Polarization of Neutrons and Protons by Scattering*

JOSEPH V. LEPORE
Institute for Advanced Study, Princeton, New Jersey
(Received January 23, 1950)

Neutrons and protons can be polarized by scattering provided there is a spin-orbit interaction between the incident particle and the scattering nucleus. The polarization effect has been found for a nucleus of spin zero allowing for the possibility of resonance scattering. Particular attention has been paid to the resonance scattering of neutrons by He^4 where almost full polarizations can be obtained, together with large cross sections. The polarization can be detected by a double scattering experiment, since the second scattering will be axially asymmetric with respect to the direction of the first scattered neutron. The existence of this effect and a preliminary treatment of it was first pointed out by Schwinger.

The scattering of polarized neutrons by protons has been studied for potentials suggested by the symmetrical, charged and neutral meson theories. The polarization effect on the differential cross section is negligible for the symmetrical theory, but is significant for the other two.

I. INTRODUCTION

THE scattering of neutrons by He^4 has been studied by various investigators[1] who have shown that there is an anomaly in the scattering in the region of 1 Mev (incident neutron energy). This anomaly has been attributed to the existence of a P-resonance associated with the formation of an unstable He^5 nucleus. Recently, Koontz and Hall[2] measured the differential cross section for various neutron energies in the region 0.8 to 1.6 Mev and thus established the existence of strong P-scattering. This confirmed the earlier conclusions[1] that the anomaly is due to a P-resonance. Under the assumption that P-resonant scattering but only ordinary S-scattering is involved[3] Koontz and Hall were able to show that the P-resonance must be split into ${}^2P_{3/2}$ and ${}^2P_{1/2}$ components. This splitting must therefore arise from some spin-orbit interaction.[4] The scattering can be qualitatively accounted for on the basis of the following parameters.

$$E_{1/2}=1.0 \text{ Mev} \quad \sin^2\delta_0=0.5$$
$$E_{3/2}=1.3 \text{ Mev} \quad \Gamma=0.3 \text{ Mev}, \quad \delta_0<0.$$

Here $E_{1/2}$, $E_{3/2}$ are the relevant resonance energies, Γ is their common width, and δ_0 is the S phase shift.

In consequence of the large splitting between the levels, the incident neutron is effectively subjected to a strong spin-orbit force which manifests itself in a polarization of neutrons scattered through a definite angle. The spin-orbit force can cause a neutron incident with a given spin direction to reverse this direction upon being scattered. The scattered wave will thus consist of two parts which represent those particles which have reversed their spin and those scattered with spin direction unchanged. The amplitudes of these two parts of the scattered wave will be different in their angular dependence so that for a given angle of scattering there will be some net spin direction.

The polarization effect can be detected by a double scattering experiment. Thus a beam polarized by scattering can be scattered again by a second nucleus and the polarization can be detected, since the differential cross section for the second scattering will now be axially asymmetric with respect to the direction of the first scattered neutron. This asymmetry arises since the scattering is now dependent upon two space vectors of the system, i.e., the beam direction and the polarization, so that the axial arbitrariness of the first scattering is absent. This effect and a preliminary treatment of it was first given by Schwinger.[5] Other discussions of polarization effects have been given by Wolfenstein[6] and Hammermesh.[7]

II. THE POLARIZATION FORMULA

Although the foregoing remarks were directed toward the case of helium scattered neutrons, it is clear that the same type of polarization effect can be expected for any nucleus of spin zero which shows a resonance of arbitrary angular momentum for the scattering of either neutrons or protons.

The polarization resulting when charged particles of spin $\frac{1}{2}$ are scattered by a nucleus with charge Z will now be calculated. The differential cross section for this process was first found by Bloch.[8] The method to be used here is somewhat different from his, however, since the polarization effect is of primary importance. This can be most easily treated by representing the wave functions describing the scattering directly in terms of the vectors $\boldsymbol{\sigma}$, $\mathbf{k}_0$ and $\mathbf{k}$ which are the Pauli spin vector of the incident particle, and its direction of incidence and scattering, respectively. This permits an elegant derivation of the polarization formula.[9]

* The first section of this paper constituted part of a thesis submitted to Harvard University (May, 1948).
[1] Williams, Shepherd, and Haxby, Phys. Rev. **52**, 390 (1937); H. Staub and W. E. Stephens, Phys. Rev. **54**, 236 (1938) and **55**, 131 (1939); H. Staub and H. Tatel, Phys. Rev. **58**, 822 (1940).
[2] P. G. Koontz and T. A. Hall, Phys. Rev. **72**, 196 (1947).
[3] L. Eisenbud, Phys. Rev. **74**, 1206 (1948) throws doubt on this conclusion.
[4] S. M. Dancoff, Phys. Rev. **58**, 327 (1940).
[5] J. S. Schwinger, Phys. Rev. **69**, 681 (1946).
[6] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).
[7] M. Hammermesh, Phys. Rev. **75**, 1281 (1949).
[8] F. Bloch, Phys. Rev. **58**, 829 (1940).
[9] This method was kindly pointed out to me by Professor Schwinger.

The incident beam may be taken as

$$\psi_{\rm inc}=\exp i[\mathbf{k}_0\cdot\mathbf{r}-\alpha\ln 2kr]\cdot\chi_{\rm inc}. \quad (1)$$

Here $\chi_{\rm inc}$ is a spin function which represents the polarization of the incident beam

$$\mathbf{P}_{\rm inc}=(\chi_{\rm inc},\boldsymbol{\sigma}\chi_{\rm inc}). \quad (2)$$

The constant α is

$$\alpha=(Ze^2\mu)/(k\hbar^2). \quad (3)$$

It is now necessary to find a solution of the Schroedinger equation which satisfies the boundary condition of asymptotically representing an incoming wave of the form (1) together with a diverging spherical wave

$$\psi\sim\psi_{\rm inc}+\frac{\exp i[kr-\alpha\ln 2kr]}{r}f(\theta)\chi_{\rm inc}. \quad (4)$$

In virtue of the spin-orbit interaction which we wish to include in the description the vector, $\mathbf{L}$, of the orbital angular momentum is no longer a constant of the motion so that the motion must be described in terms of the total angular momentum

$$\mathbf{J}=\mathbf{L}+\tfrac{1}{2}\boldsymbol{\sigma}. \quad (5)$$

The required set of commuting constants of the motion is

$$\mathbf{J}^2,\ \mathbf{J}_z,\ \mathbf{L}^2,\ \boldsymbol{\sigma}^2. \quad (6)$$

The incident beam (1) can now be decomposed, asymptotically, into its constituent orbital angular momenta

$$\psi_{\rm inc}\sim\sum_{l=0}^{\infty}(2l+1)^{\frac{1}{2}}i^l\frac{\sin(kr-l\pi/2-\alpha\ln 2kr)}{kr}\times Y_l^0(\cos\theta)\chi_{\rm inc}. \quad (7)$$

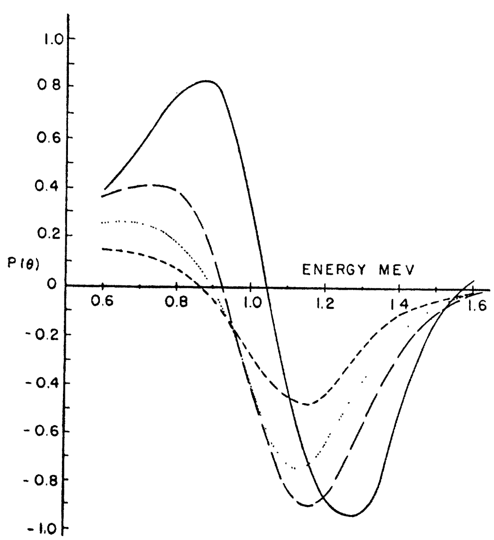

FIG. 1. The calculated degree of polarization, $P(\theta)$, as a function of the incident neutron energy, for scattering angles. $\theta=30°$, 45°, 60°, 90°. ——— Polarization at $\theta=90°$; — — Polarization at $\theta=60°$; ······ Polarization at $\theta=45°$; ‐ ‐ ‐ ‐ Polarization at $\theta=30°$.

It is now necessary to express this incident beam in terms of the proper functions of the operators (6). This resolution is most easily accomplished through the medium of the following projection operators.

$$\Pi_l^+=\frac{l+1+\boldsymbol{\sigma}\cdot\mathbf{L}}{2l+1},\quad \Pi_l^-=\frac{l-\boldsymbol{\sigma}\cdot\mathbf{L}}{2l+1}. \quad (8)$$

Π_l^+ is so constructed that when applied to a function of the type $Y_l^0\chi_{\rm inc}$ it destroys all those states within this function for which $J=l-\frac{1}{2}$ and selects out only those states with $J=l+\frac{1}{2}$. Π_l^- similarly destroys all states with $J=l-\frac{1}{2}$. These operators clearly satisfy the condition

$$\Pi_l^++\Pi_l^-=1. \quad (9)$$

We can now attempt to find a solution to the scattering problem by writing the wave function for the system in the asymptotic form

$$\psi\sim\sum_{l=0}^{\infty}(2l+1)^{\frac{1}{2}}i^l\left[A_l^+\Pi_l^+\frac{u_l^+(kr)}{kr}+A_l^-\Pi_l^-\frac{u_l^-(kr)}{kr}\right]Y_l^0\chi_{\rm inc}. \quad (10)$$

Here A_l^+ and A_l^- are constants to be determined by the condition that ψ have the form (4). The radial functions are

$$u_l^\pm(kr)=\sin(kr-l\pi/2-\alpha\ln 2kr+\delta_l^\pm), \quad (11)$$

where $\delta_l^\pm$ is a phase shift which describes the effect of Coulomb, nonresonance, and resonance scattering and may be written in this order as

$$\delta_l^\pm=\alpha_l+\beta_l^\pm+\gamma_l^\pm. \quad (12)$$

α_l is the ordinary phase shift due to the Coulomb field

$$\alpha_l=\arg\Gamma(l+i\alpha). \quad (13)$$

$\beta_l^\pm$ is the shift due to ordinary potential scattering and $\gamma_l^\pm$ is that due to the presence of the resonance which may be taken into account by writing

$$\tan\gamma_l^\pm=\Gamma_l^\pm/(E_l^\pm-E). \quad (14)$$

E is the energy of the incident particle, $E_l^\pm$ is the resonance energy and $\Gamma_l^\pm$ is its width.

After the coefficients $A^\pm$ of Eq. (10) have been determined the scattered wave

$$\psi_{\rm scatt}=\psi-\psi_{\rm inc} \quad (15)$$

is given by

$$\psi_{\rm scatt}\sim\frac{\exp[i(kr-\alpha\ln 2kr)]}{kr}\sum_{l=0}^{\infty}\frac{1}{(2l+1)^{\frac{1}{2}}}\times\{[(l+1)\exp(i\delta_l^+)\sin\delta_l^++l\exp(i\delta_l^-)\sin\delta_l^-]Y_l^0+(\exp(i\delta_l^+)\sin\delta_l^+-\exp(i\delta_l^-)\sin\delta_l^-)\boldsymbol{\sigma}\cdot\mathbf{L}Y_l^0\}\chi_{\rm inc}. \quad (16)$$

If $\mathbf{k}$ is the direction in which the scattering is observed it can easily be verified that the term $\boldsymbol{\sigma}\cdot\mathbf{L}Y_l^0$ can be written as

$$-i\sin\theta\frac{\partial}{\partial(\cos\theta)}Y_l^0\boldsymbol{\sigma}\cdot\mathbf{n}. \tag{17}$$

The vector $\mathbf{n}$ is the normal to the plane in which the scattering occurs defined by

$$\mathbf{k}\times\mathbf{k}_0=\mathbf{n}k^2\sin\theta. \tag{18}$$

The angular function $f(\theta)$ defined by (4) may now be written as

$$f(\theta)=A(\theta)+\boldsymbol{\sigma}\cdot\mathbf{n}B(\theta), \tag{19}$$

where

$$A(\theta)=\frac{1}{k}\sum_{l=0}^{\infty}\frac{1}{(2l+1)^{\frac{1}{2}}}\times[(l+1)\exp(i\delta_l^+)\sin\delta_l^+ + l\exp(i\delta_l^-)\sin\delta_l^-]Y_l^0$$

$$B(\theta)=-\frac{i\sin\theta}{k}\frac{\partial}{\partial(\cos\theta)}\sum_{l=0}^{\infty}\frac{1}{(2l+1)^{\frac{1}{2}}}\times[\exp(i\delta_l^+)\sin\delta_l^+ - \exp(i\delta_l^-)\sin\delta_l^-]Y_l^0. \tag{20}$$

From (19) we determine the differential cross section to be

$$\sigma(\theta)=(AA^*+BB^*)\left[1+\frac{A^*B+B^*A}{AA^*+BB^*}\mathbf{P}_{\text{inc}}\cdot\mathbf{n}\right]. \tag{21}$$

The second term within the bracket gives the effect of a polarized incident beam.

The polarization of the scattered beam is given by

$$\mathbf{P}=\frac{(f(\theta)\chi_{\text{inc}},\ \boldsymbol{\sigma}f(\theta)\chi_{\text{inc}})}{(f(\theta)\chi_{\text{inc}},\ f(\theta)\chi_{\text{inc}})}. \tag{22}$$

Using (19) this becomes

$$\mathbf{P}=\frac{AA^*\mathbf{P}_{\text{inc}}+(AB^*+B^*A^*)\mathbf{n}+i(A^*B-B^*A)\mathbf{P}_{\text{inc}}\times\mathbf{n}+BB^*(2\mathbf{P}_{\text{inc}}\cdot\mathbf{nn}-\mathbf{P}_{\text{inc}})}{AA^*+BB^*+(A^*B+B^*A)\mathbf{P}_{\text{inc}}\cdot\mathbf{n}}. \tag{23}$$

If the incident beam is unpolarized this reduces to

$$\mathbf{P}=\frac{AB^*+BA^*}{AA^*+BB^*}\mathbf{n}=P(\theta)\mathbf{n}. \tag{24}$$

Thus in this case the resulting polarization is directed along the normal to the scattering plane and is dependent upon the interference between the two parts of the scattered wave. Large polarizations can come about only when this interference term is comparable to the cross section itself.

III. DOUBLE SCATTERING

The question of a double scattering experiment is easily treated by comparing (24) and (21). If the first scattering takes place at an angle θ_1 and in a plane defined by $\mathbf{n}_1$ and if θ_2, $\mathbf{n}_2$ are the corresponding quantities for the second scattering the differential cross section becomes

$$\sigma(\theta)=(AA^*+BB^*)[1+P(\theta_1)P(\theta_2)\mathbf{n}_1\cdot\mathbf{n}_2] \tag{25}$$

for an originally unpolarized beam.

The asymmetry of the second scattered beam can be clearly shown by considering the case when both scatterings take place in the same plane. Then either $\mathbf{n}_1=\mathbf{n}_2$ or $\mathbf{n}_1=-\mathbf{n}_2$ and the ratio R of the scattered intensities for these two cases is

$$R=\frac{1+P(\theta_1)P(\theta_2)}{1-P(\theta_1)P(\theta_2)}. \tag{26}$$

Thus, if large polarizations can be obtained R can be quite large.

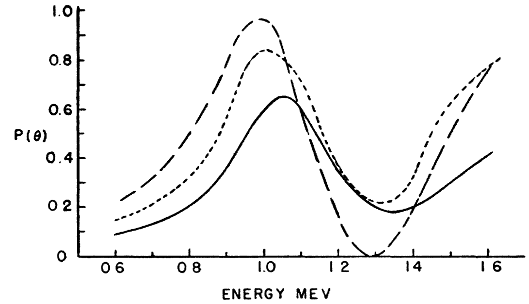

FIG. 2. The calculated degree of polarization, $P(\theta)$, as a function of the incident neutron energy, for scattering angles $\theta=120°$, 135°, 150°. — — Polarization at $\theta=120°$; - - - - Polarization at $\theta=135°$; ——— Polarization at $\theta=150°$.

IV. APPLICATION TO SCATTERING OF NEUTRONS BY HELIUM

If we apply our results to helium scattered neutrons, assuming resonant P scattering and ordinary non-resonant S scattering, the functions A and B of Eq. (20) become

$$A(\theta)=\frac{1}{k}\left[\exp(i\delta_0)\sin\delta_0+\left(\frac{\Gamma}{E_{3/2}-E-\frac{1}{2}i\Gamma}+\frac{1}{2}\frac{\Gamma}{E_{1/2}-E-\frac{1}{2}i\Gamma}\right)\cos\theta\right]$$

$$B(\theta)=\frac{\sin\theta}{2ik}\left(\frac{\Gamma}{E_{3/2}-E-\frac{1}{2}i\Gamma}-\frac{\Gamma}{E_{1/2}-E-\frac{1}{2}i\Gamma}\right). \tag{27}$$

If we substitute these expressions in (21) they yield the

TABLE I. P—phase shifts.

	η_0	η_1	η_2
Symmetric theory	0.074	−0.054	−0.017
Charged theory	0.531	−0.114	−0.046
Neutral theory	−1.02	0.995	0.073

ordinary Bloch formula[6] for the differential cross section when the incident neutron beam is unpolarized.

The polarization to be expected for an initially unpolarized beam can be obtained from (24). If we introduce the abbreviations

$$x=(E_{3/2}-E_{1/2})/\Gamma,\quad x_{1/2}=(E_{1/2}-E)/\Gamma,\quad x_{3/2}=(E_{3/2}-E)/\Gamma,\quad d_{1/2}=x_{1/2}^2+\tfrac{1}{4}\quad d_{3/2}=x_{3/2}^2+\tfrac{1}{4} \tag{28}$$

the degree of polarization[10] $P(\theta)$ can be written as

$$P(\theta)=\frac{\left[\left(\frac{1}{d_{1/2}}-\frac{1}{d_{3/2}}\right)\frac{\sin 2\delta_0}{4}-\left(\frac{x_{3/2}}{d_{3/2}}-\frac{x_{1/2}}{d_{1/2}}\right)\sin^2\delta_0-\frac{3}{4}\frac{x}{d_{1/2}d_{3/2}}\cos\theta\right]\sin\theta}{\left\{\sin^2\delta_0+\frac{1}{4}\left(\frac{1}{d_{1/2}}+\frac{1}{d_{3/2}}\right)-\frac{1}{2}\left(\frac{x_{1/2}x_{3/2}+\frac{1}{4}}{d_{1/2}d_{3/2}}\right)+\left[\left(\frac{x_{3/2}}{d_{3/2}}+\frac{1}{2}\frac{x_{1/2}}{d_{1/2}}\right)\sin 2\delta_0+\left(\frac{1}{d_{3/2}}+\frac{1}{2}\frac{1}{d_{1/2}}\right)\sin^2\delta_0\right]\cos\theta+3\left[\frac{1}{4}\cdot\frac{1}{d_{3/2}}+\frac{1}{2}\frac{(x_{1/2}+x_{3/2}+\frac{1}{4})}{d_{1/2}d_{3/2}}\right]\cos^2\theta\right\}}. \tag{29}$$

This expression has been used to calculate the polarization effects due to helium scattering of neutrons for the following parameters

$$x=1,\quad \sin^2\delta_0=0.5,\quad E_{1/2}=1.0,\quad E_{3/2}=1.3 \text{ (Mev)}. \tag{30}$$

The results are indicated in Figs 1 and 2. It is to be noted that there are regions of almost full polarization where the polarization does not vary too rapidly with energy and angle.

The effects of a double scattering experiment can be estimated directly from the graphs provided that the energy loss of the neutron at the first encounter

$$E_{\text{loss}}=[2m_n m_\alpha/(m_n+m_\alpha)^2]E(1-\cos\theta) \tag{31}$$

is taken into account. The result for the intensity ratio R can be determined from Eq. (26). Thus, for the case of neutron energy 1.2 Mev and scattering angle $\theta_1=90°$ (c.g. system) $P(\theta_1)=-0.85$. The energy loss at this angle of scattering is $E_{\text{loss}}=0.385$ Mev so that if the neutron is scattered again through $\theta_2=90°$, $P(\theta_2)=0.8$ the ratio R becomes

$$R=\frac{1+(0.85)(0.8)}{1-(0.85)(0.8)}=5.25. \tag{32}$$

V. SCATTERING OF POLARIZED NEUTRONS BY PROTONS

The scattering of neutrons by protons at 15.3 Mev has been discussed by Rarita and Schwinger[11] for various types of meson theories. It is of some interest to investigate the effect on their results if the incident neutrons were polarized. The results of their paper can be taken over almost completely, except that in performing averages over the spin we must average only over the spin direction of the scattering nucleus. Determination of the polarization effect can also be simplified by neglecting the states $^3D_1$ and $^3F_1$ and the variation of the $^3S_1$ phase shift with the magnetic quantum number. Neglect of the $^3D_1$ state is permissible for, on averaging over the spin of the nucleus, all polarization effects due to $S-D$ interference vanish. Polarization effects due to $P-D$ interference are, of course, very small compared with those arising from $S-P$ interference.

According to reference 11 the $^3P_0$ and $^3P_1$ components of the incident beam

$$3i\frac{g_1(kr)}{kr}\frac{\mathbf{k}_0\cdot\mathbf{r}}{kr}\chi_1^m \tag{33}$$

can be written as

$$^3P_0:\ \frac{1}{2}i\frac{g_1(kr)}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{k}_0}{k}\chi_1^m$$
$$^3P_1:\ \frac{3}{2}i\frac{g_1(kr)}{kr}\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{k}_0}{k}\chi_1^m. \tag{34}$$

The $^3P_2$ component can be expressed as the difference between (33) and the sum of these two expressions.

The result for the scattering amplitude is, after the inclusion of $^3S_1$ scattering,

$$f(\theta)=\frac{1}{k}\left[S+A\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{k}_0}{k}+B\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{r}}{r}\times\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\frac{\mathbf{k}_0}{k}+C\frac{\mathbf{k}_0\cdot\mathbf{r}}{kr}\right]\chi_{\text{inc}}^T, \tag{35}$$

[10] This expression was originally found by Schwinger and was applied to the case of 90° scattering (reference 5). I wish to thank Professor Schwinger for letting me see his result in this case.

[11] W. Rarita and J. Schwinger, Phys. Rev. 59, 556 (1941).

where $\chi_{\text{inc}}{}^T$ is the triplet part of the incident spin function

$$\chi_{\text{inc}}{}^T=\tfrac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\chi_{\text{inc}}. \tag{36}$$

The coefficients S, A, B, C depend on the phases η_0, η_1, η_2 due to $^3P_0$, $^3P_1$, $^3P_2$ scattering respectively and the S phase shift δ.

$$\begin{aligned} A&=(e^{i\eta_0}\sin\eta_0-e^{i\eta_2}\sin\eta_2)\\ B&=\tfrac{3}{2}(e^{i\eta_1}\sin\eta_1-e^{i\eta_2}\sin\eta_2)\\ C&=3e^{i\eta_2}\sin\eta_2 \quad S=e^{i\delta}\sin\delta. \end{aligned} \tag{37}$$

The differential cross section for triplet scattering can now be found from the relation

$$\sigma(\theta)=(f(\theta)\chi_{\text{inc}}{}^T,\quad f(\theta)\chi_{\text{inc}}{}^T). \tag{38}$$

Insertion of $f(\theta)$ in this expression yields:

$$\begin{aligned}\sigma(\theta)=\sigma_0(\theta)+&\frac{\mathbf{P}_{\text{inc}}\cdot\mathbf{n}\sin\theta}{4k^2}\\ &\times\{[\tfrac{3}{2}\sin\eta_2\sin\eta_1\sin(\eta_2-\eta_1)\\ &+\sin\eta_2\sin\eta_0\sin(\eta_2-\eta_0)\\ &+3\sin\eta_1\sin\eta_0\sin(\eta_1-\eta_0)]\cos\theta\\ &+\sin\delta[-5\sin\eta_2\sin(\eta_2-\delta)\\ &+3\sin\eta_1\sin(\eta_1-\delta)\\ &+2\sin\eta_0\sin(\eta_0-\delta)]\}. \end{aligned} \tag{39}$$

Here $\sigma_0(\theta)$ is the ordinary cross section for unpolarized neutrons given in reference 11. The effect of the polarization is represented by the term in (39) proportional to the incident polarization, $\mathbf{P}_{\text{inc}}$.

The values for the P phase shifts calculated in reference 11 are indicated in Table I. The value of the $^3S_1$ phase shift δ has been taken as the average of the real parts of the $^3S_1$ phase shifts for magnetic quantum numbers $m=\pm1$ and 0. This value is $\delta=-1.67$. The contribution of the polarization to the cross section has been calculated from (39) and added to the ordinary unpolarized cross section found in reference 11 for $^3S_1+{}^3D_1$, ^1S, and ^1P and $^3S_1$, ^3P interference. The results are:

Symmetric Theory

$$\begin{aligned}\sigma(\theta)d\omega=0.606[&(1-0.080\cos\theta+0.77\cos^2\theta)\\ &+\mathbf{P}_{\text{inc}}\cdot\mathbf{n}\sin\theta(0.017+0.001\cos\theta)]d\omega/4\pi.\end{aligned}$$

Charged Theory

$$\begin{aligned}\sigma(\theta)d\omega=0.657[&(1+0.126\cos\theta+0.042\cos^2\theta)\\ &+\mathbf{P}_{\text{inc}}\cdot\mathbf{n}\sin\theta(-0.183+0.030\cos\theta)]d\omega/4\pi.\end{aligned} \tag{40}$$

Neutral Theory

$$\begin{aligned}\sigma(\theta)d\omega=0.852[&(1+0.932\cos\theta+0.457\cos^2\theta)\\ &+\mathbf{P}_{\text{inc}}\cdot\mathbf{n}\sin\theta(0.047-0.513\cos\theta)]d\omega/4\pi.\end{aligned}$$

Thus the asymmetry is appreciable only for the two latter cases.

VI. CONCLUSION

Although the formulas derived in the first sections of this paper have not been applied to the case of proton scattering, a few general remarks can be made about this effect, since it is fully covered by our results. The large background scattering provided by the Coulomb field will tend to block the realization of full polarization. This can be achieved for neutron scattering, since the denominator of (24) (which is proportional to $\sigma(\theta)$) can become comparable to its numerator. For charged particles $\sigma(\theta)$ will contain the Rutherford scattering term and its interference with the specifically nuclear scattering so that full polarization may not be conveniently realized.

Polarization effects can come about regardless of whether a scattering resonance exists (e.g., $N-P$ scattering). One can expect such effects for He^4 scattering of neutrons in the region of 2.5 to 3.1 Mev since Barschall and Wheeler[12] have shown that the scattering at these energies can only be understood on the basis of a spin-orbit interaction which splits the P phases. Of course, the presence of a resonance is desirable, since it insures large cross sections.

The large intensity ratios which can be expected for the case of scattered neutrons in a double scattering experiment come about because in some of the cases (e.g., 90°), the polarization roughly reverses direction as one passes from one resonance level to the next, so that if the level splitting is comparable to the energy loss at the first encounter large polarizations occur at both scatterings and R is large.

Finally, it may be remarked that similar results might be expected for neutron scattering from carbon which appears to exhibit a resonance[13] of the helium type at energies of 3.6 and 4.1 Mev. For this nucleus, the energy loss for 90° scattering is $\frac{1}{6}$ of the energy of the incident neutron, so that for an incident neutron of energy 4.1 Mev the scattered neutron's energy is 3.4 Mev. Thus, one might hope that carbon exhibits effects similar to that of helium.

In conclusion, I wish to acknowledge my indebtedness to Professor Schwinger for suggesting the general problem of the production and detection of polarized neutrons and protons by helium scattering, as well as for showing me some of his earlier results and for his kind advice.

Note:† The referee has kindly called to my attention recent work[14,15] on helium scattered neutrons which casts serious doubt on its interpretation in terms of a doublet resonance with splitting comparable to the widths involved, and hence on the applicability

[12] J. A. Wheeler and H. H. Barschall, Phys. Rev. **58**, 682 (1940).
[13] Goldsmith, Ibser, and Feld, Rev. Mod. Phys. **19**, 266 (1947).
† This note was added March 16, 1950.
[14] T. A. Hall, Phys. Rev. **77**, 411 (1950).
[15] Bashkin, Petree, Mooring and Peterson, Phys. Rev. **77**, 748 (1950).

of the polarization curves herein presented. These results were prepared, however, not only to exhibit the effects to be expected in this case but were intended also to be illustrative of the general behavior of the polarization in the neighborhood of such a resonance.

The author does believe, however, that the situation in regard to helium scattering is still fluid in spite of the careful work of Bashkin *et al.* on the total cross section and will so remain until crucial experiments on the differential cross section have been performed.

PHYSICAL REVIEW VOLUME 79, NUMBER 1 JULY 1, 1950

Electrical Resistance of Thoria

W. E. DANFORTH AND F. H. MORGAN
Bartol Research Foundation of The Franklin Institute, Swarthmore, Pennsylvania

(Received March 6, 1950)

Measurements of the electrical conductivity of thoria were taken in vacuum at temperatures up to 2073°K. Activation by passage of current resulted in values of resistance as low as 1 ohm-cm at 1900°K and 10 ohm-cm at 1000°K. Activation energies between 3.2 volts and 0.58 volt were found. The density of impurity centers was computed as 10^{18} per cc and was found to be independent of the degree of activation by current, a result which is inconsistent with the hypothesis of the electrolytic origin of impurity centers.

I. INTRODUCTION

STUDIES of the electrical conductivity of thoria have obvious practical interest as regards cathode design and performance. In addition, there are several objectives of a theoretical nature. An understanding of the conduction mechanism is a necessary step in the understanding of the mechanism of thermionic emission and also of the causes of the disintegration of cathode material by passage of current. Previous investigations have been made by Foex[1] whose values of resistance are much higher than those we have found and by Wright[2] who briefly reported some data which have been found to fit satisfactorily into the general pattern of our observations.

II. EXPERIMENTAL PROCEDURE

Our measurements have been taken on sintered sleeves of thoria, mounted between molybdenum end pieces as indicated in Fig. 1. The material was molded to size and sintered. It was not pressed. The density was approximately 7 g/cm^2. Currents up to 7.6 amp./cm^2 were passed through the specimens and potentials were were measured at the probe leads, by oscilloscope for pulsed data, and high resistance voltmeter for the dc runs. Pyrometer readings were made on the inside of the specimen through a hole in one of the end pieces.

The specimens were mounted in a small vacuum furnace whose main features are shown in Fig. 2. The thoria piece, whose length was about 2 cm and outside diameter 3 mm, was supported within a coil of 60 mil tungsten.

The data to be presented were taken with this furnace mounted in a water-cooled copper jacket which was continuously pumped on a mercury system. Since the system contained one soft-soldered joint, it could not be baked at high temperature but was maintained at 100°C for 24 hours. Pressures were below 10^{-7} mm with a cold specimen but would rise to 5×10^{-6} mm at a specimen temperature of 1800°C, the maximum used at present.

III. RESULTS

Figure 3 shows the results of measurements taken on a specimen prepared from C.P. thoria from Eimer and Amend. Before any current was passed through the thoria it was outgassed at 1800° for several hours. Then runs of resistance *vs.* temperature were taken using single pulses, produced by a simple condenser discharge arrangement, and an oscilloscope.

For each one of these curves, several runs were taken, a smooth curve was drawn through the data, and points from this average curve were transferred to the logarithmic plot. The probable error estimated from the spread of the data is about 10 percent.

As shown in Fig. 3a, it was found that the data from pulse measurements require two exponential components, one with activation energy of 0.86 volt and the other of 3.2 volts. Activation energy is here defined as the quantity E in the Eq. $\sigma=\sigma_0 e^{-E/kT}$.

One must remember, of course, that we have a porous, sintered, specimen and not a crystal. It is perhaps appropriate, therefore, (following Loosjes and Vink[3]) to ascribe the low activation energy, the one predominant at the low temperature, to conductance through the solid particles, and the high temperature portions of the curve with a work function of 3.2 to thermionic emission across the interstices.

The pore-conduction hypothesis, however, is by no means established in the present case. It would, in fact,

[1] M. Foex, Comptes Rendus **215**, 534 (1942).
[2] D. A. Wright, Proc. Phys. Soc. London **B62**, 188 (1949).
[3] R. Loosjes and H. J. Vink, J. App. Phys. **20**, 884 (1949).