Magnetoresistance of Germanium Samples between 20° and 300°K*

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The resistivity of "pure" germanium samples and of samples with small additions of aluminum, antimony, and indium was measured at various temperatures and at various orientations in external magnetic fields. The samples chosen had sufficiently low carrier densities to remain within the limits of classical statistics. The results were compared with those following from the theoretical investigation of a classical electron gas in combined electric and magnetic fields in an isotropic medium and in a medium possessing cubic symmetry. In the transverse orientation, the relative change of the electrical conductivity was found to be proportional to the square of the magnetic field strength at low fields. The effect tends to become linear as the field strength increases. The magnitude of the effect was found to increase with the purity of the material. In the longitudinal orientation, the change of conductivity is of the expected order of magnitude in p-type samples, but is much larger than the theoretical results indicate in n-type samples. The angular dependence of the effect is as expected. Carrier mobilities calculated from magnetoresistance measurements in pure samples agree reasonably well with those calculated from Hall effect and conductivity of single crystals. In the case of samples with additions, there is fair agreement between the values of mobilities calculated from measurements of magnetoresistance and of the Hall effect on the same samples.

I. INTRODUCTION

IT has been pointed out by several authors that the theoretical treatment of the effect of magnetic fields on the electrical resistance of conductors involves considerable difficulties.[1,2] It has also been pointed out that these difficulties are much more easily resolved in the case of semiconductors than for metals. This is due to the fact that the concentration of the free electrons in electronic semiconductors may be so low that the electron gas is no longer degenerate as it is in metals, but obeys Maxwell-Boltzmann statistics.[3] It therefore seems to be of interest to compare the results of the theory of the magnetoresistance of semiconductors based on the classical theory of electric conductance with experimental measurements.

II. THEORETICAL DISCUSSION

The first theoretical investigation of the behavior of a classical electron gas in combined electric and magnetic fields is due to Gans.[4] He considered the electron gas in an isotropic medium, and carried the solution to terms in the current which depend on the first power of the electric field strength and on arbitrary powers of the magnetic field strength. He obtained a solution in closed form for the transverse Hall effect and for the conductivity as functions of the magnetic field. These calculations have been extended by Harding, Sengupta, and Davis[5] to include the case of an anisotropic medium. Equations in a closed form have recently been obtained by Seitz[6] for a system possessing cubic symmetry. They include quadratic terms in the dependence of the conductivity on the magnetic field, but neglect higher terms. Within the limits of this approximation, and neglecting the anisotropic terms, Seitz' work leads to the following vector equation for the current density $\mathbf{I}$

$$\mathbf{I}=\sigma_0\mathbf{E}+\alpha\mathbf{E}\times\mathbf{H}+\beta\mathbf{E}H^2+\gamma\mathbf{H}(\mathbf{E}\cdot\mathbf{H}), \qquad (2.1)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, σ_0 is the conductivity in the absence of a magnetic field, α is the Hall coefficient, and β and γ are constants of the material.

We shall now discuss the effect of the relative orientation of $\mathbf{E}$ and $\mathbf{H}$ on the resistivity.

For the longitudinal position, in which $\mathbf{E}$ is parallel to $\mathbf{H}$, the vector equation (2.1) reduces to

$$I_x=[\sigma_0+(\beta+\gamma)H^2]E_x. \qquad (2.2)$$

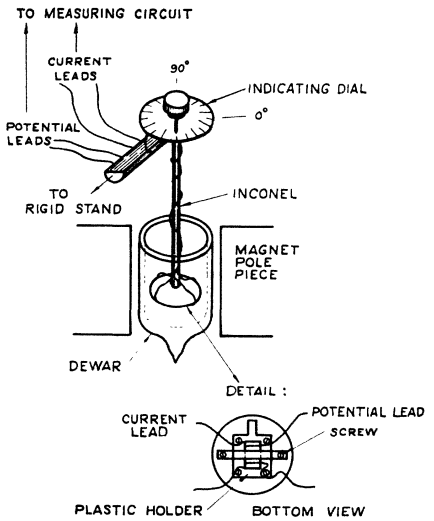

FIG. 1. Experimental arrangement.

* Assisted by the ONR. Results were presented at the Chicago Meeting of the American Physical Society, November, 1949; Phys. Rev. **77**, 759 (1950).

† Abstract of thesis submitted by Anna Foner in partial fulfillment of the requirements for the degree of Doctor of Science at Carnegie Institute of Technology.

[1] J. W. Harding, Proc. Roy. Soc. **140**, 205 (1933).

[2] M. Sengupta, Ind. J. Phys. **11**, 319 (1937).

[3] A. H. Wilson, Proc. Roy. Soc. **133**, 458 and **134**, 277 (1931).

[4] R. Gans, Ann. d. Physik **20**, 293 (1906).

[5] L. Davis, Phys. Rev. **56**, 93 (1939); see also A. Sommerfeld and N. H. Frank, Rev. Mod. Phys. **3**, 1 (1931); and A. Sommerfeld and H. A. Bethe, *Handbuch der Physik* (1934), Vol. 24, p. 2.

[6] F. Seitz, Phys. Rev. **79**, 372 (1950), following paper.

TABLE I. Characteristic properties of the germanium samples.

T °K	ohm-cm	Hall const. cm³/coulomb	n carrier/cm³
	Sample 35P, p-type, 0.006 atomic percent Al added		
300	0.032	26	2.8×10^{17}
77	0.027	40	1.8×10^{17}
20	0.064	13.1	5.6×10^{17}
	Sample 501, p-type, 0.001 atomic percent In added		
292	0.063	69	1.07×10^{17}
77	0.043	100	7.37×10^{16}
20	0.212	86	8.6×10^{16}
	Sample 33E, n-type, 0.04 atomic percent Sb added		
295	0.044	67	1.1×10^{17}
77	0.057	173	4.3×10^{16}
20	0.20	354	2.1×10^{16}
	Sample 481, n-type, no additions		
300	10.8	2.6×10^{4}	2.8×10^{14}
77	2.42	3.5×10^{4}	2.1×10^{14}
20	8.1	8.3×10^{4}	8.4×10^{14}
	Sample 491, n-type, no addition, apparently single crystal		
295	2.4	5.5×10^{3}	1.3×10^{15}
77	0.49	6.7×10^{3}	1.06×10^{15}
20	0.72	3.3×10^{4}	2.15×10^{14}

For the transverse position ($\mathbf{E}=E_x$, $\mathbf{H}=H_z$) Eq. (2.1) yields the following pair of equations

$$I_x=\sigma_0E_x+\alpha E_yH_z+\beta E_xH^2,$$
$$0=\sigma_0E_y-\alpha E_xH_z+\beta E_yH^2. \tag{2.3}$$

Solving for E_y, we obtain

$$E_y=\alpha E_xH_z/(\sigma_0+\beta H^2),$$

and

$$I_x=[\sigma_0+\alpha^2H^2/(\sigma_0+\beta H^2)+\beta H^2]E_x, \tag{2.4}$$

and, neglecting terms with a higher power of H than the second we get

$$I_x=\left[\sigma_0+\left(\frac{\alpha^2}{\sigma_0}+\beta\right)H^2\right]E_x$$
$$=\sigma_0\left[1+\left(\frac{\alpha^2}{\sigma_0^2}+\frac{\beta}{\sigma_0}\right)H^2\right]E_x. \tag{2.5}$$

The parameters α, β, γ, and σ_0 for an isotropic medium under the assumptions of Maxwell-Boltzmann energy distribution and a mean free path λ independent of velocity have the following values:

$$\sigma_0=ne\mu, \tag{2.6}$$

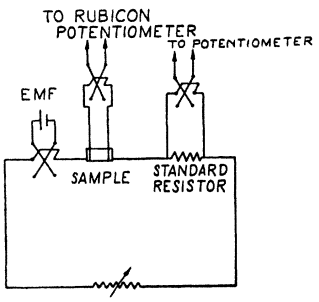

FIG. 2. Measuring circuit.

$$\alpha=-3\pi ne\mu^2/8c=-3\pi\mu\sigma_0/8c, \tag{2.7}$$

$$\beta=-9\pi\mu^3ne/16c^2=-9\pi\mu^2\sigma_0/16c^2, \tag{2.8}$$

$$\gamma=-\beta, \tag{2.9}$$

where n is the number of conducting particles (electrons or holes) per cm³, e their charge, c the velocity of light, and μ the mobility in electrostatic units defined by

$$\mu=(8e\lambda/3\pi m\bar{v}). \tag{2.10}$$

Here m is the effective mass, and $\bar{v}$ is the average velocity of the conduction electrons,

$$\bar{v}=4(kT/2\pi m)^{\frac{1}{2}}. \tag{2.11}$$

From these equations, we obtain for the conductivity σ_H in a magnetic field H for the longitudinal orientation

$$\sigma_H{}^l=(I_x/E_x)^l=\sigma_0, \tag{2.12}$$

and in the transverse orientation

$$\sigma_H{}^t=(I_x/E_x)^t=\sigma_0[1-(9\pi\mu^2/16c^2)(1-\tfrac{1}{4}\pi)]H^2. \tag{2.13}$$

For orientations other than those discussed above, σ_H will also be a function of the angle θ between $\mathbf{I}$ and $\mathbf{H}$. From Eq. (2.1) we obtain

$$\sigma_H=\sigma_0\left[1+\left(\frac{\beta}{\sigma_0}+\frac{\gamma\cos^2\theta}{\sigma_0}+\frac{\alpha^2\sin^2\theta}{\sigma_0^2}\right)H^2\right]$$
$$=\sigma_0\left[1=\left(\frac{9\pi\mu^2}{16c^2}(1-\tfrac{1}{4}\pi)H^2\sin^2\theta\right)\right] \tag{2.14}$$

or

$$(\sigma_0-\sigma_H)/\sigma_0=(9\pi\mu^2/16c^2)(1-\tfrac{1}{4}\pi)H^2\sin^2\theta. \tag{2.15}$$

For an anisotropic system possessing cubic symmetry, Seitz chooses a Cartesian coordinate system with axes x, y, z coinciding with the axes of the crystal. Equation (2.1) has then to be replaced by the three component equations

$$\left.\begin{aligned} I_x&=\sigma_0E_x+\alpha(E_yH_z-E_zH_y)+\beta E_xH^2\\ &\quad+\gamma H_x(\mathbf{E}\cdot\mathbf{H})+\delta E_xH_x^2,\\ I_y&=\sigma_0E_y+\alpha(E_zH_x-E_xH_z)+\beta E_yH^2\\ &\quad+\gamma H_y(\mathbf{E}\cdot\mathbf{H})+\delta E_yH_y^2,\\ I_z&=\sigma_0E_z+\alpha(E_xH_y-E_yH_x)+\beta E_xH^2\\ &\quad+\gamma H_z(\mathbf{E}\cdot\mathbf{H})+\delta E_zH_z^2, \end{aligned}\right\} \tag{2.16}$$

where δ accounts for the anisotropic terms, vanishing for an isotropic medium. For the longitudinal position, this results in

$$I_x=[\sigma_0+(\beta+\gamma+\delta)H^2]E_x, \tag{2.17}$$

or

$$\sigma_H{}^l-\sigma_0=(\beta+\gamma+\delta)H^2. \tag{2.18}$$

The most pertinent results of the theory are that for the isotropic case, there is no change in the conductivity if $\mathbf{I}\parallel\mathbf{H}$, and that if the angle between $\mathbf{I}$ and $\mathbf{H}$ is θ, the relative change in conductivity is proportional to $H^2\sin^2\theta$, as long as higher terms in H are neglected. For

a cubic system, there is also a change of couductivity proportional to H^2 for the longitudinal case, as shown in Eq. (2.18).

III. SURVEY OF PREVIOUS EXPERIMENTS

Most of the measurements of magnetoresistance reported in the literature[7] were carried out with metals, and are, therefore, outside the range of the assumptions in the preceding paragraphs. Kapitza's measurements on semiconductors are restricted to germanium, silicon, tellurium, and graphite. It appears that germanium should be a good system for the test of the theory, since it is a typical electronic semiconductor with a cubic lattice. The results obtained by Kapitza with germanium are very similar to those obtained with metals (except bismuth, which shows a much larger effect[8]). At low fields, the relative change of resistance $\Delta R/R_0$ obeys a quadratic law, and at high fields, the dependence on H is linear. Kapitza defines a "critical field" H_K at which the quadratic law changes into the linear law; it is about 45 kilogauss at room temperature and 55 kilogauss at liquid nitrogen temperature. The highest values of $\Delta R/R_0$ observed at a magnetic field of 300 kilogauss were 0.5 at room temperature and 1.28 at liquid nitrogen temperature. All measurements were taken for $\mathbf{E} \perp \mathbf{H}$.

A closer investigation of the data supplied by Kapitza shows, however, that the germanium samples which he used cannot be treated as semiconductors in which the electron gas obeys Maxwell-Boltzmann statistics. The resistivity of his material was 2.6×10^{-3}

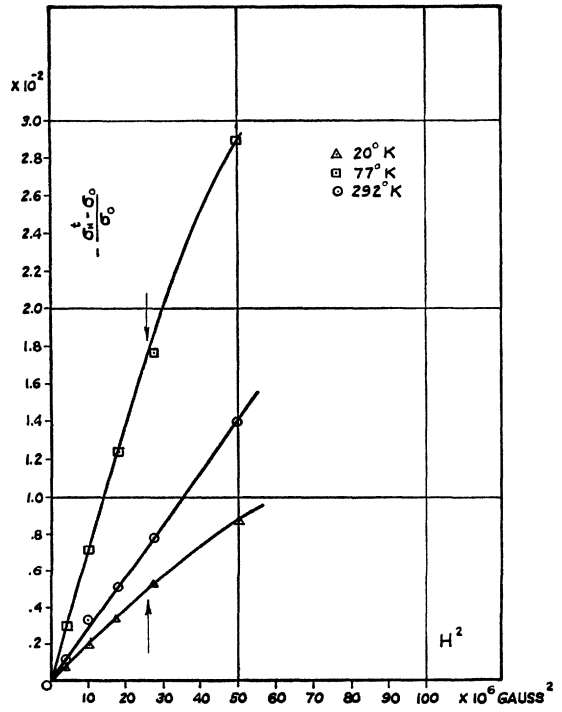

FIG. 3. Relative change of conductivity *vs.* H^2, transverse orientation, sample 501.

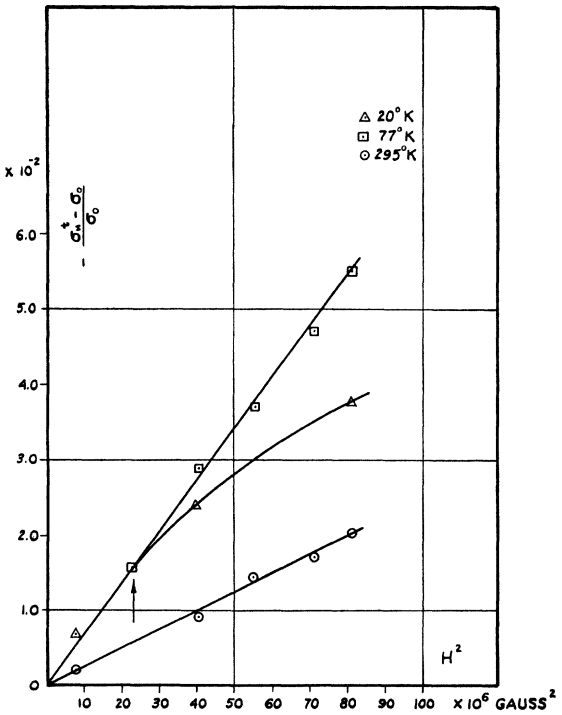

FIG. 4. Relative change of conductivity *vs.* H^2, transverse orientation, sample 33E.

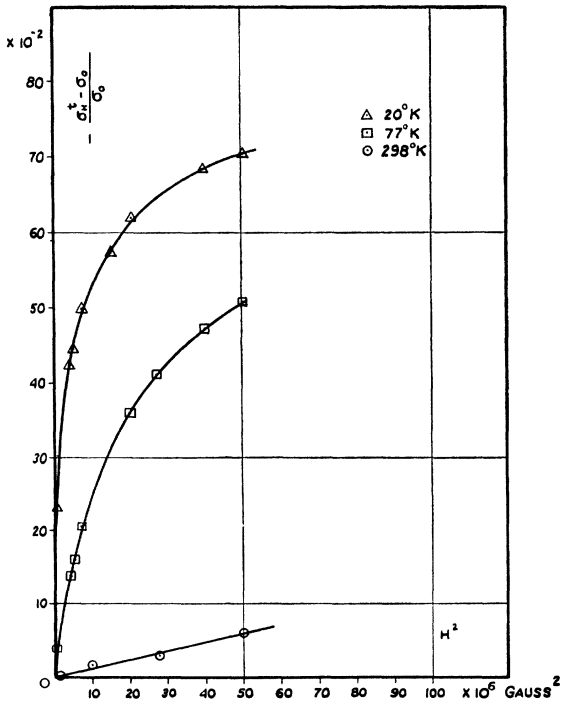

FIG. 5. Relative change of conductivity *vs.* H^2, transverse orientation, sample 481.

[7] P. Kapitza, Proc. Roy. Soc. **123**, 292 (1929). A bibliography of the earlier literature is given in this reference.
[8] P. Kapitza, Proc. Roy. Soc. **119**, 387 (1928).

TABLE II. Upper limits for the quadratic region of magneto-resistance.

Sample	Temp. (°K)	H_K (Gauss)
35P	301	8.5×10^3
	77	9.0
	20	4.5
33E	295	9.0
	77	9.0
	20	4.7
481	298	7.0
	77	1.6
	20	0.85
491	295	7.0
	77	1.4
	20	
501	292	7.0
	77	5.0
	20	5.0

ohm-cm, and the amount of impurity was given as less than 0.01 percent. More recent investigations[9] of the resistivity of germanium at high temperatures have shown that its intrinsic resistivity at room temperature should be about 60 ohm-cm, and actual samples have shown resistivities as high as 30 ohm-cm.[10] It is, therefore, quite obvious that Kapitza's measurements were carried out with a rather impure material. By comparison with other samples of similar resistivity, for which the Hall effect has been measured, one can conclude that the electron gas in Kapitza's samples had a density of about 3×10^{18} electrons per cm^3, and consequently a degeneracy temperature of $\sim$120°K, which means that even at room temperature the assumption of classical statistics is not too good. In view of this situation it was decided to carry out new measurements of the magneto-resistance with much purer germanium samples, and to include the dependence of $\Delta R/R$ on the angle θ between **I** and **H**, which had not been reported before.

IV. EXPERIMENTAL PROCEDURE

A. Method of Measurements

The purpose of this investigation was to measure the dependence of the conductivity σ on H and θ, to calculate values of the mobility μ from these measurements, and to compare these values with mobility values obtained by other methods and with the theoretical results given in the preceding section. Since the interesting quantity Q,

$$Q(H,\theta)=(\sigma_0-\sigma_H)/\sigma_0=-(\rho_H-\rho_0)/\rho_H \\ =-(R_H-R_0)/R_H, \quad (4.1)$$

is independent of the dimensions of the sample, the problem is reduced to an accurate determination of the resistance R as a function of field strength and angle.

Within the region of validity of the quadratic approximation in H, the magnetic fields are rather small and the quantity Q is frequently less than one percent. The dependence of Q on the angle θ is even smaller. Consequently, the resistance measurements had to be carried out with extreme care. Since the resistivity of the germanium samples is strongly temperature-dependent, small variations of the temperature of the samples may obscure the effect of the variation of the angle θ completely. Consequently, the measurements were carried out only at temperatures where good temperature baths were available. These baths were contained in a special Dewar vessel and consisted of a kerosene bath for room temperature, and baths of liquefied methane, nitrogen, and hydrogen boiling at constant pressure for the other temperatures.

Measurements were made in magnetic fields from 260 to 9300 gauss and through a total angle of 240°, usually in steps of 15° or 30° with check points at 0° (longitudinal position) and 90° (transverse position). In some instances experiments at certain temperatures were repeated on different days, and the reproducibility was always very good. Figure 1 shows the experimental arrangement for mounting and turning the sample in a Dewar vessel in the magnetic field.

The resistance measurements were made by measuring the potential drops through the sample and through a standard resistance in series with the sample with a Rubicon Model B potentiometer. The samples were thin rectangular parallelopipeds with current electrodes

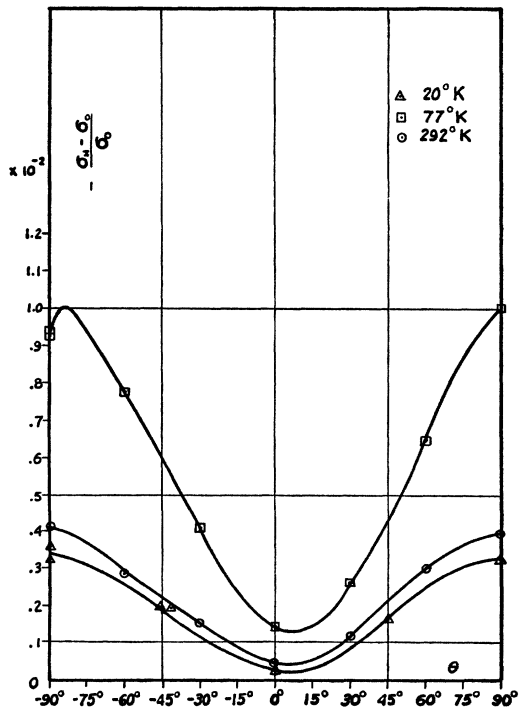

FIG. 6. Relative change of conductivity as function of orientation, sample 501, $H=3900$ gauss. 0° is longitudinal orientation.

[9] See, for instance, K. Lark-Horovitz, Report N.D.R.C.-14-585 (March, 1942–November, 1945).

[10] Estermann, Foner, and Randall, Phys. Rev. **72**, 530 (1947); G. L. Pearson, Phys. Rev. **76**, 179 (1949); K. Lark-Horovitz, reference 9.

attached to the end surfaces and potential electrodes to the top surface. The long dimension of the rectangles was taken as the direction of the applied electric field (x direction), and the sample was rotated about a vertical axis (y direction) between the pole pieces of an electromagnet. The field of the magnet was calibrated against a standard solenoid. The circuit is shown schematically in Fig. 2.

In some germanium samples, particularly at very low temperatures, it was found that the current was not proportional to the applied voltage even in the absence of a magnetic field. In all the measurements used below, it was made certain that Ohm's law was obeyed, which means that the current was proportional to the applied electromotive force.

Each measurement of the resistance at a given angle θ required six potential measurements across the potential probes for a given constant current and magnetic field. These were two potential drop measurements in zero field with current in forward and reversed directions, two similar measurements with the magnetic field on in one direction, and two with the magnetic field reversed.

The dependence of Q as function of H was obtained from a similar set of six potential measurements, but with θ always 90°, that is with the sample in the transverse orientation.

B. Choice of Samples

The applicability of the Maxwell-Boltzmann statistics to the electron gas in a semiconductor depends on the density of the conducting particles. Previous work has shown that below room temperature this density is determined even in the purest samples by the number of impurity centers. The electron density (or hole density) in all samples was determined from Hall effect measurements over the whole temperature range concerned. Only those for which the Fermi energy was small compared to kT at all temperatures and for which, therefore, Maxwell-Boltzmann statistics is applicable, are considered in the following sections. Their important characteristics are given in Table I. The samples include "pure" material from different sources, and n- and p-type material with small additions of Al, In, and Sb, all of which remained non-degenerate at all temperatures, except where otherwise noted.

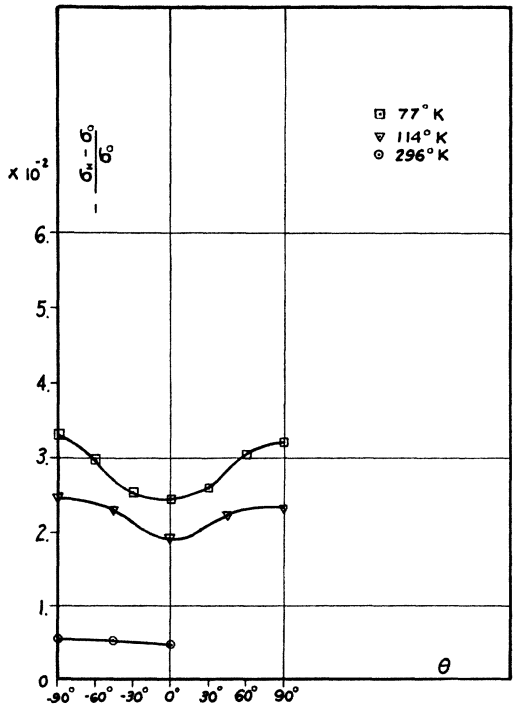

FIG. 7. Relative change of conductivity as function of orientation, sample 33E, $H=4700$ gauss.

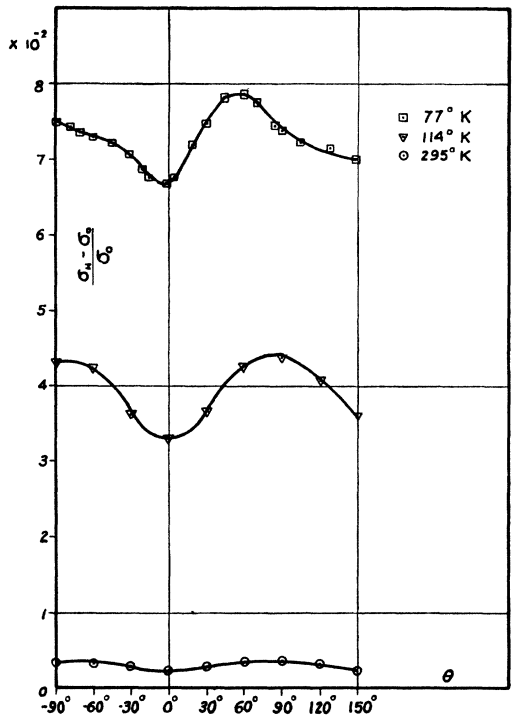

FIG. 8. Relative change of conductivity as function of orientation, sample 481, $H=1280$ gauss.

V. RESULTS

A. Dependence of the Electrical Conductivity on the Magnetic Field Strength

For these measurements the samples were mounted in the transverse orientation. As the results given in Figs. 3–5 show, the relative change of the conductivity $\Delta\sigma/\sigma_0$ at low fields is proportional to H^2, changing into a more nearly linear dependence at higher fields. This behavior is in general agreement with Kapitza's observations. The magnitude of the observed effect is, however, quite different, and varies from sample to sample with the purity of the sample. In all cases, $\Delta\sigma/\sigma_0$ is much larger than the values reported by Kapitza. On the other hand, the upper limits of the

TABLE III. Mobilities calculated by various methods.

Sample	Temp. °K	$\frac{\sigma_0-\sigma_H^t}{\sigma_0 H^2}$	$\frac{\sigma_0-\sigma_H^l}{\sigma_0 H^2}$	Mobility, μ, in cm²/volt-sec. from Eq. (6.1)	Eq. (6.4)	Eq. (6.5)	Eq. (6.6)
35P	292	1.64×10^{-10}	2.0×10^{-11}	2.07×10^{3}	1.9×10^{3}	0.69×10^{3}	0.91×10^{3}
	77	3.74	3.8	3.12	2.9	1.16	0.74
	20	1.28	7.9	1.83	1.1	0.21	
501	292	2.84×10^{-10}	2.6×10^{-11}	2.72×10^{3}	2.6×10^{3}	0.92×10^{3}	1.7×10^{3}
	77	6.78	8.6	4.20	3.9	2.00	1.7
	20	2.16	1.3	2.38	2.3	0.34	1.8
33E	296	2.36×10^{-10}	2.2×10^{-10}	2.49×10^{3}	0.604×10^{3}	1.23×10^{3}	
	114	11.1	8.7	5.39	2.51		
	77	14.7	11.2	6.20	3.02	2.61	
481	295	2.05×10^{-9}	1.39×10^{-9}	7.32×10^{3}	4.15×10^{3}	1.64×10^{3}	
	114	2.64×10^{-8}	2.01×10^{-8}	26.3	12.81	7.12	
	77	4.78×10^{-8}	4.07×10^{-8}	35.3	13.5	11.8	
	20	1.19×10^{-6}	1.04×10^{-6}	176.1	62.6		
491	295	2.48×10^{-8}	1.54×10^{-9}	8.05×10^{3}	5.37×10^{3}	1.78×10^{3}	
	77	4.74×10^{-8}	3.58×10^{-8}	35.2	17.2	11.4	
	20	4.84×10^{-7}	4.26×10^{-7}	111.2	42.1	40.6	

quadratic region, H_K, as shown in Table II, are much lower than Kapitza's values.

B. Dependence of Q on the Angle θ

In order to remain within the region of validity of the quadratic approximation in H, these measurements were carried out at fields $H<H_K$. Typical results are plotted in Figs. 6–8, and show a periodic change of Q with θ, depending in magnitude on the character of the sample and on the temperature.

VI. DISCUSSION OF THE RESULTS

A. General Observations

Within the region of the validity of the quadratic dependence of Q on H and the assumption of isotropy, the magnitude of $(Q/H^2)^t$ should be proportional to the square of the carrier mobility, μ^2. At high temperatures, this mobility is determined principally by the lattice vibrations and is, therefore, about the same for all relatively pure germanium samples. At lower temperatures, even room temperature, the mobility is in part determined by impurity scattering,[11] and at very low temperatures the impurity scattering becomes the predominant factor. In this temperature region, therefore, the effect should vary widely from sample to sample, and should have its largest magnitude in the case of the purest samples. Since the lattice scattering decreases with decreasing temperature, the mobility and the magneto-resistance should increase as the temperature is lowered, until the impurity scattering becomes dominant. We shall, therefore, expect $\Delta\sigma/\sigma_0$ to have a maximum at a certain temperature, this maximum shifting to lower temperatures as the purity of the sample increases. This maximum has been observed in the p-type samples, while in the "pure" samples, $\Delta\sigma/\sigma_0$ increases down to 20°K.

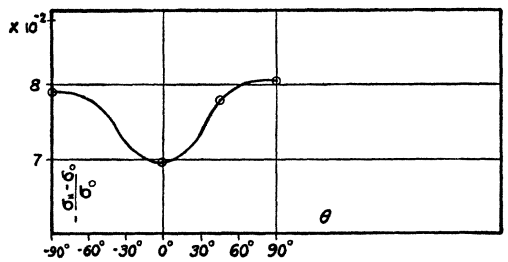

FIG. 9. Relative change of conductivity as function of orientation, sample 481, $H=260$ gauss, $T=20°$K.

[11] E. Conwell and V. F. Weisskopf, Phys. Rev. **77**, 388 (1950). (Earlier references are given in this paper.)

B. Effect of Orientation

The major difference between the simplified theory for isotropic systems and the experiments lies in the fact that even at low fields the longitudinal effect is not zero. A possible explanation for a small longitudinal effect, as observed in p-type samples, may be furnished by a consideration of the anisotropic term in Eq. (2.18), which is determined by the parameter δ, which in turn is dependent on the ratio of the "anisotropic" and the "isotropic" collision parameters[12] $(\tau_1/\tau_0)^2$. The anisotropic contribution may vary from sample to sample because well-grown single crystals were not available and the multicrystalline material may have different degrees of anisotropy. The large longitudinal effect as observed in n-type samples, cannot be explained by the theory developed by Seitz since it would lead to negative values of μ^2. (See Section VI-C.) It seems, therefore, that either this theory is still incomplete and describes adequately only the behavior of holes, that is of p-type material, or that the n-type material was too inhomogeneous to provide a

[12] See F. Seitz, reference 6, Eq. (20e).

fair test for the theory. It is known that holes can pass much more easily through internal barriers than electrons, and consequently it may be expected that inhomogeneities are less effective in p-type than in n-type semiconductors.

C. Calculation of the Mobility

Introducing numerical values into Eq. (2.13) and changing from e.s.u. to practical units, and neglecting the anisotropic terms in Eq. (2.16), we obtain

$$\mu=1.616\times10^{8}\times[(\sigma_0-\sigma_H{}^t)/\sigma_0]^{\frac{1}{2}} \times H^{-1}\ \text{cm}^2/\text{volt-sec.} \quad (6.1)$$

Values for μ for the different samples and temperatures are given in Table III, column 5.

One can attempt to make a correction for the anisotropic terms by subtracting the longitudinal effect from the effect measured at other angles. This means that one considers the contribution from the anisotropic term to be independent of θ for any given value of H. The experimental curves showing Q as function of θ can actually be represented quite well by an equation of the form

$$Q=f(H)+g(H)\sin^2\theta, \quad (6.2)$$

where from Eq. (2.15)

$$g(H)=(9\pi\mu^2/16c^2)(1-\tfrac{1}{4}\pi)H^2. \quad (6.3)$$

Under this assumption, μ can be calculated from the formula

$$\mu=1.616\times10^{8}\times[(\sigma_H{}^l-\sigma_H{}^t)/\sigma_0]^{\frac{1}{2}} \times H^{-1}\ \text{cm}^2/\text{volt-sec.} \quad (6.4)$$

Values so calculated are given in Table III, column 6 and are, of course, somewhat smaller than those calculated from Eq. (6.1), particularly for the n-type samples.

Finally, Table III contains in column 7 values for μ calculated from the Hall constant C and the electrical conductivity σ_0 according to the equation

$$\mu=0.85\times C\times\sigma_0. \quad (6.5)$$

These values are again somewhat smaller than those calculated from Eq. (6.4), but the discrepancy is usually less than a factor of two, while previous attempts to obtain numerical values for magnetoresistance on a theoretical basis lead to an incorrect order of magnitude. Moreover, values calculated according to Eq. (6.5) are likely to be too small since the directly measured values of σ_0 may easily be influenced by internal barriers in the samples. Measurements on germanium single crystals published recently by Pearson[13] have shown larger values for electron and hole mobilities than those reported for microcrystalline material, namely 2600 and 1700 cm^2/volt-sec., respectively, which are even closer to our results from magneto-resistance measurements.

[13] G. L. Pearson, Phys. Rev. **76**, 179 (1949). See also W. C. Dunlap, Jr., Phys. Rev. **77**, 759 (1950).

TABLE IV. τ_1/τ_0 for p-type samples.

Sample	Temperature °K	τ_1/τ_0
35P	292	0.24
	77	0.40
501	292	0.14
	77	0.26
	20[a]	0.10

[a] At this temperature, the sample is near the degeneracy point.

A more rigorous analysis of the anisotropic contribution in a medium of cubic symmetry can be obtained from Eq. (2.16), using the evaluation of the parameters α, β, γ, and δ given by Seitz. This analysis leads to the equation

$$\left(\frac{\sigma_0-\sigma_H{}^t}{\sigma_0H^2}\right)-\left(\frac{532}{80}\right)\left(\frac{\sigma_0-\sigma_H{}^l}{\sigma_0H^2}\right)=\frac{9\pi}{16c^2}\left(1-\frac{\pi}{4}\right)\mu^2 \quad (6.6)$$

or

$$\mu^2=2.62\times10^{16}\left(\frac{\sigma_0-\sigma_H{}^t}{\sigma_0H^2}\right) -6.65\left(\frac{\sigma_0-\sigma_H{}^l}{\sigma_0H^2}\right)(\text{cm}^3/\text{coulomb})^2$$

from which μ can be calculated. Values obtained in this way for samples 35P and 501 are also given in Table III, column 8 and agree reasonably well with those from Hall effect and conductivity, except at $T=20$°K where these samples are near the degeneracy temperature. It is obvious that Eq. (6.6) will lead to negative values for

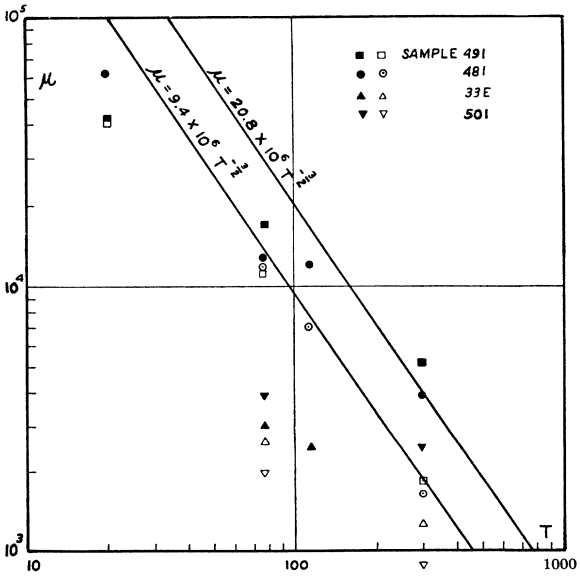

FIG. 10. Mobilities calculated from Hall effect (open symbols) and magnetoresistance according to Eq. (6.4) (filled symbols). The straight lines are adjusted to give mobilities of 1800 cm^2/volt-sec. at room temperature (our Hall effect measurements on pure samples) and 4000 cm^2/volt-sec. (measurements by Dunlap on n-type single crystals).

μ^2 if $(\sigma_0-\sigma_H{}^l)/\sigma_0H^2$ is larger than 6.65 $(\sigma_0-\sigma_H{}^t)/\sigma_0H^2$. This is unfortunately the case in our n-type samples. We feel, therefore, that the rather crude way of correcting for the anisotropic contribution used in Eq. (6.4) is a workable compromise and that mobility values computed in this way are at least of the right order of magnitude.

An estimate of the relative magnitude of the collision parameters τ_1 and τ_0 can also be obtained from our measurements. Using again the evaluation of β, γ, and δ given by Seitz, we obtain from Eq. (2.18)

$$\sigma_H{}^l-\sigma_0=(\beta+\gamma+\delta)H^2 = -(80/32)(\tau_1/\tau_0)^2(9\pi/16c^2)\sigma_0\mu^2H^2 \quad (6.7)$$

or

$$(\tau_1/\tau_0)^2=(\sigma_0-\sigma_H{}^l)/(\sigma_0H^2)(32/80)(16c^2)(9\pi\mu^2) = 2.26\times10^{15}(\sigma_0-\sigma_H{}^l)/(\sigma_0H^2\mu^2).$$

Values for τ_1/τ_0 computed from this equation are given in Table IV.

According to the theory of lattice scattering, the temperature dependence of the mobility should go with $T^{-\frac{3}{2}}$. As is shown in Fig. 10, this holds in the purest cases down to liquid oxygen temperature, but at lower temperatures there is a systematic deviation toward smaller mobilities due to the effect of impurity scattering. The same deviation is, of course, also present in the case of samples with added impurities where it reduces the mobility even at room temperature.

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Note on the Theory of Resistance of a Cubic Semiconductor in a Magnetic Field

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The theory of the behavior of a classical electron gas in combined electric and magnetic fields, developed by Gans and extended by Sommerfeld and Davis, is applied to a system possessing cubic symmetry. The problem is limited to that case in which terms involving the magnetic field to higher than the second power are neglected.

I. INTRODUCTION

THE density of carriers in relatively pure semiconductors and in all photo-conductors is sufficiently low that they may be treated as if distributed in accordance with classical rather than quantum statistics. The problem of determining the resistance of such a system of carriers in the presence of a magnetic field and the Hall coefficient was first treated by Gans[1] on the basis of Lorentz' transport theory. He assumed that the carriers are perfectly free, that the medium is isotropic, and that the mean free path is independent of energy. He treated the case in which the applied electric and magnetic fields are normal and in which no current is permitted to flow in the direction normal to these two directions (case of normal transverse Hall effect). He examined his results to the first power of the applied electrostatic field and to arbitrary powers of the magnetic field, and found that he could add the series corresponding to sums over various powers of the magnetic field. Sommerfeld[2] subsequently obtained the same results more concisely by use of an appropriate assumption concerning the initial form of the distribution function in the presence of a magnetic field. Harding[3] has also applied more modern techniques to the problem and has noted an error in Gans' paper. Sengupta,[4] in turn has criticized Harding's analysis of measurements of Kapitza on germanium.

More recently Davis[5] generalized Gans' treatment of the problem for the case in which the electric and magnetic fields are arbitrarily oriented relative to one another, the electrons are not perfectly free, the medium is not necessarily isotropic, the mean free path is an arbitrary function of the wave number of the conduction electrons, and the distribution function is degenerate. Naturally, Davis' treatment is sufficiently general that the results are not given in closed form, but are expressed instead in terms of series in powers of the magnetic field, the coefficients being integrals which must be evaluated in particular cases.

We shall discuss here the solution of Davis' equations in the particular case in which the following assumptions are valid. (1) The electrons behave as if perfectly free, although their effective mass may be different from the true electron mass. (2) The collision frequency is not perfectly isotropic, but contains terms

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