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Investigation of Nuclear Energy Levels in Sulfur*

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The total cross section of sulfur has been measured for neutrons of energies from 15 to 1450 kev with energy spreads between 1.5 and 9 kev. Below 750 kev, nine well-defined maxima are observed, three of which are interpreted as caused by *s*-neutrons; the others are believed to be produced by *p*- or *d*-neutrons. The *s*-resonances have natural widths of about 15 kev, while one of the other resonances which was studied in detail has a width of 1.5 kev. In the case of the latter resonance, the compound nucleus appears to be formed with spin of $\frac{3}{2}$. Above 750 kev neutron energy the density of observed levels increases, and no interpretation of the resonances was attempted.

I. INTRODUCTION

In order to study individual nuclear energy levels by investigating the variation of the total neutron cross section with neutron energy, it is necessary to use neutrons which have an energy spread small compared to the spacing of the energy levels, and comparable to the widths of the levels. For light nuclei, which have large level spacings and level widths, this requirement is easily met. In general, however, as the nuclear mass increases, the level widths and spacings decrease and it is difficult to obtain neutrons with sufficiently well-defined energies. In the present paper a description of measurements on the levels in sulfur will be given. Sulfur was chosen because preliminary experiments had indicated that its energy levels might be fully resolved with a neutron energy spread of the order of 1 kev, in contrast to aluminum, for example.

II. EXPERIMENTAL PROCEDURE

As in previous experiments,[1] total neutron cross sections were determined from transmission measurements. Neutrons were obtained from the $Li^7(p, n)Be^7$ reaction using the electrostatic generator to accelerate the protons. It has recently been found[2] that a second neutron group from this reaction is present if the energy of the main group is greater than 650 kev. The number of neutrons of lower energy appears to be so small, however, that this group did not produce observable resonance effects in the present experiment.

$B^{10}F_3$ proportional counters served as neutron detectors. One of the counters employed has been described previously;[1] a second counter, shown in Fig. 1, was used for some of the work carried out with high resolution and consequent low neutron intensity. The efficiency of the counter shown in Fig. 1 was 2×10^{-3} for neutrons from a Ra—Be source, or about five times that of the first counter.

In order to produce neutrons with a small energy spread, the protons striking the lithium target must be well defined in energy, the target must be thin, and the angle subtended by the detector at the target must be small.[1] As the neutron energy spread is reduced, the neutron flux decreases rapidly. Therefore, high resolution was used only in energy regions where structure had previously been located with a larger energy spread.

The energy of the protons was defined by passing them through the one-meter radius electrostatic analyzer.[3] The resolution function, or the distribution in energy of the protons, for this analyzer is triangular, and the effective proton energy spread is taken as the full width, at half-maximum, of the resolution function. This energy spread will be expressed in percent of the proton energy. With analyzer slit settings corresponding to a proton energy spread of 0.1 percent, about 2 μa of beam current were available, while with an energy resolution of 0.033 percent, 0.3 μa of proton current could be maintained.

In order to estimate the neutron energy spread used, it is necessary to know the thickness of the Li target.

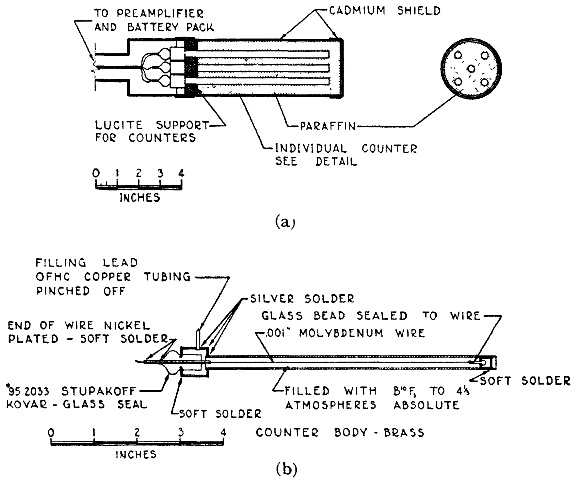

Fig. 1. Neutron detector. (a) Counter assembly. (b) Detail of single counter.

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[1] Barschall, Bockelman, Peterson, and Adair, Phys. Rev. **76**, 1146 (1949).

[2] Johnson, Wilson-Laubenstein, and Richards, Phys. Rev. **77**, 413 (1950).

[3] Warren, Powell, and Herb, Rev. Sci. Inst. **18**, 559 (1947).

TABLE I. Estimated effective neutron energy spreads used in the study of nuclear energy levels in sulfur.

Target thickness (kev)	Analyzer resolution (percent)	Effective neutron energy spread (kev)
8	0.1	9
5.5	0.1	6.5
4	0.1	5.5
2.5	0.1	4
1.3	0.1	3
	0.05	2.5
	0.07	3.2
0.6	0.033	1.5

The previously described rise method[1,4] for determining target thickness may not be reliable for very thin targets, since it is not known how the cross section for the Li(*p*, *n*) reaction varies just above the threshold energy. Target thicknesses were therefore estimated on the basis of the neutron yields. A target thick enough to be measured by the rise method (about 8 kev) was prepared, and its neutron yield at a proton energy of 1.93 Mev was determined relative to a Ra—Be source. Thinner targets were then measured against this standard, assuming that the thickness of the target is proportional to its yield. This assumption is not valid if there is appreciable oxidation of the targets.

In order to help prevent the deposition of oil from the vacuum system onto the target, the oil-sealed rotating target previously used was replaced by a stationary target assembly. A jet of heated air was directed at the tantalum backing of the lithium target to reduce the rate of oil deposition. To prevent local heating of the target, the proton beam was swept in a small circle on the target by passing it between two pairs of plates placed about a meter from the target; two transformers, phased 90° apart, were connected to the plates. With these precautions, it was possible to use a target less than 1 kev thick for several hours with no loss of lithium, and with very little oil deposition.

During the course of the experiment, targets ranging in thickness from 0.6 to 8 kev were used. The analyzer resolution was adjusted to keep the energy spread of the incident proton beam comparable to or less than the target thickness. For all but the thinnest target, the detector was placed 25 cm from the target; the angle subtended by the counter at the target then introduced maximum neutron energy spreads of 2, 3.5, and 6 kev at neutron energies of 200, 700, and 1400 kev, respectively. With the 0.6-kev target, the counter was placed 33 cm from the target, and the maximum neutron energy spread introduced by the finite size of the detector was 2 kev at 600 kev neutron energy. The effective energy spread introduced by the variation of the neutron energy with angle was taken as 0.6 of these maximum values. This factor was obtained by assuming that all the incident neutrons were detected with equal

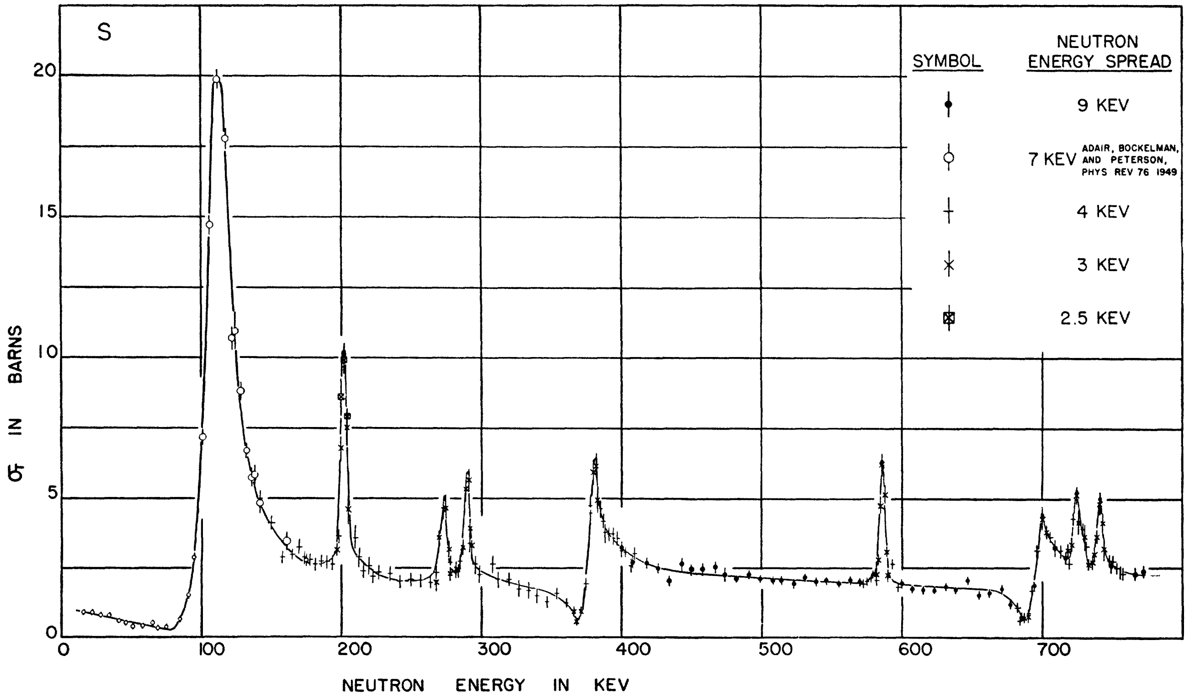

FIG. 2. The total neutron cross section of sulfur as a function of neutron energy. The heights of the symbols are equal to the standard statistical errors in the cross section. Different symbols represent the different neutron energy spreads used in determining the cross section.

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[4] R. F. Taschek and A. Hemmendinger, Phys. Rev. 74, 373 (1948).

efficiency and by taking as the effective energy spread that energy interval which would include three-fourths of all the counted neutrons. The actual variation of sensitivity across the face of the counter is probably such that this estimate of the energy spread associated with the detector is too large. Therefore, the estimated over-all effective neutron energy spreads listed in Table I will probably be too large also, particularly for the cases of the thinner targets.

The resolution obtained with a given target and a particular analyzer setting is nearly independent of neutron energy. The Doppler effect contribution to the neutron energy spread is negligible.

III. MEASUREMENTS BELOW 750-KEV NEUTRON ENERGY

Results

Figure 2 shows the total neutron cross section of sulfur as a function of neutron energy for energies up to 750 kev. The energy spreads of the neutrons used are indicated in the legend of the graph. Maxima in the cross section were observed at 111, 203, 274, 290, 375, 585, 700, 725, and 742 kev neutron energy. There are pronounced minima preceding the 111-, 375-, and 700-kev maxima. At energies near maxima and minima, preliminary data obtained with the larger neutron energy spreads are not shown on the graph. It was found, however, that the observed heights of the maxima increased as the energy resolution was improved; this effect was less pronounced in the case of the broader maxima. The cross sections at the minima did not decrease as the neutron energy spread was reduced.

Figure 3 shows the results obtained by studying the 585-kev peak (Fig. 2) in greater detail, using an estimated neutron energy spread of 1.5 kev.

All the data presented on the graphs were corrected for neutron background and for the effect of neutrons scattered into the detector by the sulfur cylinders; the latter correction was based on the assumption of isotropic scattering from the sulfur. The thicknesses of the sulfur samples were adjusted to make the transmission approximately 60 percent at all energies.

Discussion

S^{32} is the main constituent (95 percent) of normal sulfur. The observed maxima in the total cross section should therefore be caused by levels of the compound nucleus S^{33}. For sulfur, the contribution of inelastic scattering to the total cross section should be small, since the lowest reported level in S^{32} occurs[5] at about 2.3 Mev, and so cannot be excited in the present measurements. The cross section for absorption of neutrons is small even at thermal energies and would not be expected to contribute appreciably to the total cross section at higher energies; the exothermic reaction $S^{32}(n, \alpha)Si^{29}$ has a cross section of only 0.07 barn at a neutron energy[6] of 2.8 Mev so that this process should be unimportant. The predominant interaction is therefore elastic scattering. In view of the shapes and separation of the cross-section maxima found for neutrons with energies up to 750 kev, it is assumed that these maxima show the effect of individual levels in S^{33}. It should therefore be possible to assign angular momenta to these levels on the basis of the heights of the maxima in the cross section.[7]

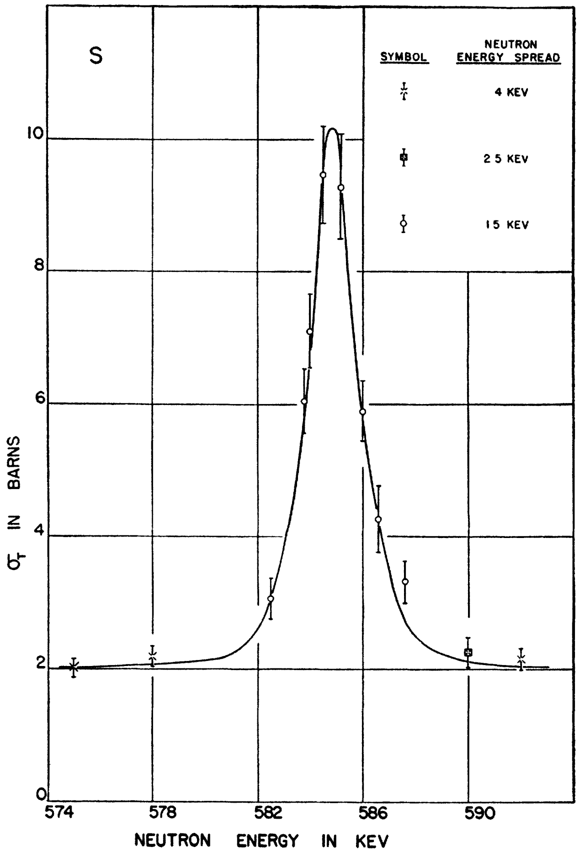

FIG. 3. The total neutron cross section of sulfur near the 585-kev resonance. The statistical errors in the cross section are given by the heights of the symbols.

The resonance at 111 kev has been discussed previously.[8] It was shown to be produced by neutrons of zero orbital angular momentum forming a compound nucleus with spin J equal to $\frac{1}{2}$, and the natural width of the level was estimated to be about 18 kev. The minimum preceding the peak was interpreted in terms of interference between resonance and potential scattering. At neutron energies of a few hundred kev, such interference is expected only near resonances caused by neutrons of zero orbital angular momentum,

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[5] Beghian, Grace, Preston, and Halban, Phys. Rev. **77**, 286 (1950).

[6] P. Huber, Helv. Phys. Acta **14**, 163 (1941).

[7] Adair, Barschall, Bockelman, and Sala, Phys. Rev. **75**, 1124 (1949).

[8] Adair, Bockelman, and Peterson, Phys. Rev. **76**, 308 (1949).

since at such energies all but a few percent of the potential scattering is due to *s*-neutrons. Therefore, the minima preceding the 375- and 700-kev maxima indicate that these resonances are also caused by *s*-neutrons forming a compound nucleus with J equal to $\frac{1}{2}$. The heights of these peaks, as measured from minimum to maximum, are within 10 percent of the values expected on this assumption. The residual cross sections of about 0.5 barn at the minima are approximately accounted for by the presence of the five percent of isotopes other than S^{32} and the contribution to the potential scattering by neutrons of higher orbital angular momenta. The natural widths of the 375- and 700-kev resonances are estimated to be 12±1.5 kev.

The estimates of the natural widths of the *s*-resonances were obtained from the positions of their maxima and minima;[9] the result does not depend strongly on assumptions regarding the distribution in energy of the neutrons. The total neutron cross section at an energy E near an *s*-resonance at energy E_r, is given by

$$\sigma_t = 4\pi k^{-2}\sin^2\delta \tag{1}$$

where

$$\delta = \tan^{-1}[\tfrac{1}{2}\Gamma/(E_r - E)] + \varphi. \tag{2}$$

Γ is the width of the resonance, k is the neutron wave number in the center-of-mass system, and φ is the phase shift for potential scattering. At the energies E_{max} and E_{min}, corresponding to the observed positions of the maximum and minimum of the cross section, δ has the values $\pi/2$ and zero, respectively. The substitution of these values of δ and of the associated values of E into Eq. (2) yields two relations from which E_r may be eliminated, with the result that

$$\Gamma = (E_{\text{min}} - E_{\text{max}})\sin 2\varphi. \tag{3}$$

The phase shift φ is obtained from the relation

$$\sigma_{\text{pot}} = 4\pi k^{-2}\sin^2\varphi \tag{4}$$

by setting σ_{pot}, the cross section for potential scattering, equal to the depth of the dip in the cross-section curve.

The maxima at 203, 274, 290, 725, and 742 kev show no interference effects and are therefore caused by neutrons of greater than zero orbital angular momentum. Since at the lower energies the widths of *d*-resonances should be about 30 times less than those of *p*-resonances,[10] *d*-neutrons might not be expected to produce experimental cross sections as high as those found. Consequently, it is believed that the 203-, 274-, and 290-kev resonances are caused by *p*-neutrons. *P*-neutrons may form a compound nucleus of spin $\frac{1}{2}$ or $\frac{3}{2}$. While none of the maxima have observed heights larger than those expected for J equal to $\frac{1}{2}$, it is still possible that the levels might be associated with a higher spin of the compound nucleus, if their natural widths are appreciably less than the neutron energy spread.

At the 585-kev resonance, shown in Fig. 3, the cross section rises to about 7 barns above the off-resonance value. The height expected if the compound nucleus were formed with J equal to $\frac{3}{2}$ is 8.8 barns at this energy. Either *p*- or *d*-neutrons could therefore be responsible for the resonance. The experimental width of this resonance is 2 kev.

From the observed height and width of the resonance, and the expected theoretical height, it is possible to obtain both the natural width of the level and the width of the neutron energy distribution. This calculation will depend only in an insensitive way on the shape of the function describing the distribution in energy of the neutrons. If this function is taken to be Gaussian, the natural width of the 585-kev resonance is found to be 1.5 kev, and the neutron energy spread 1.1 kev. This value of the energy resolution should be compared with the energy spread of 1.5 kev listed in Table I. As was previously mentioned, the energy spread introduced by the finite size of the detector may have been overestimated, and the widths given in Table I may be too large.

For purposes of comparison, the widths of the resonances may be expressed in terms of their reduced widths,[11] which are independent of energy and the effects of centrifugal barriers. The reduced width of a resonance with a natural width Γ is

$$\gamma^2 = \Gamma/2kT. \tag{5}$$

In Eq. (5), k is the neutron wave number at the resonance energy and T is a barrier penetration factor. T has the value unity for a resonance due to *s*-neutrons; for a *p*-resonance, it is[10]

$$k^2a^2/(1+k^2a^2) \tag{6}$$

where a is the nuclear radius, taken to be equal to $1.5A^{\frac{1}{3}}\cdot 10^{-13}$ cm. The reduced widths of the 111-, 375-, and 700-kev *s*-resonances are found to be approximately 125, 45, and 30×10^{-13} kev-cm, respectively. Under the assumption that the 585-kev resonance is produced by *p*-neutrons, its reduced width is 12×10^{-13} kev-cm, while its reduced width is 145×10^{-13} kev-cm if it were produced by *d*-neutrons.

If Fig. 2 shows all the *s*- and *p*-resonances which occur in the energy interval from 15 to 750 kev, the level spacing in S^{33}, for an excitation energy between 9 and 10 Mev, is of the order of 100 kev. This spacing is appreciably closer than that found for S^{33} at excitation energies of approximately 6 Mev in studies[12] of the $S^{32}(d, p)S^{33}$ reaction. While it is possible that with the poorer energy resolution used in the latter experiment some levels may not have been observed, the higher excitation energy of the S^{33} nucleus in the present experiment may be responsible for the closer level spacing.

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[9] M. R. MacPhail, Phys. Rev. **57**, 669 (1940).

[10] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947). See Section 5, Eq. (45a).

[11] E. P. Wigner, Am. J. Phys. **17**, 99 (1949).

[12] P. W. Davison, Phys. Rev. **75**, 757 (1949).

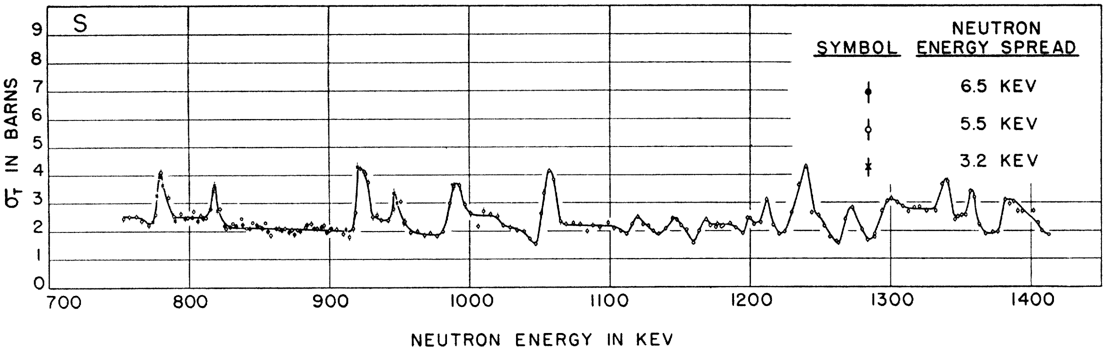

FIG. 4. The total neutron cross section of sulfur in the region of 750- to 1450-kev neutron energy. The statistical errors in the cross section are equal to the heights of the symbols.

IV. MEASUREMENTS ABOVE 750-KEV NEUTRON ENERGY

The total cross section of sulfur for neutrons of energies in the range from 750 to 1450 kev is shown in Fig. 4. A detailed comparison of these data with the earlier measurements of Freier *et al.*[13] is made difficult by the different energy resolutions used in the two experiments; there is, however, qualitative agreement between the two sets of measurements.

For neutron energies above about 1 Mev, several factors make it difficult to identify the energy levels responsible for the observed maxima. The theoretical heights of the resonances decrease with increasing neutron energy, so that the assignment of spins to resonances on the basis of their amplitudes becomes less reliable. Also, competition from the $S^{32}(n, p)P^{32}$ reaction, which has a threshold at a neutron energy of approximately 1 Mev, will reduce the expected height of the maxima, although this effect may not be important in view of the low cross section reported for this reaction by Klema and Hanson.[14] The distinction between *s*- and *p*-resonances will become less clear at higher neutron energies, since the contribution of neutrons of one unit of orbital angular momentum to the potential scattering will increase, and interference effects between resonance and potential scattering will be expected near both *s*- and *p*-resonances. An example of such interference for a *p*-resonance occurs below the 1055-kev maximum; the depth of the dip observed is consistent with the estimated 15 percent contribution of *p*-neutrons to the potential scattering cross section at this energy. Furthermore, at neutron energies in the region above 1 Mev, the widths of levels excited by *p*- and *d*-neutrons should differ only by a factor of the order of five, so that the effect of *p*- and *d*-neutrons can no longer be distinguished by considering level widths. As might be expected, the increased excitation energy of the compound nucleus and the greater widths expected for *d*-resonances increase the number of observable levels so that they are no longer well separated. It therefore does not seem possible to analyze most of the levels shown in Fig. 4.

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[13] Freier, Fulk, Lampi, and Williams, Phys. Rev. **78**, 508 (1950).

[14] E. D. Klema and A. O. Hanson, Phys. Rev. **73**, 106 (1948).

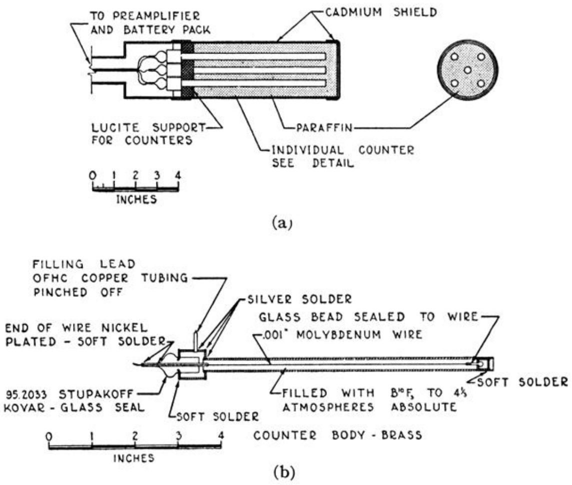

FIG. 1. Neutron detector. (a) Counter assembly. (b) Detail of single counter.