TABLE I. Calculated values of ζ_d from the multiplets of $3d^24s$ and $3d^3$.

	Term	ζ_d
	$a^4F_{3/2,5/2}$	112.73
	$a^4F_{5/2,7/2}$	112.74
	$a^4F_{7/2,9/2}$	111.83
	$b^4F_{3/2,5/2}$	91.01
	$b^4F_{5/2,7/2}$	88.64
	$b^4F_{7/2,9/2}$	85.58
$3d^24s$	a^2F	115.28
$3d^3$	a^2G	89.23
$3d^24s$	$b^4P_{1/2,3/2}$	115.74
	$b^4P_{3/2,5/2}$	112.80
$3d^3$	b^2D	82.15
$3d^3$	a^2H	88.93
$3d^24s$	b^2P	109.46
$3d^3$	b^2F	102.67

from the two ^4F terms we conclude that a^4F should be assigned to $3d^24s$ and b^4F to $3d^3$, in agreement with Russell's original assignments.

It is interesting to note that the levels of a^4F obey quite accurately Landè's interval rule, while those of b^4F deviate slightly from this rule. In order to see whether second-order effects can explain these deviations, the spin-orbit interactions were taken into account. By assuming that b^4F arises from $3d^3$, we found that the second-order effects brought about an improvement in the agreement with the interval rule, while by assuming that b^4F arises from $3d^24s$, the agreement becomes worse. This again verifies Russell's original assignments.

I am grateful to Professor G. Racah for suggesting the approach to the problem discussed in this paper.

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Remarks on the Variational Method for Scattering Problems

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IN a recent note[1] Huang has made an interesting analysis of the variational principle for continuous spectra.[2] The main point of his method is to replace the equation $\mathfrak{L}=0$ by

$$\partial\mathfrak{L}/\partial\lambda=k.$$

The last equation, involving derivatives of $\mathfrak{L}$ with respect to the phase parameter, is of the first degree in the parameters. By this means a system of solely linear equations is obtained, which greatly simplifies the numerical treatment. At the same time the ambiguity connected with the use of the quadratic equation $\mathfrak{L}=0$ is removed. It may be pointed out however that these inconveniences have been already overcome in a revised version of the principle,[3] which is more closely connected with familiar methods of the variational calculus.

As the condition $\mathfrak{L}=0$ has been given up in Huang's treatment the phase shift is ***not stationary***. This may explain the somewhat poor agreement of Huang's numerical phase shifts with the earlier values. On the other hand, the expression $\mathfrak{L}-k\lambda$ is stationary, whence a first-order correction for λ can be obtained from the equation

$$\delta\mathfrak{L}=k\delta\lambda,$$

[see reference 3, Section 4]. For the exact solution we have $\mathfrak{L}=0$, hence $\delta\mathfrak{L}=\mathfrak{L}$ for the approximate solution and

$$\delta\lambda=(\mathfrak{L}/k).$$

Therefore, if λ is the value obtained from Huang's equations, then

$$\lambda_0=\lambda-(\mathfrak{L}/k)$$

should be a better value. To check this the values λ_0 corresponding to the λ-values obtained by Huang were calculated. For comparison the figures are tabulated in Table I. It is clear that the results thus obtained agree very well with the values of the original paper although trial functions containing powers or polynomials do not seem to be ideal for this problem. Moreover, this manner of correcting the phase shifts involves very little extra work, since the expression for $\mathfrak{L}$ can be shortened considerably by the use of Euler's theorem for homogeneous functions and the equations satisfied by the parameters [see reference 3, p. 9].

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TABLE I. Comparison of the procedures of Huang and of Hulthén. The values of the parameters have been recalculated and show good agreement with the values obtained by Huang.* For comparison the last row contains Huang's numerical phase shift values.

$k=0.8,\quad l=-1.5$

	c_1, c_2	c_1, c_3	c_1, c_2, c_3	c_1, c_3, c_4	c_1, c_2, c_3, c_4	Most probable value for η_0 (ref. 2)
c_1	1.13543	0.03493	0.59699	0.05206	0.02261	
c_2	−0.56617	—	−0.28911	—	0.01502	
c_3	—	0.90190	0.44074	0.89689	0.92092	
c_4	—	—	—	−0.06516	−0.06417	
λ	1.11888	1.10430	1.11190	1.11525	1.11469	
$\mathfrak{L}/k$	0.00998	−0.00460	0.00303	0.00623	0.00566	
λ_0	1.10890	1.10890	1.10887	1.10902	1.10903	
η_0	0.83699	0.83699	0.83698	0.83705	0.83705	0.83708
η_H	—	0.83492	0.83832	0.83982	0.83958	

$k=0.8,\quad l=-2.1$

	c_1, c_2	c_1, c_3	c_1, c_2, c_3	c_1, c_3, c_4	c_1, c_2, c_3, c_4	Most probable value for η_0 (ref. 2)
c_1	4.75407	0.52837	4.87421	0.65668	0.61438	
c_2	−2.17329	—	−2.23510	—	0.02157	
c_3	—	3.47191	−0.09840	3.45159	3.48610	
c_4	—	—	—	−0.49604	−0.49462	
λ	3.27890	3.22313	3.28046	3.31117	3.31036	
$\mathfrak{L}/k$	0.00655	−0.05128	0.00811	0.02972	0.02892	
λ_0	3.27235	3.27441	3.27235	3.28144	3.28145	
η_0	1.27422	1.27447	1.27422	1.27499	1.27499	1.27515
η_H	—	1.26996	1.27489	1.27749	1.27742	

* There are some minor divergencies, e.g. c_1, c_2, c_3 and c_1, c_2, c_3, c_4 for $l=-2.1$. However, the corrected phase shifts η_0 are quite insensitive to these variations.

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Photo-Fission of Bismuth*

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WE report here some features of the fission of bismuth from irradiation at the University of Chicago betatron. The betatron was constructed by the General Electric Company and its constructural and operational features are essentially those of the 100-Mev instrument described by Westendorp and Charlton.[1] The machine was operated at a maximum energy of 85 Mev and the operating intensity was about 600 roentgens per minute, 1 meter from the tungsten target.

The fission of bismuth with high energy *particles* from the 184-in. Berkeley cyclotron has been demonstrated both by radiochemical and physical measurements.[2] The nature of the fission process with 190-Mev deuterons has been studied in detail by Goeckermann and Perlman[3] by radiochemical techniques. They found that the fission products formed are, for the most part, β^--decaying species in the light group, and β^+-decaying or