TABLE I. Calculated values of ζ_d from the multiplets of $3d^24s$ and $3d^3$.

	Term	ζ_d
	$a^4F_{3/2,5/2}$	112.73
	$a^4F_{5/2,7/2}$	112.74
	$a^4F_{7/2,9/2}$	111.83
	$b^4F_{3/2,5/2}$	91.01
	$b^4F_{5/2,7/2}$	88.64
	$b^4F_{7/2,9/2}$	85.58
$3d^24s$	a^2F	115.28
$3d^3$	a^2G	89.23
$3d^24s$	$b^4P_{1/2,3/2}$	115.74
	$b^4P_{3/2,5/2}$	112.80
$3d^3$	b^2D	82.15
$3d^3$	a^2H	88.93
$3d^24s$	b^2P	109.46
$3d^3$	b^2F	102.67

from the two ^4F terms we conclude that a^4F should be assigned to $3d^24s$ and b^4F to $3d^3$, in agreement with Russell's original assignments.

It is interesting to note that the levels of a^4F obey quite accurately Landè's interval rule, while those of b^4F deviate slightly from this rule. In order to see whether second-order effects can explain these deviations, the spin-orbit interactions were taken into account. By assuming that b^4F arises from $3d^3$, we found that the second-order effects brought about an improvement in the agreement with the interval rule, while by assuming that b^4F arises from $3d^24s$, the agreement becomes worse. This again verifies Russell's original assignments.

I am grateful to Professor G. Racah for suggesting the approach to the problem discussed in this paper.

[1] H. N. Russell, Astrophys. J. **66**, 283 (1927).
[2] A. Many, Phys. Rev. **70**, 511 (1946).
[3] H. N. Russell, Phys. Rev. **74**, 689 (1948).
[4] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1935), Chapter 57; H. H. Marvin, Phys. Rev. **47**, 521 (1935).

Remarks on the Variational Method for Scattering Problems

L. HULTHÉN AND P. O. OLSSON
Royal Institute of Technology, Stockholm, Sweden
May 31, 1950

IN a recent note[1] Huang has made an interesting analysis of the variational principle for continuous spectra.[2] The main point of his method is to replace the equation $\mathfrak{L}=0$ by

$$\partial\mathfrak{L}/\partial\lambda=k.$$

The last equation, involving derivatives of $\mathfrak{L}$ with respect to the phase parameter, is of the first degree in the parameters. By this means a system of solely linear equations is obtained, which greatly simplifies the numerical treatment. At the same time the ambiguity connected with the use of the quadratic equation $\mathfrak{L}=0$ is removed. It may be pointed out however that these inconveniences have been already overcome in a revised version of the principle,[3] which is more closely connected with familiar methods of the variational calculus.

As the condition $\mathfrak{L}=0$ has been given up in Huang's treatment the phase shift is *not stationary*. This may explain the somewhat poor agreement of Huang's numerical phase shifts with the earlier values. On the other hand, the expression $\mathfrak{L}-k\lambda$ is stationary, whence a first-order correction for λ can be obtained from the equation

$$\delta\mathfrak{L}=k\delta\lambda,$$

[see reference 3, Section 4]. For the exact solution we have $\mathfrak{L}=0$, hence $\delta\mathfrak{L}=\mathfrak{L}$ for the approximate solution and

$$\delta\lambda=(\mathfrak{L}/k).$$

Therefore, if λ is the value obtained from Huang's equations, then

$$\lambda_0=\lambda-(\mathfrak{L}/k)$$

TABLE I. Comparison of the procedures of Huang and of Hulthén. The values of the parameters have been recalculated and show good agreement with the values obtained by Huang.* For comparison the last row contains Huang's numerical phase shift values.

$k=0.8,\quad l=-1.5$

	c_1, c_2	c_1, c_3	c_1, c_2, c_3	c_1, c_3, c_4	c_1, c_2, c_3, c_4	Most probable value for η_0 (ref. 2)
c_1	1.13543	0.03493	0.59699	0.05206	0.02261	
c_2	−0.56617	—	−0.28911	—	0.01502	
c_3	—	0.90190	0.44074	0.89689	0.92092	
c_4	—	—	—	−0.06516	−0.06417	
λ	1.11888	1.10430	1.11190	1.11525	1.11469	
$\mathfrak{L}/k$	0.00998	−0.00460	0.00303	0.00623	0.00566	
λ_0	1.10890	1.10890	1.10887	1.10902	1.10903	
η_0	0.83699	0.83699	0.83698	0.83705	0.83705	0.83708
η_H	—	0.83492	0.83832	0.83982	0.83958	

$k=0.8,\quad l=-2.1$

	c_1, c_2	c_1, c_3	c_1, c_2, c_3	c_1, c_3, c_4	c_1, c_2, c_3, c_4	Most probable value for η_0 (ref. 2)
c_1	4.75407	0.52837	4.87421	0.65668	0.61438	
c_2	−2.17329	—	−2.23510	—	0.02157	
c_3	—	3.47191	−0.09840	3.45159	3.48610	
c_4	—	—	—	−0.49604	−0.49462	
λ	3.27890	3.22313	3.28046	3.31117	3.31036	
$\mathfrak{L}/k$	0.00655	−0.05128	0.00811	0.02972	0.02892	
λ_0	3.27235	3.27441	3.27235	3.28144	3.28145	
η_0	1.27422	1.27447	1.27422	1.27499	1.27499	1.27515
η_H	—	1.26996	1.27489	1.27749	1.27742	

* There are some minor divergencies, e.g. c_1, c_2, c_3 and c_1, c_2, c_3, c_4 for $l=-2.1$. However, the corrected phase shifts η_0 are quite insensitive to these variations.

should be a better value. To check this the values λ_0 corresponding to the λ-values obtained by Huang were calculated. For comparison the figures are tabulated in Table I. It is clear that the results thus obtained agree very well with the values of the original paper although trial functions containing powers or polynomials do not seem to be ideal for this problem. Moreover, this manner of correcting the phase shifts involves very little extra work, since the expression for $\mathfrak{L}$ can be shortened considerably by the use of Euler's theorem for homogeneous functions and the equations satisfied by the parameters [see reference 3, p. 9].

We are indebted to Mr. V. Grinvalds and Dr. H. Richter for skilled numerical assistance. Financial support from the Swedish Atomic Energy Committee is gratefully acknowledged.

[1] S. S. Huang, Phys. Rev. **76**, 1878 (1949).
[2] L. Hulthén, Kungl. Fysiogr. Sällsk. i Lund Förhandl. **14**, No. 21 (1944).
[3] L. Hulthén, Arkiv Mat. Astron. Fysik **35A**, No. 25 (1948).

Photo-Fission of Bismuth*

NATHAN SUGARMAN
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
June 2, 1950

WE report here some features of the fission of bismuth from irradiation at the University of Chicago betatron. The betatron was constructed by the General Electric Company and its constructural and operational features are essentially those of the 100-Mev instrument described by Westendorp and Charlton.[1] The machine was operated at a maximum energy of 85 Mev and the operating intensity was about 600 roentgens per minute, 1 meter from the tungsten target.

The fission of bismuth with high energy *particles* from the 184-in. Berkeley cyclotron has been demonstrated both by radiochemical and physical measurements.[2] The nature of the fission process with 190-Mev deuterons has been studied in detail by Goeckermann and Perlman[3] by radiochemical techniques. They found that the fission products formed are, for the most part, β^--decaying species in the light group, and β^+-decaying or

TABLE I. Photo-fission yields of bismuth.

Fission product	Photo-fission yield, %	Deuteron-fission yield,[d] %	Predicted total mass yield,[d] %
12-hr. Ge^{77}	~0.3	1.5	1.9
40-hr. As^{77}	<0.4		
35-hr. Br^{82}	<0.5	1.0	3.0
2.4-hr. Br^{83}	1.2	1.7	3.2
33-min. Br^{84}	<1.4	—	3.4
9.7-hr. Sr^{91}	2.8[a]	4.4[e]	4.6
2.7-hr. Sr^{92}	2.8[a]	—	4.7
17-hr. Zr^{97}	3.0	—	5.0
4.5-hr. Ru^{105}	5.0[b]	3.1	4.9
13-hr. Pd^{109}	~6.4[c]	4.6	4.6
7.5-day Ag^{111}	~2.8	3.4	4.2
5.3-hr. Ag^{113}	3.0	[f]	4.0
54-min. I^{134}	<0.2	—	0.2
85-min. Ba^{139}	<0.1	—	0.07

[a] Yields of 2.7-hr. Sr^{92} and 9.7-hr. Sr^{91} assumed equal for analysis of complex decay curve.
[b] Assumed yield, other yields normalized to this value.
[c] Uncertain because of contribution of 21-hr. Pd^{112}.
[d] See reference 3.
[e] Measured yield of 57-day Y^{91}.
[f] Not observed because of high independent yield of 3.2-hr. Ag^{112}.

K-electron capture species in the heavy group. The yield-mass curve is symmetric around mass 99 and the maximum fission yield is about 5 percent.

Baldwin and Klaiber[4] studied photo-fission in heavy elements at the 100-Mev betatron at Schenectady, using a balanced ionization chamber and thin sources of uranium, thorium, bismuth, lead, and other elements. No fissions were observed in the elements tested other than uranium and thorium. On the basis of a maximum cross section of 5×10^{-26} cm^2 for the photo-fission of uranium, a maximum value of 10^{-29} cm^2 was set for the cross section for photo-fission of bismuth.

In the present experiments, samples of bismuth nitrate and bismuth metal of about 30 g were irradiated with high energy photons for about 4 hr., about 55 cm from the tungsten target. The samples were dissolved after the irradiation and radiochemical analyses were made for various elements regarded as probable fission products of bismuth. Analyses were made for germanium, arsenic, bromine, strontium, zirconium, ruthenium, palladium, silver, iodine, and barium by standard procedures.[5] The isolated samples contained the radioactivities expected for fission except that no activity of iodine or barium was observed. The counting rates of the products formed in high yield were about 100 c/min., where the chemical yield was about 50 percent and the geometric factor for counting with the standard end-window bell-shaped counter was about 25 percent.

Detailed studies were made in the case of one of the isolated products, 4.5-hr. Ru^{105}, to establish that the observed radioactivity originated in the fission of bismuth and not in the activation of impurities by photons. From irradiations of ruthenium, rhodium, palladium, silver, and cadmium, it was found that either the impurity level of any of these elements had to be many times that known to be present in the bismuth in order to give the observed counting rate, or the ruthenium activity isolated had radioactive properties different from those of 4.5-hr. Ru^{105}.

The results of the measurements are given in Table I, where the fission yields of the species are arbitrarily normalized to a yield of 5 percent for Ru^{105}. For comparison purposes the yield values of Goeckermann and Perlman[3] for fission with 190-Mev deuterons are also given (column 3), as well as their predicted total yield values for the mass numbers (column 4).

Comparison of the photo-yield data with those of 190-Mev deuterons indicates that the yield-mass curve for the photo-fission of bismuth is narrower than that of deuterons and that the primary fission products are probably neutron-excessive and decay by β^--emission. This conclusion is based on the relatively low yield of mass 77 to Ru^{105}, and the presence of 5.3-hr. Ag^{113} in high yield relative to 3.2-hr. Ag^{112} formed independently of its 21-hr. Pd^{112} parent.

The above results show that fission of bismuth occurs at the betatron. However, it still remains to be established definitely that the fission process studied at the betatron is not the result of secondary fast particles. Rough estimates indicate that the fission process is more likely photo-fission, but because of the absence of information on the number and energy of neutrons and protons emitted from bismuth, particle-fission cannot be excluded. In this respect it should be mentioned that the irradiation of bismuth produces bismuth activities in high yield of about the same half-lives as those reported[6] for Bi^{204} and Bi^{206}. This suggests that the emission of several neutrons from bismuth occurs in high probability.

A single experiment was performed on the fission of uranium by photons. Several fission products were isolated including 85-min. Ba^{139}. If a fission yield of 6 percent is taken for Ba^{139}, the integrated cross section under the resonance is about 1×10^{-24} cm^2 Mev, roughly five times that found by Baldwin and Klaiber,[4] and is in better agreement with the Goldhaber-Teller[7] resonance theory. The fission rate per gram of bismuth is about 1/1000 that of uranium.

The author gratefully acknowledges the assistance of Messrs. C. R. McKinney and B. C. Cook of the Betatron Group in these experiments.

* Assisted by the joint program of the ONR and AEC.
[1] W. F. Westendorp and E. E. Charlton, J. App. Phys. **16**, 581 (1945).
[2] Perlman, Goeckermann, Templeton, and Howland, Phys. Rev. **72**, 352 (1947); E. L. Kelly and C. Wiegand, Phys. Rev. **73**, 1135 (1948).
[3] R. H. Goeckermann and I. Perlman, Phys. Rev. **76**, 628 (1949).
[4] G. C. Baldwin and G. S. Klaiber, Phys. Rev. **71**, 3 (1947).
[5] See papers of Part VI, Vol. 9, Div. IV, National Nuclear Energy Series, McGraw-Hill Book Company, Inc. New York (in preparation).
[6] I. Perlman and G. T. Seaborg, Rev. Mod. Phys. **20**, 585 (1948).
[7] M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948).

Polaron States in Ionic Crystals*

HERBERT B. CALLEN
Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
June 2, 1950

A CONDUCTION electron in a polar crystal induces an ionic polarization which, in turn, modifies the electronic wave function. The existence of this effect was first noted by von Hippel and Landau, and the contribution to the energy from this electron-ion interaction has been treated semiclassically by Pekar and by Markham and Seitz. Very recently Fröhlich, Pelzer, and Zienau[1] (F.P.Z.) have given a quantum-mechanical treatment of this energy and have also computed the effective mass of a polaron (electron plus surrounding ionic polarization). Despite the certain reality of the effect, however, no definite experimental observations have been correlated with it, and it would therefore seem desirable to survey the properties of polarons as they relate to experimental observability.

In a polar crystal the longitudinal optical phonons produce regions of positive and negative charge accumulation at alternate nodes. A conduction electron decreases its energy by concentrating its wave function at alternate nodes, thereby also altering the form and energy of the phonon; the net effect being a lowering of each of the states of the conduction band. As F.P.Z. have stressed, the polaron states are the only states in the conduction band; they are not additional states existing below the conduction band. Thus the usual proposal of observation of an infra-red absorption corresponding to optical excitation from polaron to conduction levels must necessarily fail. Furthermore, F.P.Z. have shown that the effective mass of a polaron is essentially equal to that of a conduction electron in a non-polar crystal, so that observations of mobility are not apt to be fruitful.

Consider the thermal excitation of an electron to the conduction band. After the excitation both electron and hole are surrounded by ionic polarization. If E_G is the gap width neglecting the polaron effect, and ΔE_t is the energy shift of an electron due to the polaron effect, the energy required for thermal excitation will thus be approximately $E_G-2\Delta E_t$. But ΔE_t, and hence the effective gap width, will depend upon temperature which is an effect which