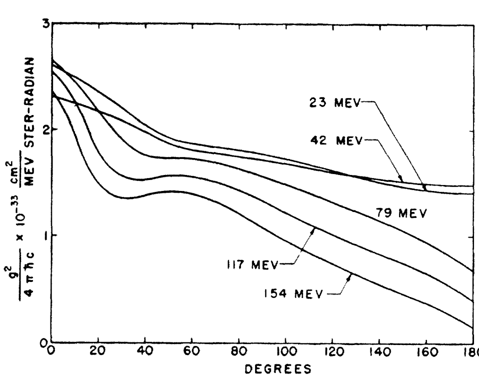

FIG. 3. Angular distribution of positive pseudoscalar mesons from dK/K photon spectrum.

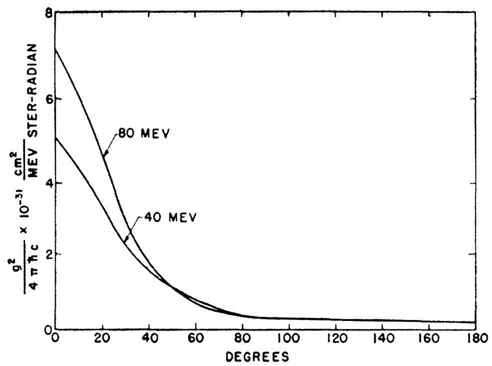

FIG. 4. Angular distribution of positive vector mesons from dK/K photon spectrum.

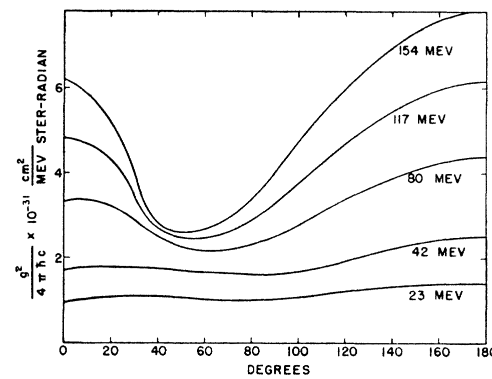

FIG. 5. Angular distribution of positive pseudovector meson from dK/K photon spectrum.

for the meson will not be small compared with unity. The angular distribution will show a large asymmetry about 90 degrees due to the presence in the differential cross section of the denominator

$$(1-\beta\cos\theta)^{-2} \tag{6}$$

which appears because of the retardation effects in the interaction of charge with the electromagnetic field. However, if the absorption of the photon is through its coupling with the nucleon, $v/c \ll 1$ for the nucleon and the angular distribution will be nearly symmetric about 90 degrees. One would therefore expect that the degree of symmetry would indicate which particle interacts most strongly with the photon.

If the interactions are primarily of the form $e\mathbf{v}\cdot\mathbf{A}$ then, since the photon field is transverse, the angular spectra must fall off to zero at 0 or 180 degrees. Spectra which do not exhibit this behavior must be due to magnetic moment-like interactions.

III. THE PRINCIPAL FEATURES OF THE VARIOUS THEORETICAL RESULTS

The calculations, unless otherwise specified, are for the first non-vanishing order in which the process can take place. The nucleons are treated as Dirac particles and the effects of their recoil are fully taken into account. The scalar, pseudoscalar, vector, and pseudovector theories are considered, using the couplings with the nucleon field which do not involve derivatives of the meson field.

The spectra shown in Figs. 2 to 5 are for positive mesons produced by a dK/K photon spectrum. This approximates the bremsstrahlung energy distribution at high energies which is used in the laboratory to produce mesons. The relation between the meson and the photon energy, as a function of angle, is given in Fig. 6.

A. Scalar Meson

The scalar meson characteristically shows a dipole angular distribution (Fig. 2) at low energies which is strongly distorted in the forward direction at energies above a few Mev by the factor of Eq. (6). This is due to the predominance of the photon-meson interaction and the absence of a magnetic moment for the meson. This simple result is quite analogous to the low energy photoelectric effect in an atom, since the mesons are spherically symmetric in distribution about the nucleon, and the photon interacts relatively weakly with the nucleon. This is in agreement with the fact that the meson cloud about the nucleon extends to distances of the order $\hbar/\mu c$, while the circulating currents associated with the magnetic moment of the nucleon are distributed over a region of order $\hbar/Mc$ and therefore give contributions which are smaller in the ratio $(\mu/M)^2$. The plus-to-minus ratio from the lowest order calculations is the same as that obtained by the classical argument, as would be expected since the nucleons are treated as Dirac particles and the meson has no magnetic moment interaction.

B. Pseudoscalar Meson

The pseudoscalar theory shows a roughly isotropic angular distribution (Fig. 3) indicating the predominance of the coupling of the photon to the magnetic moment of the nucleon. The unimportance of the electric-dipole terms in the interaction, i.e., the coupling to the linear motion of charge, is due to the close binding of the meson cloud to the nucleon. The probability of

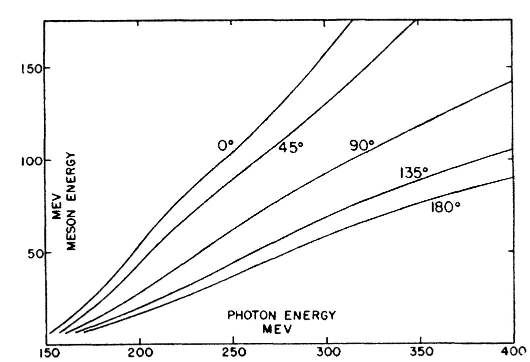

FIG. 6. Relation between meson and photon energy as a function of the angle of the meson with the photon beam direction.

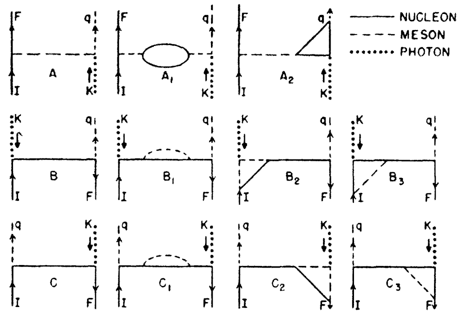

FIG. 7. Diagrams for photo-meson production. Diagrams A, B, C are for production in lowest order ge; diagrams with subscripts are for production in order g^3e.

finding a meson at a distance $\hbar/\mu c$, which is the wavelength of the photon, is relatively small. However, the direct coupling of the photon to the magnetic moment of the nucleon, preceded or followed by the emission of the final meson, is relatively probable. Examination of the matrix elements involved in these transitions indicates that this behavior is due to the existence of intermediate states in which the nucleon undergoes transitions to negative energy states. For such processes the matrix elements of the couplings to the electromagnetic and meson fields are of the order unity instead of v/c for the nucleons. The ratio of negative to positive mesons given by the lowest order calculation is the same as that for the scalar theory.

Preliminary experimental results indicate that the spectrum of mesons observed is roughly isotropic in angular distribution as predicted by the pseudoscalar theory. Therefore this theory was selected for examination of the corrections of higher order in the nucleon-meson coupling constant g. This involves another emission and reabsorption of a virtual meson, as indicated in Fig. 7. It was hoped that the investigation of the corrections would answer questions concerning the contribution of the anomalous magnetic moments of the nucleons and the possible large effects of corrections to processes in which the photon is coupled directly to the meson or its associated Dirac vacuum field. Examination of the effects of higher order processes would also indicate the probability that the expansion in powers of $g^2/4\pi$ would actually lead to a convergent series.

The calculation of the next higher order terms shows that they give contributions about as large as the first-order terms, therefore casting grave doubts on the convergence of the expansion and the validity of the application of perturbation methods of this kind.

The largest contribution to the corrections comes from the anomalous magnetic moments of the nucleons, Figs. 7-B_1, 7-B_2, and 7-B_3. The anomalous moments differ considerably from those which the nucleons exhibit in a static field, as calculated by Case.[5] Non-static effects occur in the interaction which decrease the magnitudes of the moments and also change the sign of the proton moment. This result indicates that it may not be correct to assume that the nucleons interact, with their static moments, in high energy processes. The actual magnitude of these corrections, however, probably can be believed only in a very qualitative way.

Corrections in which the meson creates virtual nucleon pairs (Fig. 7-A_1), one of which may interact directly with the photon (Fig. 7-A_2), give negligible contributions and so do not affect the predominance of the coupling of the photon to the nucleon through the Dirac and anomalous moments.

C. Vector and Pseudovector

The vector theory shows a strongly asymmetric angular distribution (Fig. 4), indicating the predominance of the photon-meson interaction. This distribution is also peaked markedly forward showing that the interaction is due at least in part to the magnetic moment of the meson. The pseudovector theory shows a nearly isotropic angular spectrum (Fig. 5). This, however, is not due to the largeness of the nucleon photon coupling but to the large magnetic moment interaction of the meson. The negative to positive ratio (Fig. 8) also reflects the large effects of the magnetic moment of the spin-one mesons, differing considerably from the result obtained by ignoring the meson moment. Also very characteristic of vector and pseudovector meson is the rapid increase of the cross section with energy (Figs. 4 and 5). This is due both to the strong energy dependence of the electromagnetic field coupling with longitudinally polarized mesons and to the magnetic moment interactions.

Higher order corrections probably would not particularly affect these results, since processes involving production of virtual nucleon pairs by the mesons seem to give negligible contributions. Higher order processes involving the coupling of the photon to the nucleon and its associated meson field would give corrections to terms which are already relatively unimportant in the process.

IV. CONCLUSIONS

We have seen that certain features of the process of production of mesons by photons are nearly independent of the nature of the coupling of mesons to nucleons. In particular, one can expect through the measurement of the distribution in energy and angle of photo-mesons at energies above a few Mev: (1) to determine with some confidence the spin of the meson, through the characteristic behavior of particles with a magnetic moment in the electromagnetic field. The strong energy dependence of the coupling of spin-one mesons to photons has been used previously by Christy and Kusaka[6] to demonstrate that cosmic-ray mesons cannot be of this

[5] K. M. Case, Phys. Rev. 76, 1 (1949).

[6] R. F. Christy and S. Kusaka, Phys. Rev. 59, 414 (1941).

type; (2) to examine the distribution of mesons about the nucleon, since if the distribution extends to distances of the order $h/\mu c$, one would expect that the electric-dipole terms would be predominant in the coupling to the electromagnetic field. Only if the meson distribution is singular, i.e., closely bound to the nucleons, can one expect the interaction with the circulating currents of the magnetic moments to become relatively important; (3) through a measurement of the plus-to-minus ratio to determine the effects of a meson magnetic moment and the nature of the anomalous moments for the nucleons. The anomalous moments will be nearly independent of the frequency of the electromagnetic field only if they are confined to regions small compared with the wave-length of the radiation.

The further details of the calculation made with perturbation theory can probably not be accepted quantiatively since higher order corrections are not negligible. For example, explicit calculation of the higher effects for the pseudoscalar theory predicts large anomalous magnetic moment interactions for the nucleons including corrections, as large as the lowest order terms, corresponding to polarization of the vacuum by the nucleons. The anomalous moments are considerably smaller than those which the nucleons exhibit in a static field and different in sign for the proton. The importance of the corrections for the pseudoscalar theory appears to be the result of the close binding of the mesons to the nucleons, which in turn leads to the characteristic predominance of the nucleon magnetic moment interactions in the production process. Higher order effects, however, do not change the general features of the lowest order results. There, therefore, seems to be ground for hope that the lowest order calculations for all of the theories may give qualitatively correct results.

The experimental results obtained by the Berkeley workers,[2] although still preliminary, indicate that mesons of 40 to 100 Mev produced by photons on hydrogen are nearly isotropic in angular distribution from 45 to 135 degrees in the laboratory system. The cross section for mesons produced in this energy and angular range from hydrogen appears to decrease slowly with increasing meson energy. The magnitude of the total cross section, about 10^{-28} cm^2, can be fitted for the theories giving roughly isotropic angular spectra at low energies (Figs. 3 and 5) with a value of the coupling constant $g^2/4\pi hc$ of about 40 for pseudoscalar mesons and 0.4 for pseudovector mesons. The mesons produced from carbon show an excess of negative over positive mesons in the ratio of 1.7 ± 0.2 in the energy range of 30 to 100 Mev observed at 90 degrees to the photon beam direction. This is in agreement with the ratio given in Fig. 1, with the assumption that the neutron does not interact with the electromagnetic field. However, it is not clear that the complications of the binding of the nucleons in the carbon nucleus are unimportant. A more detailed study of the dependence on energy and angle of the negative-to-positive ratio is being carried on at present.

These results all seem to indicate that the general features of the pseudoscalar meson theory are correct, i.e., the zero spin of the meson and the close binding of the meson cloud to the nucleons. It is hoped that the degree of validity of these conclusions will be indicated as the experimental work is completed.

V. EFFECTS OF MAGNETIC MOMENT INTERACTIONS ON THE NEGATIVE TO POSITIVE RATIO

The elementary argument given in Section II must be generalized considerably before it can be applied to magnetic moment interactions. One can formulate the argument in the following manner: The interaction with the electromagnetic field leading to the ejection of mesons is of the form

$$I = A_\mu \int j_\mu(\mathbf{r}', t) \exp[i(\mathbf{K}\cdot\mathbf{r}' - K_0 t)] dr' dt, \qquad (7)$$

where j_μ is the total current carried by the interacting particles. The current can be separated into curl-free and divergence-free parts; the former corresponds to a linear motion of the charge and the latter to circulating currents. Thus

$$j_\mu = q v_\mu + \partial M_{\mu\nu}/\partial x_\nu, \qquad (8)$$

where v_μ is the relativistic velocity with spacial components $\mathbf{v}/(1-\beta^2)^{\frac{1}{2}}$, q the charge, and $M_{\mu\nu}$ is an anti-symmetric tensor. The interaction then is

$$I = A_\mu \int (q v_\mu + \partial M_{\mu\nu}/\partial x_\nu) \exp[i(\mathbf{K}\cdot\mathbf{r}' - K_0 t)] dr' dt$$

$$= A_\mu \int (q v_\mu - i M_{\mu\nu} K_\nu) \exp[i(\mathbf{K}\cdot\mathbf{r}' - K_0 t)] dr' dt. \qquad (9)$$

We can easily evaluate this integral, considering each interacting particle separately. If the wave-length of the radiation is large compared with the region over which the charges and currents are distributed, or if one assumes a delta-function of position for the spacial

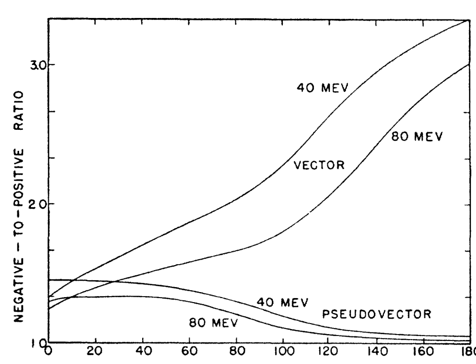

FIG. 8. Ratio of cross sections for production of positive and negative mesons, vector, and pseudovector theory.

distribution, the dependence on r' can be given by

$$\begin{aligned} v_\mu(\mathbf{r}',t)&=\mathbf{v}(t)\delta[\mathbf{r}'-\mathbf{r}(t)]\\ &=v_\mu(t)\delta[\mathbf{r}'-\mathbf{r}(t)](1-\beta^2)^{\frac{1}{2}},\\ M_{\mu\nu}(\mathbf{r}',t)&=M_{\mu\nu}(t)\delta[\mathbf{r}'-\mathbf{r}(t)](1-\beta^2)^{\frac{1}{2}}. \end{aligned} \tag{10}$$

The integral for each particle then is

$$I=A_\mu\int[qv_\mu(t)-iM_{\mu\nu}(t)K_\nu](1-\beta^2)^{\frac{1}{2}}\times\exp i[\mathbf{K}\cdot\mathbf{r}(t)-K_0t]dt. \tag{11}$$

A change of variable and a partial integration gives

$$I=A_\mu\int\frac{d}{ds}\left[\frac{v_\mu(s)-iM_{\mu\nu}(s)K_\nu}{K_0(1-\beta\cos\theta)}\right]e^{is}(1-\beta^2)^{\frac{1}{2}}. \tag{12}$$

If the interaction takes place over a time much shorter than the period of the radiation, the variation of the exponential term can be ignored. This gives

$$I=-\Delta\left[\frac{A_\mu P_\mu+\frac{1}{2}mM_{\mu\nu}F_{\mu\nu}}{K\cdot P}\right], \tag{13}$$

where the four-vector product is

$$K\cdot P=-K_0P_0+\mathbf{K}\cdot\mathbf{P},$$

and $\Delta[\ \ ]$ denotes the change during the interaction.

In this expression for the interaction of the electromagnetic field with the charges and currents of the meson-nucleon system, we have not included the dependence of the meson emission on such factors as the strength of the coupling of the mesons to the nucleons and the spins of the nucleons. We shall assume that a simple multiplicative factor, U, gives the spin dependence of the forces and the strength of the coupling. When we consider the application of field theory to this process, we shall see that this actually is the case for spin-zero mesons. The quantum nature of the process also has not been considered. We actually need the matrix element between the initial and final nucleon states of the operator which we have derived. The interaction therefore can be written for each particle as

$$\begin{aligned} I&=\psi_F^+\left\{U\Delta\left[\frac{q\mathbf{A}\cdot\mathbf{P}+\frac{1}{2}mF_{\mu\nu}M_{\mu\nu}}{K\cdot P}\right]\right\}\psi_I\\ &\equiv\langle U\rangle\Delta\left(q\frac{\mathbf{A}\cdot\mathbf{P}}{K\cdot P}\right)+\tfrac{1}{2}mF_{\mu\nu}\Delta(\langle UM_{\mu\nu}\rangle). \end{aligned} \tag{14}$$

We can use this expression to evaluate the ratio of the cross sections for negative and positive mesons. For simplicity we shall take the mesons to have spin zero (i.e., no magnetic moment), and shall first assume that the nucleons interact only in the proton state. When a positive meson is produced the proton is the initial nucleon at rest, and so interacts only through its magnetic moment with the transverse photon.

$$I(+)=-e\frac{(\mathbf{A}\cdot\mathbf{q})\langle U\rangle}{K\cdot q}+\tfrac{1}{2}m\frac{\langle UM_{\mu\nu}\rangle}{K\cdot I}F_{\mu\nu}, \tag{15}$$

where q is the meson 4-momentum, and I is the initial nucleon 4-momentum. When a negative meson is produced, the proton is the final recoil nucleon carrying current $e\mathbf{F}/m$

$$I(-)=e\left(\frac{\mathbf{A}\cdot\mathbf{q}}{K\cdot q}-\frac{\mathbf{A}\cdot\mathbf{F}}{K\cdot F}\right)\langle U\rangle-\tfrac{1}{2}m\frac{\langle UM_{\mu\nu}\rangle}{K\cdot F}F_{\mu\nu}, \tag{16}$$

where F is the final nucleon 4-momentum. If we now use over-all 4-momentum conservation, we find that

$$\mathbf{A}\cdot\left(\frac{\mathbf{q}}{K\cdot q}-\frac{\mathbf{F}}{K\cdot F}\right)=\frac{\mathbf{A}\cdot\mathbf{q}}{K\cdot q}\frac{K\cdot I}{K\cdot F}$$

and

$$\begin{aligned} I(-)&=-\frac{K\cdot I}{K\cdot F}\left[e\frac{\mathbf{A}\cdot\mathbf{q}}{K\cdot q}\langle U\rangle+\tfrac{1}{2}m\frac{\langle UM_{\mu\nu}\rangle}{K\cdot I}F_{\mu\nu}\right]\\ &=-[K\cdot I/K\cdot F]I(+). \end{aligned} \tag{17}$$

Therefore, under these assumptions, the plus-to-minus ratio would be

$$\begin{aligned} \sigma(+)/\sigma(-)&=(K\cdot F/K\cdot I)^2\\ &=[1-(q_0/Mc^2)(1-v/c\cos\theta)]^2. \end{aligned} \tag{18}$$

This is the same result as was obtained in Eq. (3) when magnetic moment terms were ignored. It is interesting to observe that the ratio of the linear current interactions is the same as the ratio of the magnetic moment interactions, so that the plus-to-minus ratio is independent of the values of

$$\langle U\rangle=\psi_F^+U\psi_I,\quad \langle UM_{\mu\nu}\rangle=\psi_F^+UM_{\mu\nu}\psi_I.$$

If the nucleons interact not only in the proton state, but also with anomalous moments, we can write

$$\begin{aligned} M_{\mu\nu}(\text{proton})&=\gamma_PM_{\mu\nu},\\ M_{\mu\nu}(\text{neutron})&=-\gamma_NM_{\mu\nu}, \end{aligned} \tag{19}$$

where γ is the magnitude of the magnetic moment of the nucleon in Bohr magnetons, with

$$\gamma_P=2.87,\quad \gamma_N=1.91. \tag{20}$$

This gives for the ratio of the interactions

$$\frac{I(+)}{I(-)}=\frac{-e\dfrac{\mathbf{A}\cdot\mathbf{q}}{K\cdot q}\langle U\rangle-\tfrac{1}{2}mF_{\mu\nu}\left(\dfrac{\gamma_P}{K\cdot I}+\dfrac{\gamma_N}{K\cdot F}\right)\langle M_{\mu\nu}U\rangle}{e\dfrac{\mathbf{A}\cdot\mathbf{q}}{K\cdot q}\dfrac{K\cdot I}{K\cdot F}\langle U\rangle+\tfrac{1}{2}mF_{\mu\nu}\left(\dfrac{\gamma_P}{K\cdot F}+\dfrac{\gamma_N}{K\cdot I}\right)\langle M_{\mu\nu}U\rangle}. \tag{21}$$

If $\langle M_{\mu\nu}U\rangle=\langle M_{\mu\nu}\rangle\langle U\rangle$ then the factor U cancels. The ratio of the cross section averaged over photon polarization and over nucleon-moment orientation gives

$$\sigma(+)/\sigma(-)=(\boldsymbol{K}\cdot\boldsymbol{F}/\boldsymbol{K}\cdot\boldsymbol{I})^2[1+0(\mu^2/M^2)].$$

Therefore, if the interaction is of this type, the ratio of the cross sections is nearly unchanged by the anomalous moments, since $\mu^2/M^2\approx 2$ percent. This is a result of the predominance of the coupling of the electromagnetic field to the linear motion of charge; i.e., to the electric dipole formed by the meson-nucleon charges.

If $\langle M_{\mu\nu}U\rangle\gg\langle M_{\mu\nu}\rangle\langle U\rangle$, or what is equivalent, if the magnetic moment terms are predominant in the interaction, then

$$\frac{\sigma(-)}{\sigma(+)}=\left[1-\frac{\gamma_P-\gamma_N}{\gamma_P+\gamma_N}(q_0/Mc^2)(1-v/c\cos\theta)\right]^{-2}. \quad (22)$$

These results are discussed in Section II.

VI. METHOD OF CALCULATION (LOWEST ORDER)

The calculation of matrix elements can be simplified greatly by use of the Feynman-Dyson methods.[7] The necessary operators can be derived by a technique due to Feynman, the correctness of which can be demonstrated by Dyson's methods. The calculations can also be carried out by the older methods of perturbation theory to give exactly the results derived here. The meson couplings to the nucleon field that are used are those which do not involve derivatives of the meson field, since these introduce non-renormalizable singularities in the higher order processes.

In the following, the notation used is such that $h=c=M=1$, where M is the nucleon mass. Therefore, all energies and momenta will be measured in units of the nucleon mass. All products of the form $\boldsymbol{A}\cdot\boldsymbol{B}$ will be understood to be 4-vector products, with $\boldsymbol{A}\cdot\boldsymbol{B}=-A_0B_0+\mathbf{A}\cdot\mathbf{B}$. We also use the notation $\boldsymbol{U}\equiv U_\mu\gamma_\mu$ with the Dirac matrixes $\gamma_i=ia_i\beta(i=1,2,3)$, $\gamma_4=\beta$. The adjoint operator ψ^+ is related to the complex conjugate by the condition $\psi^+=i\psi^*\gamma_4$.

The equations of motion for the fields are

(*A*) *Free particles*

$$(\boldsymbol{P}-i)\psi=0, \quad \text{(Dirac)} \quad (23)$$

$$(\Box-\kappa^2)\phi=0, \quad \text{(spin zero)} \quad (24)$$

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial\phi_\nu}{\partial x_\mu}-\frac{\partial\phi_\mu}{\partial x_\nu}\right)-\kappa^2\phi_\nu=0. \quad \text{(spin one)} \quad (25)$$

The last equation can also be written as

$$(\Box-\kappa^2)\phi_\nu=0, \quad \text{(spin one)}$$

with the divergence condition

$$\partial\phi_\mu/\partial x_\mu=0. \quad (26)$$

[7] R. P. Feynman, Phys. Rev. **76**, 769 (1949); F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

(*B*) *Interaction of mesons with Dirac field*

$$(\Box-\kappa^2)\phi=g\psi^+U\psi, \quad U=(-1)^{\frac{1}{2}} \quad \text{(scalar)} \quad (27)$$
$$=\gamma_5 \quad \text{(pseudoscalar)}$$

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial\phi_\nu}{\partial x_\mu}-\frac{\partial\phi_\mu}{\partial x_\nu}\right)-\kappa^2\phi=g\psi^+U_\nu\psi, \quad (28)$$

$$U_\nu=\gamma_\nu \quad \text{(vector)}$$
$$=\gamma_5\gamma_\nu. \quad \text{(pseudovector)}$$

The last equation can be written

$$(\Box-\kappa^2)\phi_\nu=\left(\delta_{\mu\nu}-\kappa^{-2}\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right)g\psi^+U_\mu\psi, \quad (28')$$

$$(\boldsymbol{P}-i)\psi=g\phi U\psi \quad \text{(spin zero)} \quad (29)$$
$$=g\phi_\nu U_\nu\psi. \quad \text{(spin one)}$$

(*C*) *Interaction with electromagnetic field*

$$(\boldsymbol{P}+e\boldsymbol{A}-i)\psi=0, \quad \text{(Dirac)} \quad (30)$$

$$\left[\left(\frac{\partial}{\partial x_\mu}+ieA_\mu\right)\left(\frac{\partial}{\partial x_\mu}+ieA_\mu\right)-\kappa^2\right]\phi=0, \quad \text{(spin zero)} \quad (31)$$

$$\left(\frac{\partial}{\partial x_\mu}+ieA_\mu\right)\left[\left(\frac{\partial}{\partial x_\mu}+ieA_\mu\right)-\left(\frac{\partial}{\partial x_\nu}+ieA_\nu\right)\phi_\mu\right]-\kappa^2\phi_\nu=0. \quad \text{(spin one)} \quad (32)$$

To order e, these can be written

$$(\boldsymbol{P}-i)=-e\boldsymbol{A}\psi, \quad \text{(Dirac)} \quad (30')$$

$$(\Box-\kappa^2)\phi=-2ceA_\mu\partial\phi/\partial x_\mu, \quad \text{(spin zero)} \quad (31')$$

$$(\Box-\kappa^2)\phi_\nu=-ie\left(2A_\lambda\frac{\partial}{\partial x_\lambda}\delta_{\mu\nu}-A_\nu\frac{\partial}{\partial x_\mu}-\frac{\partial A_\nu}{\partial x_\mu}-A_\mu\frac{\partial}{\partial x_\nu}\right)\phi_\mu. \quad \text{(spin one)} \quad (32')$$

If we consider the direct solution of these equations of motion, following Feynman's general arguments, we find the following expressions to be inserted into the Feynman-Dyson diagram:

(*B'*) *Emission of a meson of momentum* P_μ

gU (spin zero) (33)
$g(\delta_{\mu\nu}+P_\mu P_\nu/\kappa^2)U_\mu$. (spin one, polarization ν)

(*C'*) *Absorption of a photon*

$-e\boldsymbol{A}$ (Dirac)
$2e\boldsymbol{A}\cdot\boldsymbol{P}'$ (spin zero, momentum P_μ') (34)
$e[2\boldsymbol{A}\cdot\boldsymbol{P}'\delta_{\mu\nu}-A_\nu P_\mu'-K_\mu A_\nu-A_\mu q_\nu]$.

(spin one, momentum P_μ', initial polarization μ, final polarization ν).

(D) Propagation of an intermediate particle with momentum P_μ'.

$$1/(\boldsymbol{P}'-i) \qquad \text{(Dirac)} \quad (35)$$
$$-1/(\boldsymbol{P}'^2+\kappa^2) \qquad \text{(boson, mass } \kappa).$$

Exponential factors of the form $\exp[i(\boldsymbol{P}\cdot\boldsymbol{x})]$ have been omitted, since after spacial integrations have been carried out, they simply give 4-momentum conservation at each point of the diagram and for the over-all process.

Using these expressions we can now write down matrix elements directly. A virtual meson emitted by a nucleon can absorb a photon and go into a free meson. This is represented by diagram A of Fig. 7, and gives the matrix elements

$$ge\psi_F{}^+\left[2\boldsymbol{A}\cdot\boldsymbol{q}\frac{-1}{(\boldsymbol{q}-\boldsymbol{K})^2+\kappa^2}U\right]\psi_I\phi, \qquad \text{(spin zero)} \quad (36)$$

$$ge\psi_F{}^+(\delta_{\mu\nu}+\kappa^{-2}q_\mu'q_\nu')U_\mu \times\left(\frac{-2\boldsymbol{A}\cdot\boldsymbol{q}\delta_{\lambda\nu}+A_\lambda q_\nu'+K_\nu A_\lambda+A_\nu q_\lambda'}{(\boldsymbol{q}-\boldsymbol{K})^2+\kappa^2}\right)\psi_I\phi_\lambda. \qquad \text{(spin one)} \quad (37)$$

The nucleon can emit a real meson, going into a virtual intermediate state, and then absorb the photon. The photon absorption can also come first, followed by the meson emission. These processes are represented by diagrams B and C of Fig. 7, giving the matrix elements

$$-ge\psi_F{}^+\left(A\frac{1}{\boldsymbol{F}-\boldsymbol{K}-i}U\right)\psi_I\phi, \qquad \text{(spin zero)} \quad (38)$$

$$-ge\psi_F{}^+\left[A\frac{1}{\boldsymbol{F}-\boldsymbol{K}-i}(\delta_{\mu\nu}+\kappa^{-2}/q_\mu q_\nu)U_\mu\right]\psi_I\phi_\nu \qquad \text{(spin one)}. \quad (39)$$

If the nucleons are treated as Dirac particles, the photon can be absorbed only by a proton. Therefore, this diagram represents the production of a negative meson. For production of a positive meson, simply replace I by F and invert the order of the operators.

Combining these two contributions from diagram A and B, we obtain for the lowest order matrix element for the transition

$$M_2=ge\psi_F{}^+U\left(\frac{\boldsymbol{A}\cdot\boldsymbol{q}}{\boldsymbol{K}\cdot\boldsymbol{q}}-\frac{\boldsymbol{A}\cdot\boldsymbol{I}}{\boldsymbol{K}\cdot\boldsymbol{I}}-\frac{\boldsymbol{K}\boldsymbol{A}}{2\boldsymbol{I}\cdot\boldsymbol{K}}\right)\psi_I\phi. \qquad \text{(spin zero)} \quad (40)$$

For negative mesons replace I by F in the bracket. This is of the form (Eq. (11)) derived above, with the moment tensor for the Dirac field

$$M_u=-\tfrac{1}{4}ie(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)=\tfrac{1}{2}e\sigma_{\mu\nu},$$

$$M_2=ge\psi_F U_\nu\left[\frac{2\boldsymbol{A}\cdot\boldsymbol{q}\phi_\nu+A_\nu\boldsymbol{\phi}\cdot\boldsymbol{K}-K_\nu\boldsymbol{\phi}\cdot\boldsymbol{A}}{2\boldsymbol{q}\cdot\boldsymbol{K}}-\phi_\nu\frac{\boldsymbol{A}\cdot\boldsymbol{I}}{\boldsymbol{K}\cdot\boldsymbol{I}} -\phi_\nu\frac{\boldsymbol{K}\boldsymbol{A}}{2\boldsymbol{I}\cdot\boldsymbol{K}}-\frac{F_\nu-I_\nu}{\kappa^2}\left(\frac{\boldsymbol{q}\cdot\boldsymbol{K}\,\boldsymbol{\phi}\cdot\boldsymbol{A}-\boldsymbol{A}\cdot\boldsymbol{q}\,\boldsymbol{\phi}\cdot\boldsymbol{K}}{2\boldsymbol{q}\cdot\boldsymbol{K}}\right)\right]\psi_I. \qquad \text{(spin one)} \quad (41)$$

For negative mesons replace $\boldsymbol{I}$ by $\boldsymbol{F}$ in the bracket and

$$U_\nu\boldsymbol{K}\boldsymbol{A} \text{ by } -\boldsymbol{A}\boldsymbol{K}U_\nu. \quad (42)$$

In these expressions we can demonstrate gauge-invariance by substituting

$$A_\mu=A_\mu'+\partial\Lambda/\partial x_\mu=A_\mu'+iK_\mu\Lambda \quad (43)$$

which should leave the matrix element for the transition M_2 unchanged. This is equivalent to showing that replacing A_μ by K_μ reduces M_2 to zero. That this is so can be seen by inspection, using the condition

$$\boldsymbol{K}\boldsymbol{K}=K_\mu K_\mu=0.$$

If we now specialize to the transverse vector potential, $A\cdot I$ is zero since the momentum I_μ is along the direction of the photon momentum. We then have for the spin-zero mesons

$$M_2(\text{positive})=ge\psi_F{}^+\phi U\frac{\mathbf{A}\cdot\mathbf{q}}{\boldsymbol{K}\cdot\boldsymbol{q}}-\frac{\boldsymbol{K}\boldsymbol{A}}{2\boldsymbol{I}\cdot\boldsymbol{K}}\psi_I, \quad (44)$$

$$M_2(\text{negative})=-ge\psi_F{}^+\phi U\left[\mathbf{A}\cdot\left(\frac{\mathbf{q}}{\boldsymbol{K}\cdot\boldsymbol{q}}-\frac{\mathbf{F}}{\boldsymbol{F}\cdot\boldsymbol{K}}\right)-\frac{\boldsymbol{K}\boldsymbol{A}}{2\boldsymbol{F}\cdot\boldsymbol{K}}\right]\psi_I =\frac{\boldsymbol{K}\cdot\boldsymbol{I}}{\boldsymbol{K}\cdot\boldsymbol{F}}M_2(\text{positives}).$$

This is the same result as that obtained above (Eq. (18)).

The differential cross section in the laboratory system then can be obtained from the formula

$$d\sigma=2\pi|M_2|^2\rho_F, \quad (45)$$

where M_2 is to be summed over meson and final nucleon spins and averaged over initial nucleon spin and photon polarization. The sum over the meson spin can be performed easily. If ϵ_μ is the 4-vector representing the direction of polarization of the meson which satisfies

$$\epsilon_\mu q_\mu=0, \quad \text{(divergence condition)} \quad (46)$$

$$\epsilon_i^2+\epsilon_4^2=1/2q_0 \quad \text{(normalization condition)} \quad (47)$$

then

$$\sum_{\text{spin}}\boldsymbol{\varepsilon}\cdot\boldsymbol{A}\,\boldsymbol{\varepsilon}\cdot\boldsymbol{B}=(1/2q_0)\left[\boldsymbol{A}\cdot\boldsymbol{B}-\frac{\boldsymbol{A}\cdot\boldsymbol{q}\,\boldsymbol{B}\cdot\boldsymbol{q}}{\boldsymbol{q}\cdot\boldsymbol{q}}\right]. \quad (48)$$

Carrying out the indicated sums and averages gives, after simplification of the resulting expressions in the laboratory system, for positive mesons

$$\left[\frac{g^2}{4\pi}\frac{e^2}{4\pi}\frac{\pi}{2}\frac{1}{K_0^2}\right]^{-1}\frac{d\sigma}{dq_0}$$

$$=\frac{q^2\sin^2\theta(2-\frac{1}{2}\kappa^2)}{(\boldsymbol{q}\cdot\boldsymbol{K})^2}+1-\frac{\boldsymbol{q}\cdot\boldsymbol{K}}{\boldsymbol{I}\cdot\boldsymbol{K}} \quad \text{(scalar)} \quad (49)$$

$$=-\tfrac{1}{2}\kappa^2\frac{q^2\sin^2\theta}{(\boldsymbol{q}\cdot\boldsymbol{K})^2}+1-\frac{\boldsymbol{q}\cdot\boldsymbol{K}}{\boldsymbol{I}\cdot\boldsymbol{K}} \quad \text{(pseudoscalar)} \quad (50)$$

$$=\frac{1}{(\boldsymbol{q}\cdot\boldsymbol{K})^2}\Bigg[(\boldsymbol{I}\cdot\boldsymbol{K})^2-3\boldsymbol{I}\cdot\boldsymbol{K}\,\boldsymbol{q}\cdot\boldsymbol{K}+15/4(\boldsymbol{q}\cdot\boldsymbol{K})^2+1/2(\boldsymbol{q}\cdot\boldsymbol{K})^3\left(\frac{-4}{\boldsymbol{K}\cdot\boldsymbol{I}}\frac{-1}{\kappa^2}\right)+\frac{q^2\sin^2\theta}{2\kappa^2}[(\boldsymbol{I}\cdot\boldsymbol{K})^2-2\kappa^4-4\kappa^2]\Bigg]. \quad \text{(vector)} \quad (51)$$

The cross section for pseudovector mesons can be obtained from the result for vector mesons by the addition of the expression

$$\frac{1}{(\boldsymbol{q}\cdot\boldsymbol{K})^2}\left[-\frac{2(\boldsymbol{q}\cdot\boldsymbol{K})^2}{\kappa^2}-\frac{2(\boldsymbol{q}\cdot\boldsymbol{K})^3}{\kappa^4}+6q^2\sin^2\theta\right]. \quad (52)$$

Finally, if we wish to apply these expressions to the calculation of the spectrum of mesons produced by a photon beam which is the result of the bremsstrahlung of high energy electrons, we can represent the distribution of energies in the photon beam by

$$\phi(K)dK/K, \quad (53)$$

where $\phi(K)$ is a factor, which is nearly unity over the part of the spectrum of interest, and which indicates the degree of departure from the simple dK/K distribution.[8] At a given meson energy, the distribution in photon energies leads to a distribution in meson angles, with the energies and angles related by the conservation laws of energy and momentum

$$K=(q_0-\mu^2/2)/(1-q_0+q\cos\theta). \quad (54)$$

If

$$d\sigma/dq_0=f(\theta,q_0) \quad (55)$$

then

$$\frac{d\sigma}{dq_0d\Omega}=f(\theta,q_0)\frac{1}{K}\frac{dK}{d\Omega}\phi(K)=\frac{Kq}{q_0-\kappa^2/2}\frac{\phi(K)}{2\pi}f(\theta,q_0).$$

This spectrum is that shown in Figs. 2–5, with $\phi(K)$ set equal to unity.

[8] W. Heitler, *The Quantum Theory of Radiation*, (Oxford University Press, London, 1944), p. 170.

VII. HIGHER ORDER CORRECTIONS

The perturbation calculations carried out in Section II are for the lowest non-vanishing order in the coupling constants g and e. The corrections of order e^3 and higher are considered negligible, which is probably justified for non-relativistic energies because of the smallness of the expansion parameter $e^2/4\pi=1/137$. Therefore, only terms of order eg^3 are calculated. The pseudoscalar theory is considered because it gives prediction in qualitative agreement with the experimental results of McMillan *et al.*,[1] and because of the simplicity of the theory. Pseudoscalar coupling is used, which is equivalent to pseudovector coupling in lowest order but not in the higher order processes. The calculations made are for energies near threshold where the momenta of the particles can be ignored relative to the rest energies.

The possible diagrams for the corrections to the lowest order result for the production of positive mesons are given in Fig. 7. The diagrams B_3, C, C_1 are forbidden since the nucleons indicated as interacting with the photon are in the neutron state. Diagrams involving neutral mesons are also omitted. The evaluation of these corrections can be carried out using the straightforward methods outlined by Dyson.[7] We simply quote the results given by these calculations. We find for the fourth-order matrix element for photon energies near threshold

$$M_4(A_1)=g^2/16\pi^2(2/15)\kappa^2M_2(A),$$
$$M_4(A_2)=g^2/16\pi^2(4/3)\kappa^4M_2(A),$$
$$M_4(B_1)=-1.26g^2/16\pi^2M_2(B),$$
$$M_4(B_2)=0.88g^2/16\pi^2M_2(B),$$
$$M_4(C_2)=-1.47g^2/16\pi^2M_2(C),$$
$$M_4(C_3)=0.32g^2/16\pi^2M_2(C),$$

where $M_2(A)$, $M_2(B)$, and $M_2(C)$ are the second-order matrix elements corresponding to diagrams A, B, and C of Fig. 8.

If we examine these matrix elements, it is apparent that the corrections from diagrams B_1, B_2, C_2 and C_3 of Fig. 7 are equivalent to an interaction by the nucleons with an anomalous moment in Bohr magnetons

proton $g^2/16\pi^2(0.88-1.26)=-0.38g^2/16\pi^2$,
neutron $-g^2/16\pi^2(1.47-0.32)=-1.15g^2/16\pi^2$.

These results are to be contrasted with those of Case,[5] who also calculated the anomalous moments for a static field, using pseudoscalar meson theory. He found that for charge symmetric theory, the anomalous moments were

proton $0.65g^2/16\pi^2$
neutron $-1.61g^2/16\pi^2$,

where the notation has been adjusted to agree with that used here.

We find therefore that the correction of order eg^3 is the sum of the contributions from the graphs A_1, A_2, B_1, B_2, C_2, C_3. The first two give corrections which are less than 1 percent and can be neglected relative to the others. The remaining correction is

$$M_4 = g^2/16\pi^2[0.88-1.26]M_2(B) + g^2/16\pi^2[-1.47+0.32]M_2(C) = 0.62g^2/16\pi^2 M_2(B).$$

Since $g^2/4\pi$ is about 6, we see that the contribution from the fourth-order terms is about two-thirds as large as that fromthe lowest order. The smallness of the total effect, however, is due to near cancellation of the large contributions from the individual effects. This cancellation is probably for tuitous and cannot be expected to be repeated for the next order processes. We also see that the corrections do not affect the predominance of nucleon moment interactions, characteristic of the pseudoscalar theory, and that this is probably true if effects of even higher order are included.

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