to the rotational temperatures measured by Vegard<sup>20</sup> and his collaborators<sup>21</sup> from the intensity distribution in the R branch of the 4278A (0, 1) band of  $N_2^+$  in the aurora at Trömso, Norway. Vegard and Tönsberg found  $T_{\rm eff} = 219 \pm 25^{\circ} \text{K}$  for the auroral region from 90 to 140 km. Although most of their spectra are of auroras at this altitude, studies of rays extending to 600 to 800 km show no increase in rotational temperature. In this band system the moments of inertia of the molecule are almost identical in the excited N<sub>2</sub><sup>+</sup> state and in the ground states of both  $N_2^+$  and  $N_2$ , so that B''/B' is unity. In addition, the intensities of the auroras permit the use of a much greater resolving power than in the case of night sky spectra. A third factor increasing the reliability of these measurements is that the  $N_2^+$  band involves no inertial change upon radiation, so that the shape of the unresolved band profile is much more sensitive to the temperature than are the shapes of the Herzberg and Vegard-Kaplan bands.

<sup>20</sup> L. Vegard, Geofys. Publ. 9, No. 11 (1932); Terr. Mag. 37, 389 (1932); Phil. Mag. 24, 588 (1937); Naturwiss. 26, 639 (1938). <sup>21</sup> L. Vegard and E. Tönsberg, Geofys. Publ. 11, No. 2 (1935); 12, No. 3 (1938); 13, No. 1 (1940) and No. 5 (1941). Nature 145, 588, 623 (1940). Penndorf<sup>22</sup> calls attention to the fact that Vegard's measurements are all made in the arctic winter, while the night sky experiments and radio sounding data have been primarily low latitude experiments. However, Penndorf's criticism does not explain the extension of auroral rays to 1000 km which is occasionally observed in the arctic winter and is one of the best evidences of a high mean temperature at high altitudes. Comparison of measurements made in widely different latitudes using different techniques should perhaps be interpreted with caution.

It would be very desirable to obtain independent confirmation of Vegard's results, particularly in low latitudes. In addition, a systematic investigation of the latitude dependence of the temperature at 100 km is called for using the same experimental method at all latitudes.

I am greatly indebted to Professor Otto Oldenberg who suggested this problem and assisted with many stimulating discussions. I also wish to acknowledge the generous support of a Predoctoral Research Fellowship from the National Institute of Health.

<sup>22</sup> R. Penndorf, Bull. Am. Met. Soc. 27, 331 (1946).

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# The Liquid-Solid Transformation in Helium near Absolute Zero

C. A. SWENSON\* The Clarendon Laboratory, Oxford University, England (Received March 13, 1950)

Although helium is normally a liquid to absolute zero, it can be solidified by the application of moderately high pressures. This transformation is of particular interest, since by the third law of thermodynamics the entropy difference between the liquid and the solid must disappear as the absolute zero is approached. This can most easily be shown by the disappearance of the heat of melting. Measurements from  $1.0^{\circ}$  to  $1.8^{\circ}$  of the slope of the melting pressure curve and the change in the molar volume of helium on melting are described, from which the heat of melting and the change in internal energy on melting were calculated. These results verify the predictions made by the third law. The measurements are also combined with those due to earlier workers to show the discontinuities in the various thermodynamic functions at the intersection of the  $\lambda$ -point-pressure curve with the melting pressure curve.

#### I. INTRODUCTION

SOLID helium, in contrast with liquid helium, is a relatively normal substance, although it exists only under relatively high pressures (Fig. 1). W. H. Keesom and his collaborators were the first investigators to observe solid helium,<sup>1</sup> and they have performed many fundamental experiments with it, all of which point to the fact that a transformation of the  $\lambda$ -type does not take place.

The fact that helium can be solidified by the application of pressure, although under its saturated vapor pressure it remains a liquid to absolute zero, makes the liquid-solid transformation very interesting from the thermodynamic point of view. This has been pointed out in another paper,<sup>2</sup> and it is worth noting that, by the third law of thermodynamics, the entropies of liquid and solid helium must become the same at very low temperatures. This means that both the heat of melting and dP/dT must approach zero as a limit. The fact that the melting pressure curve as first determined by Keesom (Fig. 1)<sup>3</sup> did seem to approach a positive pressure with dP/dT = 0 at absolute zero was taken as a direct confirmation of the validity of the third law.

Probably the most interesting point on the melting pressure curve, with the exception of the point at the absolute zero, is the intersection of the melting pressure and  $\lambda$ -point pressure curves<sup>3,4</sup> at 1.8°. This is a form of

<sup>\*</sup> Now at Harvard University, Cambridge, Massachusetts.

<sup>&</sup>lt;sup>1</sup>W. H. Keesom, Helium (Elsevier, Amsterdam, 1942), p. 180.

<sup>&</sup>lt;sup>2</sup> F. Simon and C. A. Swenson, Nature 165, 829 (1950).

<sup>&</sup>lt;sup>3</sup> W. H. Keesom, reference 1, p. 202.

<sup>&</sup>lt;sup>4</sup> W. H. Keesom, reference 1, p. 226.

"triple point" between liquid He I, liquid He II, and solid helium, and since it is at this point that dP/dTbegins to decrease rapidly, it would seem that the other thermodynamic quantities characteristic of the transformation— $\Delta V = V_L - V_s$ , the change in molar volume,  $\rho$ , the heat of melting, and  $\Delta U = U_L - U_s$ , the change in internal energy—would also tend to change rapidly below this temperature. All previous determinations of these quantities have been confined to temperatures above 2°.

Because of this expected very rapid decrease in the heat of melting below the  $\lambda$ -point, it is obvious that thermal measurements would soon become so inaccurate as to be impracticable as the temperature was lowered. Therefore, the most logical solution seemed to be to determine  $\Delta V$  and dP/dT from the  $\lambda$ -point to as low temperatures as possible, and to calculate  $\rho$  and  $\Delta U$  from them using the Clausius-Clapeyron equation. A general discussion of these results was published in another paper,<sup>2</sup> and it is the purpose of this paper to show how the results were obtained and to give some further data.

# **II. DESCRIPTION OF THE APPARATUS**

The liquid helium was produced by means of a Simon expansion liquefier<sup>5</sup> of about 65 cc capacity. The actual cryostats (one for the melting pressure determinations and one for the  $\Delta V$  measurements) were suspended alternately from the bottom of the liquefier expansion vessel in the vacuum jacket by thin-walled German silver tubes. The required accuracy and reproducibility in the melting pressure measurement (0.01 atmos. in 25 atmos.) were obtained using a simple piston-incylinder pressure balance which has been described elsewhere.<sup>6</sup>

A separate phosphor-bronze resistance thermometer was used for each experiment, and the thermometers were calibrated against the vapor pressure of helium (Bleaney and Simon scale) using mercury and oil manometers. The resistance wires were permanently attached to the cryostats by well-baked Bakelite varnish, and under these conditions the calibrations remained constant in spite of repeated warmings up to room temperature. Resistance measurements of sufficient accuracy were obtained by using a Tinsley vernier potentiometer, and current variations were kept to a minimum.

# III. THE MELTING PRESSURE MEASUREMENTS

The melting pressure measurements were made by the conventional blocked capillary technique.<sup>1</sup> The cryostat (shown schematically in Fig. 2) consisted of a 15 cc copper vessel, A, with a fine (0.5 mm) capillary attached to the inside of its base. The temperature of the liquid helium in A was lowered by pumping through the 3 mm German silver tube, C, and the vapor pressure was measured through the 2 mm tube, D. The phosphor-bronze wire was wound about A.

By means of other fine capillaries, the two ends of the cryostat capillary were attached at room temperature to opposite sides of a sensitive bellows manometer, E, one side of which was connected to the pressure balance. In practice the temperature of A was kept constant by throttling the pump, and by adding 10 g weights to the pressure balance piston, the pressure on the helium in the capillary was slowly raised in 0.01 atmos. steps until the capillary blocked; whereupon further increase in pressure resulted in a depression of the bellows and an electrical contact, F, was broken. The bellows sensitivity was roughly 0.01 atmos.; that is, an increase of pressure of 0.01 atmos. with the capillary blocked was enough to close the contact.

The apparatus worked very well for temperatures below 1.5° and these results are given in Fig. 3. It can be seen that the points (representing several different days' work) are reproducible to  $\pm 0.01$  atmos. Points for higher temperatures were much more difficult to obtain accurately, since dP/dT is so large that a temperature shift of only 0.001° produces a very appreciable change in *P*. In this region the data were only good enough to show that the *P vs. T* curve is continuous through the  $\lambda$ -point. The rapid change in dP/dT at the  $\lambda$ -point (to be discussed later) was shown nicely in a semiquantitative sort of way.

Fortunately, the melting points obtained above 1.4° agreed within experimental error with previous results given by Keesom and Miss Keesom.<sup>7</sup> These data were



FIG. 1. The melting pressure and  $\lambda$ -point-pressure curves for helium.

<sup>&</sup>lt;sup>5</sup> W. H. Keesom, reference 1, p. 160.

<sup>&</sup>lt;sup>6</sup> C. A. Swenson, Rev. Sci. Inst. 21, 22 (1950).



FIG. 2. The blocked capillary melting pressure cryostat.

analyzed by Haggenmacher<sup>8</sup> to fit an interpolation formula, and from this formula, he obtained the following expression for dP/dT:

$$\frac{dP/dT = 100[(T-1.14)/(32-P)] \text{ atmos./deg.}}{(1.5^{\circ} \text{ to } 1.75^{\circ} \text{ K}).}$$
 (1)

This formula is only useful as far as interpolation is concerned, since the expression for the pressure has no physical significance at absolute zero.

Below 1.5°, the curve given here is slightly below the data given by Keesom and Keesom, with a maximum deviation of about 0.1 atmos. at 1.2°. Since their curve (with the temperature corrected to the Bleaney and Simon scale) extends only to approximately this temperature, and was obtained as a supplementary part of another experiment, the data in Fig. 1 were accepted as correct and were fitted to an extrapolation curve which was used to calculate dP/dT from 1.4° to 1.0°.<sup>8a</sup> The experimental points in Fig. 1 can be represented by the following equation in this range within experimental accuracy:

$$P = 0.053T^{8} + 25.00$$
 atmos. (1.0° to 1.5° K). (2)

Therefore, dP/dT is given by:

$$dP/dT = 0.425T^7$$
 atmos./deg. (1.0° to 1.4° K). (3)

Finally, to obtain values for dP/dT above the  $\lambda$ -point, the earlier data due to Keesom<sup>3</sup> were fitted to the following curve, and dP/dT was calculated from it; giving the formula

$$P = 15.45T^{1.57} - 8$$
 atmos. (2.0° to 4.0° K). (4)

These data are summarized in Table I, column 3, and are plotted in Fig. 4. The rapid drop in dP/dT by a factor of 100 from the  $\lambda$ -point (1.77°) to the lowest temperature reached (1.0°) shows the expected effect due to the decrease in the entropy difference between liquid and solid helium. The values for dP/dT are accurate to about 2 percent below the  $\lambda$ -point, and to roughly 5 percent between 2° and 4°.

Equation (2) enables us to extrapolate the melting curve to absolute zero, yielding a value

$$P_0 = 25.00 (\pm 0.05)$$
 atmos.  $(T = 0^{\circ} \text{ K}).$ 

1	2	3	4 5	6	7	8	9	10	11
Temp. °K	P atmos.	dP/dT atmos./deg.	$\begin{array}{c} \Delta V = V_L - V_s \\ \mathrm{cm^3/mole} \\ \mathrm{meas.}  \mathrm{calc.} \end{array}$	V⊥ cm³/mole	Vs cm³/mole	(cal./ calc.	ρ mole) meas.	$P\Delta V$ calc./mole	$\Delta U = U_L - U_s$ cal./mole
0°	25.00	0	2.07 —	23.25	21.18	0		1.22	-1.22
1.00 1.10 1.20 1.30 1.40 1.50 1.70 1.75 1.77 1.78 1.80 <sup>5</sup> 1.83 <sup>5</sup> 2.0 2.5 3.0 3.5	25.05 25.11 25.22 25.43 25.81 26.37 27.16 28.4 29.25 	$\begin{array}{c} 0.425\\ 0.82\\ 1.51\\ 2.66\\ 4.50\\ 6.36\\ 9.43\\ 15.20\\ 21.1\\ 29.8(calc.)\\ 30.5\\ 30.8(calc.)\\ 31.6(calc.)\\ 35\\ 41\\ 45^{5}\\ 49^{5} \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	23.24 23.23 23.22 23.18 23.11 23.02 22.84 22.50 22.30 22.22 	21.17 21.16 21.15 21.11 21.05 20.95 20.95 20.85 20.78 20.75 	0.021 0.045 0.090 0.170 0.310 0.456 0.684 1.00 1.34 	1.89 1.98 2.07 2.33 3.34 4.35 5.45	$1.23 \\ 1.24 \\ 1.24 \\ 1.25 \\ 1.27 \\ 1.25 \\ 1.23 \\ 1.10 \\ 1.06 \\ \\ 1.06 \\ \\ 1.27 \\ 1.82 \\ 2.38 \\ 2.97 \\ 1.27 \\ 1.82 \\ 2.97 \\ 1.82 \\ 2.97 \\ 1.82 \\ 2.97 \\ 1.82 \\ 2.97 \\ 1.82 \\ 2.97 \\ 1.82 \\ 2.97 \\ 1.82$	$\begin{array}{c} -1.21 \\ -1.20 \\ -1.15 \\ -1.08 \\ -0.96 \\ -0.79 \\ -0.55 \\ -0.10 \\ +0.28 \\ - \\ +0.85 \\ - \\ +1.06 \\ +1.52 \\ +1.97 \\ +2.48 \end{array}$
4.0	129.0	53	1.0 1.26	18.40	17.25		6.50	3.55	+3.05

TABLE I. Thermodynamic data for the liquid-solid trans. in helium below 4°K.\*

\* The data above 2.0° are mostly due to Keesom, as is the whole of column 6 (see text).

<sup>8</sup> J. E. Haggenmacher, Phys. Rev. 74, 1539 (1948).

 $a_{a}$  A possible source of error in melting pressure determinations by the blocked capillary technique has been pointed out by Bridgman [Rev. Mod. Phys. 18, 28 (1946)], who noted that the shearing stress exerted on the solid by the pressure difference across it can very easily lead to a lower than normal melting point. However, in these experiments, the pressure was increased in 0.01 atmos. increments, and the pressure difference across the solid helium in the capillary was never greater than 0.02 to 0.03 atmos. Thus, this effect should be negligible.

The error here arises not so much in the extrapolation as in the absolute calibration of the pressure balance. The temperature and pressure of the intersection of the  $\lambda$ -point-pressure curve with the melting curve (called the " $\lambda$ -point" in this paper) can also be determined from these data and from the  $\Delta V$  data as

$$P_{\lambda} = 30.0(\pm 0.1)$$
 atmos.,  $T_{\lambda} = 1.77^{\circ}(\pm 0.005)$ .

These figures agree quite well with data given by Keesom.<sup>4</sup>

### IV. THE MEASUREMENT OF $\Delta V$

The cryostat shown in Fig. 5 was used to determine the volume change of solid helium on melting. Liquid helium was condensed into the upper compartment, A, and low temperatures were obtained by pumping this helium through the thin-walled German silver tube, C. As in the previous experiments, the resistance thermometer was calibrated with vapor pressures measured through the 2 mm tube, D. The resistance thermometer was wound directly on the lower or high pressure compartment, B, which contained either liquid helium or solid helium under pressure, and which was connected directly to the pressure balance. As the solid helium melted, liquid helium was pushed out through the capillary, and, if all external conditions were kept constant, this was shown directly by the rise of the pressure balance piston in its cylinder as it rose to keep the pressure of the system constant by increasing its volume.

In practice, the cryostat was cooled to approximately 1° by pumping the helium in A. The excess liquid helium was then evaporated away by an auxiliary heater wound on A. When it had all disappeared, the normal heating curve of the cryostat (temperature vs. time, with no heat applied) was taken. This was essentially governed by the natural heat influx and the specific heat of the solid helium in B. A simultaneous curve of pressure balance piston height vs. time was also taken. At the melting point, the change in volume of the solid helium as it melted resulted in a rather rapid rise of the pressure balance piston, while the heat of melting caused a plateau in the heating up curve. Thus, the melting temperature and the change in volume of the solid helium on melting could be determined at the pressure given by the weights on the pressure balance. Actually, while these determinations of temperature were accurate enough for the measurements, of  $\Delta V$ , they were not good enough for really accurate melting pressure measurements, and thus the first set of experiments was performed.

Thermal measurements were difficult because, due to the high thermal conductivity<sup>9</sup> of He II, most of the heat influx came down the capillary. This correction is very temperature sensitive, and is also complicated by the flow of "cold" liquid helium out of the cryostat. Thus, it was only around the  $\lambda$ -point, where the natural

FIG. 3. The experimental points for the melting pressure curve of helium below 1.6°.

heat influxes were relatively small, that direct determinations of the heat of melting could be made. Three of these are given in column 9 of Table I, and can be seen to check within experimental error (roughly 3 percent) the dP/dT and  $\Delta V$  values that were determined in the same set of experiments.

It might be added that this high heat influx down the capillary performed one useful function in that it made certain that the helium in the capillary melted first, and made less likely any form of superheating effects.

The values of  $\Delta V$  were calculated from the volume of *B* (2.50 cc at 1° K) and the mass of helium pushed out of the cryostat (obtained from the pressure balance rise). The densities of liquid helium along the melting curve were also needed and were deduced from the isopyncnal data given by Keesom and Keesom.<sup>8</sup> These (or, rather, the molar volumes,  $V_L = 4.003/d_L$ ) are tabulated in column 6 of Table I, and are plotted in the upper curve of Fig. 8.

The actual experimental points for  $\Delta V$  are plotted in Fig. 6. The estimated experimental error is 3 percent,



FIG. 4. dP/dT for the liquid-solid transformation in helium.

<sup>&</sup>lt;sup>9</sup> W. H. Keesom, reference 1, p. 282.

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and the measurements are easily as reproducible as this. Figure 7 gives this curve combined with previous data due to Keesom and Keesom (from 2.2° to 4.0°),10 although there is doubt about some of this earlier data. This is shown in columns 4 and 5 of Table I, where the values of  $\Delta V$  as calculated from the experimental heats of melting (column 9)<sup>11</sup> and the values for dP/dT(column 3) are compared directly with the experimental determinations. An average value has been taken for the curve of Fig. 7, although there is some uncertainty near 4°.



FIG. 5. The cryostat for measurement of  $\Delta V$ .

By use of the curves of  $V_L$  and  $\Delta V$ , the densities of solid helium on melting  $(V_s)$  were calculated. These are tabulated in column 7 of Table I, and are plotted in the lower curve of Fig. 8. The accuracy of these figures depends very much on the accuracy of the data for  $V_L$ , and can only be estimated at roughly 0.5 percent.

These figures can all be extrapolated to absolute zero



FIG. 6. The experimental data for the molar change in volume of helium on melting,  $\Delta V = V_L - V_s$ .



<sup>10</sup> W. H. Keesom, reference 1, p. 210. <sup>11</sup> W. H. Keesom and A. P. Keesom, Leiden Comm. 240b (1936).

quite easily. From Fig. 6, it can be seen that  $\Delta V$  is constant from about 1.45° to 1.2°, so it is possible to say that at absolute zero:

$$\Delta V = 2.07 (\pm 0.06) \text{ cm}^3/\text{mole}$$
  $(T = 0^\circ \text{ K}).$ 

The extrapolation of  $V_L$  and  $V_s$  from Fig. 8 is more difficult, but fortunately the melting pressure also varies very slightly with temperature in this region. If instead of plotting the molar volumes as functions of temperature, they are plotted as functions of the melting pressure, the extrapolation to 25.00 atmos. is quite simple and the following values are obtained:

> $V_{L0} = 23.25 (\pm 0.05) \text{ cm}^3/\text{mole},$  $V_{s0} = 21.18(\pm 0.1)$  cm<sup>3</sup>/mole.

#### **V. CALCULATIONS**

The heats of melting,  $\rho$ , were calculated from the Clausius-Clapeyron equation,

$$\rho = T \Delta V dP / dT. \tag{4}$$

They are tabulated in column 8, Table I, and are plotted as the upper curve in Fig. 9. Below 1.4°,  $\Delta V$  is practically constant, and dP/dT is given by Eq. (3), so that a simple analytical expression can be given for the heat of melting:

$$\rho = 0.021 T^8 \text{ cal./mole} (1.4^\circ \text{ to } 1.0^\circ \text{ K}).$$
 (5)

The figures for the heats of melting in column 9, Table I, are experimental values due to Keesom and Keesom<sup>11</sup> and Kaischew and Simon.<sup>12</sup> They extend only down to about 2.2°, but extrapolate to the heats of melting determined directly in these experiments.

From these data and the relation

$$\Delta U = \rho - P \Delta V \tag{6}$$

the change in the internal energy of liquid helium on solidification is easily calculated. This is also shown in Fig. 9 and in Table I. The interesting effect here is that, while  $\rho$  approaches zero very rapidly,  $\Delta U$  approaches a negative value at absolute zero. This is a direct consequence of the fact that the liquid-solid transformation exists with a definite  $\Delta V$  at absolute zero, as has been pointed out previously.<sup>2</sup>



FIG. 7. The change in molar volume of helium on melting,  $\Delta V = V_L - V_s.$ 





FIG. 8. The molar volumes of liquid  $(V_L)$  and solid helium  $(V_s)$  along the melting curve.

The actual internal energy of solid helium at absolute zero can also be calculated. From the work of Bleaney and Simon<sup>13</sup> on the vapor pressure of helium below 1.6°, the latent heat of vaporization of liquid helium at absolute zero can be calculated to be 14.3 cal./mole; therefore the internal energy of liquid helium at zero temperature and pressure is -14.3 cal./mole. The increase in internal energy of liquid helium upon the application of 25 atmos. pressure can be calculated from the integral  $\int PdV$ , which gives -1.1 cal./mole, using data due to Keesom and Keesom.<sup>14</sup> Finally, from Fig. 9, the internal energy difference between the liquid and solid under 25 atmos. pressure and at absolute zero can be determined as -1.25 cal./mole. These data are summarized in Table II, together with the other constants for liquid and solid helium at absolute zero.

One other calculation can be made, concerning the specific heat of liquid helium near 1° K and under a pressure of 25 atmos. Using the thermodynamic formula,

$$C_{PL} - C_{PS} = \frac{d\rho}{dT} - \frac{\rho}{T} + \frac{\rho}{\Delta V} \left( \frac{\partial (V_L - V_S)}{\partial T} \right)_P, \quad (7)$$

and the experimental results summarized in Eq. (5), the following relation is obtained for temperatures slightly above 1°, where the melting pressure is practically constant (the last term in Eq. (7) being negligible):

$$C_{PL} - C_{ps} = 0.15T^7 \text{ cal./mole deg.}$$
 (8)

In this region,  $C_{vs}$  (which differs very little from  $C_{ps}$ ) can be extrapolated roughly to<sup>15</sup>

$$C_{vs} = 0.03T^4 \text{ cal./mole deg.}$$
(9)

This gives the specific heat of liquid helium under 25 atmos. pressure, near 1°, as:

$$C_{PL} = 0.15T^4(T^3 + \frac{1}{5}) \text{ cal./mole deg.}$$
 (8a)

As a rough check, the specific heat of liquid helium as calculated from the specific heat of the liquid at 1° and



FIG. 9. The molar heat of melting,  $\rho$ ,  $P\Delta V$ , and the change in internal energy,  $\Delta U = \rho - P\Delta V$ , for helium on solidification.

TABLE II. The molar volumes and internal energies of liquid and solid helium at absolute zero.

	P (atmos.)	U (cal./mole)	V (cm <sup>3</sup> /mole)
Liquid, 0° K	0	-14.3	27.4
Solid, 0° K	25 25	-13.2 -11.9	23.25 21.2

zero pressure and

$$(\partial C_P / \partial P)_T = -T(\partial^2 V / \partial T^2)_P$$

agrees in order of magnitude with this equation quite nicely. Of course,  $(\partial V/\partial T)_P$ , much less,  $(\partial^2 V/\partial T^2)_P$ , is not very well known.

The following conclusion can be drawn. The specific heat of liquid helium near 1° and under 25 atmos. pressure varies as some power of the temperature closer to  $T^7$  than to  $T^4$  (probably  $T^6$ ). This is in good, although probably coincidental, agreement with the Leiden data<sup>16</sup> for the specific heat of liquid helium under its saturated vapor pressure in this same region.

The effect of the  $\lambda$ -transformation in liquid helium is shown quite markedly in the thermodynamic functions for the liquid-solid transformation. For a normal substance, the curves of Fig. 9 would terminate at a triple point which would occur at approximately the same temperature as does the  $\lambda$ -transformation. Helium, however, instead of losing its entropy by becoming a solid, becomes some form of degenerate liquid, and as such loses its entropy in a complicated manner. Thus, near absolute zero, the curves in Fig. 9 represent a curious type of mechanical transformation from a crystal of high spatial order to a liquid with a high degree of order of a different type.

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<sup>&</sup>lt;sup>13</sup> B. Bleaney and F. Simon, Trans. Faraday Soc. 35, 1205 (1939).

<sup>&</sup>lt;sup>14</sup> W. H. Keesom, reference 1, p. 240.

<sup>&</sup>lt;sup>15</sup> W. H. Keesom, reference 1, p. 227.

<sup>&</sup>lt;sup>16</sup> W. H. Keeson, reference 1, p. 220.