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Reflection Properties of Spin $\frac{1}{2}$ Fields and a Universal Fermi-Type Interaction

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It is pointed out that four different transformations are possible under an inversion for fields of spin $\frac{1}{2}$. The consequences are discussed, and the bearing on the possibility of a universal Fermi-type interaction is analyzed.

I. INTRODUCTION

THE transformation property under an inversion of the space coordinates of the wave function of a particle of spin zero (or one) leads to the differentiation between scalar and pseudoscalar (or vector and pseudovector) particles. It is the purpose of the present note to point out that there is also a similar differentiation of the various spin $\frac{1}{2}$ particles. In contrast to the case of integral spin, however, there are for spin $\frac{1}{2}$ particles *four* different types of transformation properties under an inversion.

The types of transformation properties to which the various known spin $\frac{1}{2}$ fields belong are physical observables and could in principle be determined experimentally from their mutual interactions and their interactions with fields of integral spin. In practice this is, of course, very difficult. This problem is discussed in Section III in connection with β-decay and with the symmetry properties of the π-mesons.

In the final section, an attempt is made to see if by properly assigning the various known fields of spin $\frac{1}{2}$ to the four types one could have a universal Fermi-type interaction that would account for the experimental information accumulated in recent years about the interactions between spin $\frac{1}{2}$ fields.

II. GENERAL THEORY

We consider first the transformation property of a *single* particle under an orthochronous proper Lorentz transformation[1] (i.e., a Lorentz transformation involving neither time reversal nor space reflection). The wave function ψ undergoes[2] the following transformation

$$\psi' = S\psi. \tag{1}$$

The matrix S is defined only up to a $(\pm)$ sign. In particular, the identity transformation is represented by

$$S_0 = \pm I, \quad I = \text{unit matrix}. \tag{2}$$

This ambiguity of sign is *necessary* because a rotation through 360° about any axis always brings about a change of sign of the wave function of a spin $\frac{1}{2}$ particle.

The space inversion P is usually represented[2] by the transformation

$$\psi' = \pm\gamma_4\psi. \tag{3}$$

With this definition one might say that there is only *one* double-valued representation of the orthochronous Lorentz group for particles of spin $\frac{1}{2}$. This, however, leads to *two* different possibilities when the transformation properties of two different fields ψ_A and ψ_B are considered at the same time:

$$\begin{cases} \psi_A' = S\psi_A, \quad \psi_B' = S\psi_B & \text{for } L \text{ (orthochronous proper Lorentz transformation)} \\ \psi_A' = \pm\gamma_4\psi_A, \quad \psi_B' = \pm\gamma_4\psi_B & \text{for } P \text{ (space inversion)} \end{cases} \tag{4}$$

or

$$\begin{cases} \psi_A' = S\psi_A, \quad \psi_B' = S\psi_B & \text{for } L \\ \psi_A' = \pm\gamma_4\psi_A, \quad \psi_B' = \mp\gamma_4\psi_B & \text{for } P. \end{cases} \tag{5}$$

If case (4) holds, the fields A and B behave exactly alike under all orthochronous Lorentz transformations. This is customarily accepted as true for all spin $\frac{1}{2}$ fields. We see now that there is also possible the other case (5) in which the two fields A and B *always differ by a rotation of 360° under a space reflection.*

Racah[3] has pointed out that for the space inversion of a single particle it is also possible to have instead of Eq. (3),

$$\psi' = i\gamma_4\psi. \tag{6}$$

This introduces two other possible transformations for a spin $\frac{1}{2}$ field under a space inversion. Altogether, we would have four kinds of fields of spin $\frac{1}{2}$ that behave differently under an inversion. Denoting by ψ_A, ψ_B, ψ_C, and ψ_D four such fields one has under a space inversion

$$\begin{pmatrix} \psi_A' \\ \psi_B' \\ \psi_C' \\ \psi_D' \end{pmatrix} = \pm \begin{pmatrix} \gamma_4 & & & 0 \\ & -\gamma_4 & & \\ & & i\gamma_4 & \\ 0 & & & -i\gamma_4 \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_D \end{pmatrix}. \tag{7}$$

It is important to notice that with the simultaneous existence of the fields A, B with C, D the representation

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[1] This nomenclature is due to H. J. Bhabha, Rev. Mod. Phys. 21, 451 (1949).

[2] W. Pauli, *Handbuch der Physik*, Vol. 24/1, pp. 220–227.

[3] G. Racah, Nuovo Cimento 14, 322 (1937).

becomes four-valued. For example, the identity transformation corresponds now to

$$\begin{pmatrix} \psi_A' \\ \psi_B' \\ \psi_C' \\ \psi_D' \end{pmatrix} = \pm \begin{pmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_D \end{pmatrix} \quad \text{or} \quad \pm \begin{pmatrix} \psi_A \\ \psi_B \\ -\psi_C \\ -\psi_D \end{pmatrix}. \tag{8}$$

The necessity of the two latter possibilities arises from the fact that if the inversion (7) is applied twice one gets a representation of the identify transformation with a relative change of sign of the fields ψ_A, ψ_B and ψ_C, ψ_D.

It is also important to notice that there is no *intrinsic* difference between the A-type fields (i.e., fields with the transformation property of ψ_A) and the B-type fields, or between the C- and D-type fields. The difference is only relative, and is there only if both types of fields exist. If, for example, only C- (or D-) type fields exist, it is impossible to tell whether it is the C-type or the D-type. Further, it would not be appropriate, for example, to call the A-type fields "fields" and the B-type fields "pseudofields." This is to be contrasted with the case of fields with integral spin, for which there is an *intrinsic* difference between fields and pseudofields.

It is easy to show that the charge conjugate field of an A-type field is B-type, and *vice versa*. On the other hand, the charge conjugate field of a C-type field (D-type) is also C-type (D-type). This fact is summarized in Table I.

TABLE I. Field type of charge conjugate field.

Field (particle)	A	B	C	D
Charge conjugate field (antiparticle)	B	A	C	D

It should be remarked that the fact that the C- and D-type fields have the same transformation properties as their charge conjugate fields has led Racah[3] to the proposal that all spin $\frac{1}{2}$ fields transform like C-type fields.

With the Majorana theory, since the field and the charge conjugate field are identical, the C- and D-type transformations are the only possibilities.

III. APPLICATIONS TO ELECTRON AND MESON FIELDS

The five densities $\phi^*\gamma_4\psi$, $\phi^*\gamma_4\gamma_\mu\psi$, $\phi^*\gamma_4(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\psi$, $\phi^*\gamma_1\gamma_2\gamma_3\gamma_\mu\psi$ and $\phi^*\gamma_1\gamma_2\gamma_3\psi$ are usually referred to as scalar, vector, tensor, pseudovector, and pseudoscalar quantities.[4] This is evidently correct if $\phi=\psi$, whatever type of field ψ is. However, if ϕ and ψ are two different fields, and if they belong to different types, it is not correct, for example, to call $\phi^*\gamma_4\psi$ a scalar quantity. In fact, if ψ is an A-type field, while ϕ is a B-type field, then $\phi^*\gamma_4\psi$ is a pseudoscalar.

[4] See, for example, H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. 8, 190 (1936). We use the same notation for the $\boldsymbol{\alpha}$-, β-, and γ_μ-matrices as these authors.

In the β-decay theory, the following five interactions

$$\psi_P^*\beta\psi_N\psi_e^*\beta\gamma_5\psi_\nu, \tag{9}$$

$$\psi_P^*\psi_N\psi_e^*\gamma_5\psi_\nu-\psi_P^*\boldsymbol{\alpha}\psi_N\psi_e^*\boldsymbol{\sigma}\psi_\nu, \tag{10}$$

$$\psi_P^*\beta\boldsymbol{\sigma}\psi_N\psi_e^*\beta\boldsymbol{\alpha}\psi_\nu+\psi_P^*\beta\boldsymbol{\alpha}\psi_N\psi_e^*\beta\boldsymbol{\sigma}\psi_\nu, \tag{11}$$

$$\psi_P^*\gamma_5\psi_N\psi_e^*\psi_\nu-\psi_P^*\boldsymbol{\sigma}\psi_N\psi_e^*\boldsymbol{\alpha}\psi_\nu, \tag{12}$$

$$\psi_P^*\beta\gamma_5\psi_N\psi_e^*\beta\psi_\nu, \tag{13}$$

are usually rejected on the ground that they are not invariant under a space inversion. We see now that this is not justifiable in view of the fact that the types of fields to which ψ_P, ψ_N, ψ_e, and ψ_ν belong are not known. If it turns out that protons, neutrons, and electrons are all of type A, while the neutrino is of type B, then these five interactions are the invariant ones instead of the usual five.

If the mass of the neutrino is not zero, the interactions (9) to (13) lead to β-spectra different from that predicted by the ordinary interactions, especially near the upper end of the electron spectra. Also, the angular correlation between the electron and the neutrino would be different.

On the other hand, if the mass of the neutrino is zero, it is experimentally impossible to differentiate between the five possibilities (9) to (13) and the usual five (unless one could measure the neutrino spin). This is so because when the square of the matrix element of the interaction Hamiltonian is summed over the spin of the neutrino, the result for (9), for example, is the same as the result for the ordinary scalar-type of Fermi interaction, provided the mass of the neutrino is zero.

It is interesting to notice that the original proposal of Fermi[5] is identical with the interaction (10), rather than with the usual vector interaction. Hence, Fermi's spectrum is different from that of later authors for the case in which the mass of the neutrino is not zero, a fact already pointed out by Konopinski and Uhlenbeck.[6]

The usual statements about the meson theory of nuclear forces need to be understood in a new light now that, for example, $\psi_P^*\beta\psi_N$ can be a pseudoscalar. In particular, contrary to the usually accepted concept, a *scalar* meson *can* have a "pseudoscalar" and a "pseudovector" interaction with the nucleons:

$$iF\psi_P^*\gamma_4\gamma_5\psi_N\phi+iG\psi_P^*\gamma_4\gamma_5\gamma_\mu\psi_N(\partial\phi/\partial\chi_\mu).$$

It is also to be remarked that if the process

$$P\rightarrow N+\pi$$

occurs, it is impossible that the proton field be of type A or B, while the neutron field be of type C or D, or *vice versa*. This fact will be used later in Section IV.

IV. A UNIVERSAL FERMI-TYPE INTERACTION

An analysis of the phenomena of:

$$\beta\text{-decay}: N\rightarrow P+e+\nu, \tag{14}$$

[5] E. Fermi, Zeits. f. Physik. 88, 161 (1934).

[6] E. J. Konopinski and G. Uhlenbeck, Phys. Rev. 48, 7 (1935).

$$\mu\text{-capture:}^{7,8}\ P+\mu^- \rightarrow N+\nu, \tag{15}$$

and

$$\mu\text{-decay:}^{8,9}\ \mu \rightarrow e+2\nu \tag{16}$$

by assuming that the charged μ-meson[10] has spin $\frac{1}{2}$, and that the interactions which lead to these processes are of the Fermi-type, gives values of the coupling constants in the three interactions of the same order of magnitude.[11] This suggests the possibility of the existence of a universal interaction of the Fermi-type between *all* particles of spin $\frac{1}{2}$,[11] provided the requirement of conservation of charge is satisfied. The difficulty immediately arises that such a universal interaction would lead to processes inconsistent with experience, such as

$$N+P \rightarrow e+\nu \quad \text{or} \quad N \rightarrow e^+ + \mu^- + \nu. \tag{17}$$

To rule out these processes, one might propose[12] that some additional conservation laws other than the conservation of charge must be fulfilled. Indeed, with the proper assignment of the spin $\frac{1}{2}$ fields to the four types discussed in Section II, one might expect to obtain such conservation laws as consequences of the principle of invariance under a space inversion.

We have attempted to carry this out but have not succeeded in finding a completely satisfactory assignment. It turns out that some additional rather arbitrary tules will still have to be introduced. In the following, a brief account of our attempt is given, together with the results obtained.

A. The Universal Interaction

To make the proposal of a universal interaction quite definite, one has to choose a definite form of Fermi interaction. We have chosen the Wigner-Critchfield interaction[13]

$$H_{rstu} = g \int d^3\mathbf{x} \sum_{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \psi_\alpha{}^r(\mathbf{x}) \psi_\beta{}^s(\mathbf{x}) \psi_\gamma{}^t(\mathbf{x}) \psi_\delta{}^u(\mathbf{x}), \tag{18}$$

where $\epsilon_{\alpha\beta\gamma\delta} = \pm 1$ is antisymmetric with respect to its four indices, and $\psi_\alpha{}^r(\mathbf{x})$ is the α-component of the q-number spinor wave function of the rth field. To avoid taking into account the explicit occurrence of the complex conjugate of the wave functions in the interaction, we consider a field and its charge conjugate field as *two separate* fields. The reason that the Wigner-Critchfield interaction is chosen is that it is the *only* interaction that is symmetrical with respect to the four fields.[14] It is also evidently invariant under an orthochronous proper Lorentz transformation.

The universal interaction is formulated thus: *Between any four fields r, s, t, u, the interaction (18) exists with the same constant g provided it is consistent with charge conservation and is invariant under a space inversion, in the sense discussed in Section I.*[14a]

This last requirement we use to eliminate the undesired interactions, such as (17). For example, the term H_{rstu} would be absent in the Hamiltonian if the fields r, s, t are of type A and u of type B, C, or D, because in any of these cases H_{rstu} is not invariant under a space inversion.

B. Restrictions on the Assignments of Spin $\frac{1}{2}$ Fields to the Different Types

We want to allow the process

$$N \rightarrow P+e+\nu \tag{19}$$

which amounts to the same thing as

$$P \rightarrow N+\bar{e}+\bar{\nu} \tag{20}$$

where the bar over e and ν means the antiparticle. At the same time we want to forbid the process

$$N+P \rightarrow \bar{e}+\bar{\nu} \quad \text{or} \quad P \rightarrow \bar{N}+\bar{e}+\bar{\nu}. \tag{21}$$

This is only possible if N and $\bar{N}$ have different transformation properties, and from Table I we conclude that the neutron field is of type A or B. Similar reasoning, with the additional requirement that we want to forbid the process

$$N+P \rightarrow \bar{e}+\nu \quad \text{or} \quad P \rightarrow \bar{N}+\bar{e}+\nu, \tag{22}$$

leads to the conclusion that the neutrino is of type C or D. Now it is generally accepted that the π-meson has integral spin and interacts with the $N-P$ field. Therefore, the proton field, like the neutron field, must be of type A or B (see Section III). Consequently, from (19) one concludes that the electron is a C- or D-type field.

From an analogous analysis of the μ-capture, we conclude that μ also belongs to the C- or D-type field.

These assignments lead immediately to a *conservation law of heavy particles* which has been noticed by many physicists.

Another feature of this assignment is that it is consistent with the double β-decay,[15] with Majorana's theory of the neutrino,[16] and with the experiments[8] on μ-decay and π-decay.

C. Results

By writing down all the Fermi-type interactions consistent with charge conservation and continuing the

[7] B. Pontecorvo, Phys. Rev. **72**, 246 (1947).
[8] For more complete references to the literature see J. Tiomno and J. A. Wheeler, Rev. Mod. Phys. **21**, 144–153 (1949). See also, Taketani, Nakamura, Ono, and Sasaki, Phys. Rev. **76**, 60 (1949), and L. Michel, Nature **163**, 959 (1949). Proc. Phys. Soc. London (to be published).
[9] Leighton, Anderson, and Seriff, Phys. Rev. **76**, 159 (1949).
[10] J. Tiomno, Phys. Rev. **76**, 858 (1949).
[11] J. Tiomno and J. A. Wheeler, reference 8; Lee, Rosenbluth, and Yang, Phys. Rev. **75**, 905 (1949).
[12] This was pointed out by Professor E. Fermi in a seminar of about a year ago.
[13] C. L. Critchfield, Phys. Rev. **63**, 416 (1943).
[14] This is so because two spin $\frac{1}{2}$ fields which are not charge conjugate to each other anticommute.
[14a] One notices that the interaction Hamiltonian consistent with this proposal is Hermitian.
[15] E. L. Fireman, Phys. Rev. **75**, 323 (1949).
[16] E. Majorana, Nuovo Cimento **14**, 171 (1937).

above analysis it is found that unless some additional conditions are imposed one cannot eliminate all the undesired interactions (in particular, those indicated in footnotes 17 and 18). To be more definite, the conclusion is as follows:

The only two possible assignments are

$$(\alpha) \quad P, N\epsilon A, \quad \mu, e, \nu\epsilon C \tag{23}$$

where ϵ reads "belong to type." Notice that this assignment is identical, for example, with the assignment

$$P, N\epsilon B, \quad \mu, e, \nu\epsilon C. \tag{24}$$

(Compare Section II.) Or

$$(\beta) \quad P\epsilon A, \quad N\epsilon B, \quad \nu\epsilon C \quad \text{and} \quad \mu, e\epsilon D. \tag{25}$$

But the additional restrictions will have to be imposed that (a) all terms in which a field and its charge conjugate field appear,[17] and (b) all terms in which four identical fields appear[18] are to be excluded from the Hamiltonian.

If experimental results should show the existence of another neutral spin $\frac{1}{2}$ particle μ_0 such as in[8]

$$P+\mu^- \rightarrow N+\mu_0, \tag{26}$$

and[8, 9]

$$\mu \rightarrow e+\mu_0+\nu, \tag{27}$$

it would be straightforward to include μ_0 in the present scheme of considerations.

The authors wish to thank Professor J. R. Oppenheimer, Professor E. P. Wigner, and Dr. A. Wightman for helpful discussions.

[17] This is to forbid such processes as $P+\mu \rightarrow P+e$ which contradicts the experimental result that no electrons are emitted in the capture of the μ^--meson by heavy nuclei.

[18] This is to forbid such processes as $N+N \rightarrow \bar{N}+\bar{N}$ which would lead to the instability of complex nuclei.

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The Origin of Cosmic Rays

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The original idea of Menzel and Salisbury concerning the origin of cosmic rays has been extended and some of its possible consequences worked out in more detail. It is concluded that low frequency electromagnetic waves (a few cycles per second) may exist in limited regions near the outer edge of the solar corona, and could accelerate ions to cosmic-ray energies. An attempt is made to explain both the "ordinary" cosmic rays and the intensity increases following solar flares in terms of the action of these waves.

I. INTRODUCTION

In a very interesting paper Menzel and Salisbury[1] have proposed a mechanism for the origin of cosmic rays, in which the active agent is assumed to be low frequency electromagnetic radiation from the sun. These waves, with frequencies of a few cycles per second, would probably be difficult to detect at the earth's surface because of their nearly total reflection by the ionosphere; there is also some difficulty concerning their propagation through interplanetary space. This comes from the fact that the refractive index of a medium containing N free particles of charge e and mass m per cubic centimeter, traversed by a wave of angular frequency ω, is equal to $(1-4\pi Ne^2/m\omega^2)^{\frac{1}{2}}$. If the usual assumption is made that interplanetary space contains at least one free electron per cubic centimeter, one finds that the refractive index becomes imaginary, leading to total reflection of the waves, for frequencies less than 9 kilocycles per second.

This situation is not improved by relativistic effects; the results of Section II of this paper show that electrons starting from rest move so that the relation between their displacement and the phase of the wave is identical with that given by classical laws, so that the refractive index formula requires no relativistic correction, except for that imposed by the initial velocities of the electrons which will not change the order of magnitude of the low frequency transmission limit.

In spite of these difficulties, it seems worth while to examine the possibilities of this type of mechanism in more detail. The initial problem is to find the relativistic motion of ions under the influence of such waves, which is done in the following section.

II. MOTION OF IONS IN A PLANE WAVE

It is desired to find the motion of a charged particle in a plane polarized electromagnetic wave. (The more general case of arbitrary polarization with different time variations of the components leads to a very similar solution; since it does not alter any of the conclusions, this extra complication has been omitted.) Let the wave be moving in the positive x-direction with the electric vector in the y-direction, and let the electric field at a fixed point vary with time like $(1/e)F(t)$, where e is the charge on the particle. The field com-

[1] D. H. Menzel and W. W. Salisbury, Nucleonics 2, No. 4, 67 (1948).