conductivity. Assuming, however, the existence of such currents and their stability at very low temperatures, it is natural to suppose that superconductivity cannot exist above a certain critical temperature T_c at which certain higher modes of the lattice become dominant in the lattice energy density spectrum. It was with this in mind that Dolecek and de Launay[1] sought to display a pattern for superconducting elements by plotting the critical temperature T_c against the Debye characteristic temperature, Θ_D.

For an element, the lattice spectra of the various isotopes at a given temperature will differ because of the different atomic weights, A. Thus the spectra will be similar, for the various isotopes of the element, at different temperatures. These spectra will be similar when the wave-length bearing the greatest energy density is the same for the various isotopes. Thus, this dominant wave-length can be used to identify the spectra and to determine the variation of T_c with A.

To determine how the dominant wave-length depends on temperature, in Debye's model of a solid, we have only to compare Debye's formula for the energy of a lattice with Planck's formula for the energy of blackbody radiation. The calculation of the dominant wave-length λ for blackbody radiation from Planck's formula leads to Wien's displacement law: $\lambda T = hc/4.965k$, where c is the velocity of light, h is Planck's constant, and k is Boltzmann's constant. The same method of analysis applied to Debye's formula also leads to Wien's displacement law where c, in the Debye case, is the velocity of sound. This relationship probably holds even for real lattices in the region of the T^3 law of specific heats.

The velocity of sound, c, besides being a function of the elastic constants and atomic volume, varies as $A^{-\frac{1}{2}}$. The elastic constants, atomic volume, and the required critical lattice spectrum characterized by the dominant wave-length λ, will not vary among the isotopes of a given element since they depend essentially on electron-ion configuration. Consequently, for the isotopes of the same element:

$$A^{\frac{1}{2}}T_c = \text{constant}. \qquad (1)$$

If T_c is plotted as ordinate and A as abscissa, then in the short range over which A can be varied physically, Eq. (1) will be represented by a straight line of slope $(-T_c/2A)$. For Pb, Sn, and Hg, this slope should be about -0.0175, -0.0156, -0.0103 °K/atmos. wt., respectively.

Recently, Maxwell[2] and Reynolds, Serin, Wright, and Nesbitt[3] have observed a shift in the temperature T_c with change in A for the isotopes of mercury. The second of these two papers presents the data of both in a plot of T_c *versus* A. The result is a straight line of slope -0.009. This result is not only in agreement with Eq. (1) in sign but is also in reasonable numerical agreement.

Referring, finally, to a T_c *versus* Θ_D plot,[1] the isotope effect of any one of the superconducting elements would be represented by a straight line through the origin and the representative point of the element. Since T_c/Θ_D is roughly constant for the set of elements Al, Cd, Ga, Hf, Ti, Zn, and Zr, it would appear that this group of elements has similar electron-ion properties insofar as the dominant wave-length λ is involved.

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Photo-Alpha-Reactions in Light Nuclei*

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June 5, 1950

A RATHER direct test of the alpha-particle picture of nuclear structure should be afforded by the study of reactions in which gamma-rays eject alpha-particles from nuclei such as C^{12}, O^{16}, and Ne^{20}. We consider the oxygen reaction, which seems to be essentially simpler. For gamma-rays between 7.2 and 12.1 Mev, alpha-emission will occur without competition, and a study of the angular distribution and energy dependence of the reaction is of interest. Millar and Cameron[1] have reported observing this reaction, while Hänni, Telegdi, and Zünti[2] and Wäffler and Younis[3] have observed the disintegration of C^{12} into three alphas. In this work we more or less follow the calculation of Telegdi and Verde[4] on the C^{12} cross section.

We assume, in the sense of the resonating group picture, that the O^{16} nucleus is composed of four pre-formed alpha-particles, each strongly bound but interacting relatively weakly with its neighbors. We further assume that a gamma-ray of energy above 7.2 Mev can excite this structure to a state in its continuum, in which one alpha emerges. For lack of better knowledge we take the ground state wave function to describe an alpha-particle bound, in an S state, to a cluster of three alpha's. The order of magnitude of the final answer is probably not particularly sensitive to the details of this wave function, except for a parameter specifying the approximate range of radii within which the single alpha is likely to be found. We assume an exponential distribution of the form $e^{-r/R}$, and leave R as an adjustable parameter.

For the final state we assume a Coulomb distorted plane wave for the wave function in the relative coordinates. That is, we neglect the effect of the nuclear forces on the wave functions of the continuum states. Because of the exact coincidence of the charge center and mass center in our assumed picture, the transition cannot be electric dipole. Because of the fact that the ground state is spherically symmetric, no magnetic dipole matrix element exists. The transition will then be pure electric quadrupole, so that only the D component of the final wave function will be involved in the calculation of the matrix element. This circumstance makes more plausible the neglect of nuclear forces in the final wave function.

These assumptions lead directly to a formula for the differential cross section:

$$\sigma(\theta)d\Omega = 0.88(R/r_0)''(4+\eta^2)(1+\eta^2)\epsilon^3\eta^{-4} \times (e^{2\pi\eta}-1)^{-1}\sin^2\theta\cos^2\theta d\Omega \text{ (barns)},$$

where R = length parameter previously introduced, r_0 = classical electron radius, $\epsilon = \gamma$-ray energy in Mev, ϵ_0 = threshold energy $= 7.2$ Mev, and $\eta = 3.26\,(\epsilon - \epsilon_0)^{-\frac{1}{2}}$.

The energy dependence should be moderately reliable near the threshold, and the angular dependence follows in a very simple manner from the fact that a quadrupole transition is involved. The extremely sensitive dependence on R means that no prediction of the magnitude of the cross section will be reliable, but it may well be that R can be determined nicely from the results of the experiments. The somewhat unusual angular distribution which we have obtained seems to be peculiar to the alpha-particle model, since in a nuclear picture without alpha-particle structure, a sizable electric dipole cross section must also be expected, with a resulting drastic change in angular distribution.

* This work supported in part by the ONR.
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Equivalence Theorems for Meson-Nucleon Couplings

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June 6, 1950

THE degree to which the scalar (pseudoscalar) and vector (pseudovector) couplings of the meson and nucleon fields are equivalent in scalar (pseudoscalar) meson theory has been discussed several times in the literature. Pointing out the incompleteness of earlier treatments[1,2] (which stressed only the equivalence of the couplings in first order), Case,[3] in a paper of the above title, set the problem up generally in order to find the exact criteria of equivalence.

The Lagrangian (in Case's notation)

$$\mathfrak{L}=-\hbar c\bar{\psi}[\gamma_\nu(\partial\psi/\partial x_\nu)+\kappa_0\psi]+ie\bar{\psi}\gamma_\nu\tfrac{1}{2}(1-\tau_3)\psi A_\nu -[(D_\nu^*\varphi^*)(D_\nu\varphi)+\kappa^2\varphi^*\varphi]-(R\varphi^*+R^*\varphi) -[\Gamma_\nu(D_\nu^*\varphi^*)+\Gamma_\nu^*(D_\nu\varphi)] \quad (1)$$

describes a charged scalar (pseudoscalar) meson field with both types of coupling to the nucleon field, both fields being coupled to an external potential A_ν. The interaction Hamiltonian is derived from $\mathfrak{L}$ and the derivative is removed from the meson field variables by the rearrangement;[1]

$$\Gamma_\nu\frac{\partial\varphi^*}{\partial x_\nu}+\Gamma_\nu^*\frac{\partial\varphi}{\partial x_\nu}=\frac{\partial}{\partial x_\nu}(\Gamma_\nu\varphi^2+\Gamma_\nu^*\varphi)-\frac{\partial\Gamma_\nu}{\partial x_\nu}\varphi^*-\frac{\partial\Gamma_\nu^*}{\partial x_\nu}\varphi, \quad (2)$$

where $\partial\Gamma_\nu/\partial x_\nu$ can be evaluated from the Dirac field equations. Case then removes the divergence expression by a sequence of canonical transformations,[2] evaluating the transformed Hamiltonian to the second order in the meson-nucleon coupling constants.

We wish to point out only that the same rearrangement (2) can be performed directly in the Lagrangian, where the divergence expression can be dropped immediately, as it does not contribute to the equations of motion. This method has the advantages that, (1) no further canonical transformations are required beyond that needed to enter the interaction representation, and (2) there is no need to approximate the resulting Hamiltonian, the criteria of equivalence being obtained exactly. From the new Lagrangian

$$\mathfrak{L}'=-\hbar c\bar{\psi}[\gamma_\nu(\partial\psi/\partial x_\nu)+\kappa_0\psi]+ie\bar{\psi}\gamma_\nu\tfrac{1}{2}(1-\tau_3)\psi A_\nu -[(D_\nu^*\varphi^*)(D_\nu\varphi)+\kappa^2\varphi^*\varphi]-(R\varphi^*+R^*\varphi) -(ie/\hbar c)(\Gamma_\nu\varphi^*-\Gamma_\nu^*\varphi)A_\nu+(ig/\kappa)[(\partial\bar{\psi}/\partial x_\nu)\gamma_\nu\gamma_5' \times(\tau_+\varphi^*+\tau_-\varphi)\psi+\bar{\psi}\gamma_\nu\gamma_5'(\tau_+\varphi^*+\tau_-\varphi)(\partial\psi/\partial x_\nu)], \quad (3)$$

we find the same field equations as before and the momenta

$$\pi\equiv-\frac{\partial\mathfrak{L}'}{\partial(\partial\psi/\partial x_\nu)}n_\nu=\hbar c\bar{\psi}\gamma_\nu n_\nu-\frac{ig}{\kappa}\bar{\psi}\gamma_\nu n_\nu\psi_5'(\tau_+\varphi^*+\tau_-\varphi),$$

$$\bar{\pi}\equiv-\frac{\partial\mathfrak{L}'}{\partial(\partial\bar{\psi}/\partial x_\nu)}n_\nu=-\frac{ig}{\kappa}\gamma_\nu n_\nu\gamma_5'(\tau_+\varphi^*+\tau_-\varphi)\psi,$$

$$\chi\equiv-\frac{\partial\mathfrak{L}'}{\partial(\partial\varphi/\partial x_\nu)}n_\nu=n_\nu(D_\nu^*\varphi^*),$$

$$\chi^*\equiv-\frac{\partial\mathfrak{L}'}{\partial(\partial\varphi^*/\partial x_\nu)}n_\nu=n_\nu(D_\nu\varphi).$$

We then construct the interaction part of the total Hamiltonian

$$T_{\mu\nu}n_\mu n_\nu=-\mathfrak{L}'+\pi\left(n_\nu\frac{\partial\psi}{\partial x_\nu}\right)+\bar{\pi}\left(n_\nu\frac{\partial\bar{\psi}}{\partial x_\nu}\right)+\chi\left(n_\nu\frac{\partial\varphi}{\partial x_\nu}\right)+\chi^*\left(n_\nu\frac{\partial\varphi^*}{\partial x_\nu}\right).$$

The result is more simply expressed in the interaction representation, which is characterized by the transformations

$$\pi\rightarrow\hbar c\bar{\psi}\gamma_\nu n_\nu,\quad \bar{\pi}\rightarrow 0,\quad \chi\rightarrow n_\nu\partial\varphi^*/\partial x_\nu,\quad \chi^*\rightarrow n_\nu\partial\varphi/\partial x_\nu.$$

In this representation the interaction Hamiltonian is

$$\mathfrak{H}=-ie\bar{\psi}\gamma_\nu\tfrac{1}{2}(1-\tau_3)\psi A_\nu-\frac{ie}{\hbar c}\left(\frac{\partial\varphi^*}{\partial x_\nu}\varphi-\varphi^*\frac{\partial\varphi}{\partial x_\nu}\right)A_\nu +(e/\hbar c)^2\varphi^*\varphi A_\mu A_\nu(\delta_{\mu\nu}+n_\mu n_\nu)+[1+(\epsilon-1)\kappa_0 g/\kappa f] \times(R\varphi^*+R^*\varphi)+(fg/\kappa\hbar c)\bar{\psi}[(1+\tau_3)-\epsilon(1-\tau_3)]\psi\varphi^*\varphi +(g^2/\kappa^2\hbar c)\bar{\psi}\gamma_\nu\tau_3\psi[(\partial\varphi^*/\partial x_\nu)\varphi-\varphi^*(\partial\varphi/\partial x_\nu)] +(2ieg^2/\kappa^2\hbar^2c^2)\bar{\psi}\gamma_\mu\tau_3\psi\varphi^*\varphi A_\nu(\delta_{\mu\nu}+n_\mu n_\nu), \quad (4)$$

where $\epsilon=1$ (scalar theory), -1 (pseudoscalar theory).

This result agrees with that of Case to first order; the remaining terms are similar to his, differing only by a canonical change of representation. It is to be noted that in the interaction representation the vector coupling is represented by terms of no higher explicit order than the second, while in the representation used by Case, explicit terms of all orders presumably appear. In addition, our result shows that scalar theories can be so formulated that contact terms *never* appear. That contact terms play no role in vector theories also can be inferred from the fact that they are explicitly surface-dependent terms, and will therefore disappear along with other surface terms when the first-order coupling is removed from the Hamiltonian by means of the Schwinger transformation.[4]

The author wishes to thank Dr. M. L. Goldberger for much helpful advice.

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The Radioactivity of Hf^{181}

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June 1, 1950

A CONSIDERABLE number of investigations of the decay of Hf^{181} have been described in the literature. Most of these have been interpreted to support a disintegration scheme of the general type first proposed by Wiedenbeck and Chu[1] with minor modifications such as the scheme recently shown by Cork *et al.*[2] We have investigated some features of this decay by means of magnetic and scintillation spectrometers and coincidence techniques. The main new result is that most of the 345-kev gamma-radiation does not belong to the 45-day Hf^{181}. Delayed coincidence measurements in which the succeeding radiation was focused by a magnetic lens spectrometer showed that less than 10 percent of the conversion electrons of the 345-kev transition follow the 20-μsec. period while all of the electrons from the 132- and 481-kev transitions follow it. A similar result is obtained when the delayed radiation spectrum is studied by means of a NaI—TlI scintillation spectrometer. Because of our limited experience with the instrument it has not yet been possible to determine whether any 345-kev radiation at all appears with the decay of the metastable state, but it could only be a small part of the total.

While the conversion electrons of the 481-kev transition show "prompt" coincidences with electrons, presumably conversion electrons of the 132-kev transition, those due to the 345-kev component show a much smaller "prompt" coincidence rate. The situation concerning the 135-kev radiation is less clear since it was not resolved from the 132-kev radiation in the coincidence experiments. However, the relative heights of the K and L conversion peaks in the "prompt" and delayed coincidence spectra favors the idea that both of these gamma-rays follow the 20-μsec. period. We have also found a small "prompt" coincidence rate for the continuous spectrum. This rate was so small (about one-tenth of that for the 481-kev line) that one cannot rule out effects of impurities.

We are thus forced to the conclusion that the 345-kev transition either proceeds from a long-lived state in Ta^{181} or that it is not related to the beta-decay of Hf^{181} at all. To test the latter view the gamma-radiations of two samples of neutron irradiated hafnium differing in age by about 18 months were compared in the scintillation spectrometer. The relative abundance of the 345-kev radiation was much greater in the older sample, proving that this gamma-ray follows a longer period than Hf^{181}. Its behavior in chemical separations showed that it belongs almost certainly to a hafnium or zirconium activity. The fact that Cork *et al.*[2] find K, L, and M conversion lines due to a 342-kev transition converted in lutecium rather than tantalum favors the conclusion that the longer period is an electron-capture activity of hafnium, perhaps Hf^{175}.