First-Forbidden Beta-Spectra and the Beta-Spectrum of $Sr^{90}-Y^{90}$

L. JACKSON LASLETT, E. N. JENSEN, AND A. PASKIN
Institute for Atomic Research and Department of Physics, Iowa State College, Ames, Iowa*
May 25, 1950

SEVERAL investigations supporting the applicability of the correction factor $(W_0-W)^2+(W^2-1)$ to first-forbidden beta-spectra have been reported[1] recently. The theoretically expected shape of such spectra, as reported by Marshak[2] and by Greuling,[3] suggests that a more precise form for this correction factor would be $a=(W_0-W)^2+\Lambda(W^2-1)$, where the coefficient Λ is not strictly independent of the electron momentum and may differ appreciably from unity when Z is large.

In terms of Greuling's notation,[3]

$$\Lambda=[(S_1+2)/(2S_0+2)][F_1/F_0];$$

a plot[4] of Λ as a function of electron momentum is shown in Fig. 1, for $Z=40$ and $Z=60$. Although the introduction of the coefficient Λ may result in no more than a very small modification of the spectral form expected for a first-forbidden beta-transition when Z is in the neighborhood of 40, recent work[5] here with Pr^{142} has indicated that the introduction of this coefficient can result in a noticeable change in the spectral shape of $Z=60$. This note is submitted at this time with the thought that the application of the modified correction factor indicated here may be of interest to other investigators.

In a previous letter[6] the beta-spectra of Sr^{90} and Y^{90} were reported as of the first-forbidden form discussed above. An indication of the rather small improvement obtained in this case by including the coefficient Λ in a comparison of the experimental data with theory is afforded by the following weighted r.m.s. relative differences between the theoretical curves and the individual counting rates observed in the Y^{90} spectrum (for the interval $2.079\leqslant W\leqslant 5.256$): with $\Lambda=1$, r.m.s. deviation = 1.5 percent; with Λ from Fig. 1, r.m.s. deviation = 0.8 percent; and from statistical counting error, expected r.m.s. deviation = 0.6 percent.

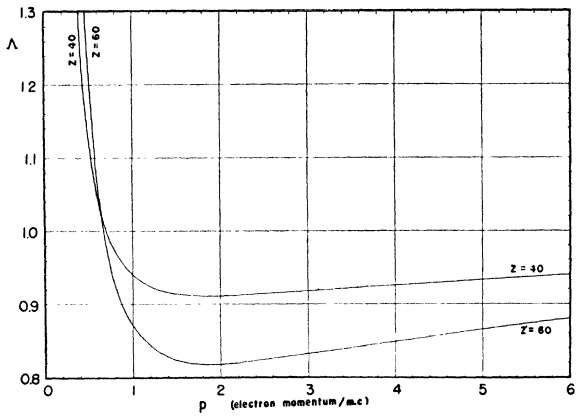

FIG. 1. Graph of the coefficient Λ, which appears in the correction factor $a=(W_0-W)^2+\Lambda(W^2-1)$.

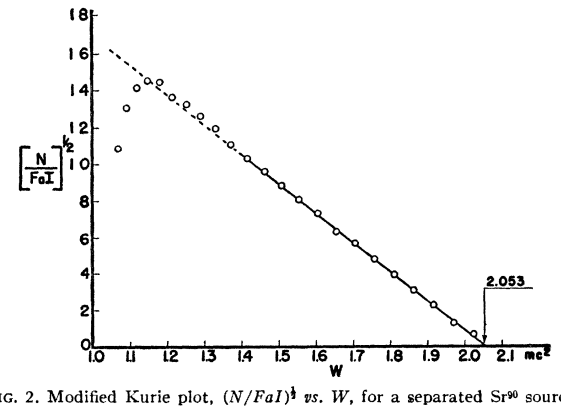

FIG. 2. Modified Kurie plot, $(N/FaI)^{\frac{1}{2}}$ *vs.* W, for a separated Sr^{90} source.

A more adequate separated source of Sr^{90} was obtained[7] since our first work on this activity was performed and a modified Kurie plot of the more recent data is shown in Fig. 2 to supplement the results previously reported.[6] Upper limits of 2.24 and 0.54 Mev (kinetic energy) are obtained for the Y^{90} and Sr^{90} beta-spectra, respectively.

The *ft*-values were calculated with the more precise correction factor a included with the Fermi function in the integral expression for f. Values of 1.7×10^9 and 2.0×10^9 were obtained for Sr^{90} (19.9 yr.[8]) and Y^{90} (62 hr.), respectively.

* Contribution No. 104. The work was performed at the Ames Laboratory of the AEC.

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[6] E. N. Jensen and L. J. Laslett, Phys. Rev. **75**, 1949 (1949).

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The Effect of Nuclear Structure on the Elastic Scattering of Fast Electrons

L. R. B. ELTON
University College, London, England
May 22, 1950

THE cross section for the scattering of electrons by atomic nuclei has been investigated at energies for which the nuclei can no longer be treated as point charges. Three nuclear models were used: (A) point charge, (B) uniform spherical charge distribution of radius R, (C) uniform spherical shell charge distribution of radius R. The radius R is taken to be[1] $R=e^2A^{\frac{1}{3}}/2mc^2$, in the usual notation.

Preliminary calculations, based on Born's approximation, showed that significant deviations from the cross sections using a point nucleus should occur for a light element, such as Al, at about 30 Mev, and for a heavy element, such as Au, at about 15 Mev. For Al, Born's approximation is valid,[2] but for Au it is not. Hence an exact calculation without any approximation was carried out for Au and 20-Mev electrons by calculating the phases for the lower order partial cross sections, using models (B) and (C). It was found that only the zero-order phases $(\eta_0-\eta_{-2})$ were significantly different for model (A) and for models (B) and (C). The calculations for model (A) by Bartlett and Watson[3] (these were done for Hg, but are sufficiently accurate also for Au) were then modified accordingly in the zero-order phases. The ratio $R_{B,C}=I_{B,C}(\theta)/I_A(\theta)$ of the differential cross section for an extended nucleus to that of a point nucleus is given in Table I, where the values for Al are calculated by Born's approximation and those for Au are calculated exactly. The energy has been adjusted to 16.5 Mev, so that comparison can be made with the experimental results,[4] R_{exp}. Because of the electrostatic repulsion of the charges inside the nucleus, the charge density is likely to increase with the radius, and so the true distribution is likely to be somewhere

TABLE I. Ratio of the differential cross section for an extended nucleus to that for a point nucleus for 16.5-Mev electrons being scattered by Al and Au nuclei.

	$\theta=$	30°			90°			150°		
Z		R_B	R_C	R_{exp}	R_B	R_C	R_{exp}	R_B	R_C	R_{exp}
Al	13	1.00	0.99	0.97	0.97	0.95	0.75	0.89	0.84	0.94
Au	79	0.99	0.97	0.88	0.45	0.30	0.45	0.29	0.19	0.38

between models (B) and (C). R_{exp} would therefore be expected to lie between R_B and R_C. It is seen that in view of the preliminary nature of the experimental results agreement is quite satisfactory.

A full account of this investigation will be published in the *Proceedings of the Physical Society*.

The author wishes to thank Professor H. S. W. Massey for his suggestion of this problem and general guidance of the work.

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Stopping Power of K-Electrons

H. A. BETHE, L. M. BROWN, AND M. C. WALSKE
Cornell University, Ithaca, New York
May 25, 1950

AN expression for the contribution B_K of the K-electrons to the stopping number has been given by Livingston and Bethe[1] in the form

$$B_K(\theta,\eta)=A(\theta)\ln\eta+B(\theta)-C_K(\theta,\eta), \quad [\eta=(mv^2/2Z_{eff}^2Ry)\gg 1]. \tag{1}$$

Here θ is the ratio of the observed ionization potential to the "ideal ionization potential" Z_{eff}^2Ry, m is the electronic mass, and v is the velocity of the incident particle; $A(\theta)$, $B(\theta)$, and $C_K(\theta,\eta)$ can be computed directly by using the excitation and ionization probabilities of the K-shell.[1,2] The coefficient of the $\log\eta$ term is described in reference 1 as "the total oscillator strength of all optical transitions from the K-shell into the continuous spectrum (and to the unoccupied discrete levels)," and the formula has been applied, with this interpretation of $A(\theta)$, to higher atomic shells.[3] It is the purpose of this letter to show that this coefficient, $A(\theta)$, is actually $1+f$ where $2f$ is the above-mentioned total oscillator strength.

The Born approximation expression for the stopping number (per K-electron) is

$$B=\sum_n{}'\int_{q_{\min(n)}}^{q_{max}}(dq/q)\varphi_n(q), \tag{2}$$

with $\varphi_n(q)=(E_n-E_1)|(e^{iqz})_{1n}|^2/q^2$, E_n being the energy of the nth atomic state and q^2 being the square of the momentum loss of the incident particle divided by $2m$ (both E_n and q^2 measured in units of Z_{eff}^2Ry). Energy-momentum considerations give $q_{max}=(4\eta)^{\frac{1}{2}}$ and $q_{min}=(E_n-E_1)(4\eta)^{-\frac{1}{2}}$; Σ_n' indicates the summation over all unoccupied states. Letting q_1 be a small value of q, independent of n and η, and $q_0=q_{min}(n=\infty)=(4\eta)^{-\frac{1}{2}}$, we can write

$$B=\sum_{\text{all states}}\int_{q_0}^{q_{max}}\varphi_n(q)dq/q - \sum_{\text{occupied levels}}\left[\int_{q_0}^{q_1}\varphi_n(0)dq/q+\int_{q_1}^{\infty}\varphi_n(q)dq/q\right] - \Sigma'\int_{q_0}^{q_{\min(n)}}\varphi_n(0)dq/q. \tag{3}$$

This expression is correct, except that in the second and last term $\varphi_n(q)$ has been replaced by $\varphi_n(0)$, neglecting higher terms in q^2 because q_0, q_1, and $q_{min}(n)$ are all small for large η. In the third term, the upper limit q_{max} has been replaced by ∞ because, for occupied states, $\varphi_n(q)$ decreases as q^{-14} with increasing q.

To evaluate the first term, we change the order of summation and integration and use the well-known rule[2] that $\Sigma\,\varphi_n(q)=1$; then we obtain simply $\ln(q_{max}/q_0)=\ln 4\eta$. Since q_0 and q_1 are both independent of n, we obtain for the second term

$$-(1-f)\ln(q_1/q_0)=-\tfrac{1}{2}(1-f)\ln 4\eta-(1-f)\ln q_1$$

where $f=\Sigma'\varphi_n(0)$ is the total oscillator strength per electron for transitions going to unoccupied states. Since q_1 is independent of η and since the third integral converges at $q=\infty$, it yields a result independent of η. The last term gives

$$-\Sigma'\,\varphi_n(0)\ln(q_{min}/q_0)=-\Sigma'\,\varphi_n(0)\ln(E_n/E_1)$$

which is also independent of η. The η-dependent part of B arises therefore from the first and second terms alone and is:

$$\ln\eta-\tfrac{1}{2}(1-f)\ln\eta=\tfrac{1}{2}(1+f)\ln\eta. \tag{4}$$

It is not possible to calculate $B(\theta)$ by this general method, but if we write Eq. (1) in the form

$$B_K(\theta,\eta)=(1+f)\ln(2mv^2/\lambda Z_{eff}^2Ry)-C_K(\theta,\eta), \tag{5}$$

a Born approximation calculation[2] using hydrogenic wave functions gives the results displayed in Table I. The first line of Table I refers to the complete hydrogen atom, the other lines to the K-shells of heavier atoms, θ increasing with Z. It is remarkable that λ is so very nearly constant; note that this makes the "effective ionization potential" close to Z_{eff}^2Ry, not to the observed ionization potential. It may also be useful to note that f is very nearly inversely proportional to θ^2 (last column of Table I).

TABLE I. Calculated results for hydrogenic wave functions.

	λ	f	$\theta^2 f$
Hydrogen	1.1018	1	——
$\theta=0.7$	0.9542	0.813	0.398
0.75	0.9392	0.722	0.406
0.8	0.9295	0.646	0.413
0.9	0.9207	0.525	0.425

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Decay of Au^{199}

R. D. HILL
Physics Department, University of Illinois, Urbana, Illinois*
May 29, 1950

AN estimate[1] given for the capture cross section of Au^{198} for slow neutrons was based on the intensity of a 159-kev transition from Au^{199}. A decay scheme for Au^{199} devised by Beach, Peacock, and Wilkinson[2] was used and a value of 3.5×10^4 barns was obtained for the cross section.

The radiations from a Au^{199} source[3] separated from Pt^{199} have since been studied and the complete identity of the spectra from Au^{199} produced by both methods has been shown. Further analysis of the Au^{199} electron conversion spectrum is given in Table I. Other lines present in the spectrum are due to the conversion of mercury K x-rays (~70 kev) and L x-rays (~10 kev). If a 230-kev γ-ray is present, as claimed by Beach *et al.*,[2] it can be estimated from the absence of its K conversion line that its intensity is less than 15 percent of the 158.5-kev transition.

The N_K/N_L ratio of 0.37 for the 158.5-kev transition suggests a 2^3-pole electric transition, for which according to Hebb and Nelson the theoretical value is 0.4. However, the N_{L111}/N_{L1} ratio for electric 2^3, according to Goodrich and Drell,[4] is of the order of 20, whereas the experimental value is $0.55\pm<0.1$. Nor will a combination of 2^2 magnetic and 2^3 electric transitions assist in bringing these values into closer accord with theory. A similar conflict for