than that used by James and Coolidge[13] in their fundamental investigation of the H_2 molecule often is needed, the authors intend to repeat the energy calculations (eventually the magnetic shielding) for the H_2 molecule by the aid of the wave function

$$\psi_0=\psi\{1+c_1r_{12}+c_2(r_{1a}-r_{2b})^2\}+\bar{\psi}\{1+c_1r_{12}+c_2(r_{1b}-r_{2a})^2\} \tag{37}$$

which should be at least as good as the corresponding third-order wave function of the He atom.

[13] H. M. James and A. S. Coolidge, J. Chem. Phys. 12, 825 (1933).

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On the Energy-Momentum Tensor of the Electron

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The first-order radiative correction to the energy-momentum tensor of the electron is investigated. In order to avoid the ambiguities connected with the occurrence of divergent integrals a regulization with auxiliary masses is introduced. It is shown that a procedure which is in accordance with the conservation laws necessarily has some of the features of an auxiliary field theory. The method of the present calculation corresponds to an introduction of an auxiliary neutral vector-meson field which is coupled to the electron-positron field with an imaginary coupling constant.

I. INTRODUCTION

On investigating the effects of the vacuum field fluctuations on the field source, difficulties arise due to ambiguities in the interpretation of divergent integrals. The possibility of obtaining a non-vanishing photon self-energy, in contradiction to the gauge invariance of the underlying theory, illustrates this situation.[1] Since these ambiguities occur within the framework of a covariant formalism, there is some hope that on insisting on the formal properties of the theory: covariance, gauge invariance, and the validity of conservation laws, we might find a way to get rid of these difficulties.

On the other hand, one can argue that since these ambiguities are connected with the occurrence of divergent expressions, an invariant limiting process that makes these expressions finite should resolve these ambiguities. The first point of view, supported especially by Schwinger,[2,3] furnishes a set of rules of interpretation; thus, for instance, gauge invariance implies the invariance of a certain integral in momentum space under translations of the integration variable. (For further detail we refer to a forthcoming paper by S. Borowitz and W. Kohn.)

An investigation along the lines of the second point of view[4] exhibits the remarkable fact that the postulates of gauge invariance and conservation laws strongly reduce the possible number of different regularization procedures. A common feature of all of the admissible regularization methods seems to be their "realistic" aspect; this means, more precisely, that apart from the condition of a hermitian interaction, which in general is violated,[5,6] these methods can be interpreted as auxiliary field theories in the sense of Pais' *f*-field.[7]

In the present paper the second point of view is applied to the problem of the radiative corrections to the energy-momentum tensor of the electron. A straightforward calculation may lead to results in contradiction to the conservation laws, which require that

$$\partial T_{\mu\nu}(x)/\partial x_\nu=0.$$

An equivalent expression of this fact is the result of Pais and Epstein.[8] The difficulty of this special problem was first resolved by Rohrlich.[9] His results are contained in the present calculation as a special case.

Section II will be concerned with the establishment of the expression for the first-order radiative correction to the one-particle part of $\tilde{T}_{\mu\nu}$. In Section III the regularization will be discussed, and finally in Section IV the actual form of $\tilde{T}_{\mu\nu}(p'p)$ will be determined.

II. CALCULATION OF THE FIRST-ORDER RADIATIVE CORRECTION

The symmetrized gauge-invariant energy-momentum tensor of the interacting photon and electron-positron fields can be written as

$$T_{\mu\nu}(x)=\tfrac{1}{2}(F_{\mu\lambda}F_{\nu\lambda}+F_{\nu\lambda}F_{\mu\lambda}-\tfrac{1}{2}\delta_{\mu\nu}F_{\lambda\sigma}{}^2)+\tfrac{1}{4}(\bar{\psi}\gamma^{(\mu}\partial_{\nu)}\psi-(\partial_{(\nu}{}^*\bar{\psi})\gamma^{\mu)}\psi). \tag{1}$$

[1] G. Wentzel, Phys. Rev. 74, 1070 (1948).
[2] J. Schwinger, Phys. Rev. 76, 790 (1949), Appendix.
[3] Borowitz, Kohn, and Schwinger, Phys. Rev. 78, 345 (1950).
[4] W. Pauli and F. Villars, Rev. Mod. Phys. 21, 434 (1949).
[5] D. Feldman, Phys. Rev. 76, 1369 (1949).
[6] W. Jost and J. Rayski, Helv. Phys. Acta 22, 457 (1949).
[7] A. Pais, Verh. Ned. Akad. Amsterdam XIX, No. 1 (1947).
[8] A. Pais and S. Epstein, Rev. Mod. Phys. 21, 445 (1949).
[9] F. Rohrlich, Phys. Rev. 77, 357 (1950).

$A_{(\mu}B_{\nu)}=A_\mu B_\nu+A_\nu B_\mu$ and $\partial=\partial/\partial x-ieA(x)$; $\partial^*=\partial/\partial x+ieA(x)$. For the purpose of the present investigation it is convenient to express (1) in terms of operators which describe the physical electron, that is, satisfy a free particle equation, in which the original mass m_0 is replaced by the total mass m. In terms of these operators, the interaction between the two fields is

$$\mathcal{H}'=-\int d^3x(j_\mu(x)A_\mu(x)+h_s(x))$$
$$=\int d^3x(h_I(x)-h_s(x)). \quad (2)$$

Expression (1) does not refer to a definite representation as long as $i(\partial/\partial x_4)$ has the signification of the total time derivative

$$\partial\Omega/\partial x_4=\partial'\Omega/\partial x_4+[\mathcal{H}',\Omega].$$

In introducing the above mentioned representation into (1), $T_{\mu\nu}$ acquires the form

$$T_{\mu\nu}{}^P+T_{\mu\nu}{}^E+\delta_{(\nu 4}\vartheta_{\mu)},$$
$$T_{\mu\nu}{}^P=\tfrac{1}{2}(F_{\mu\lambda}F_{\nu\lambda}+F_{\nu\lambda}F_{\mu\lambda}-\tfrac{1}{2}\delta_{\mu\nu}F_{\lambda\sigma}{}^2), \quad (3a)$$

$$T_{\mu\nu}{}^E=\tfrac{1}{4}\left(\bar\psi\gamma^{(\mu}\frac{\partial'\psi}{\partial x_{\nu)}}-\frac{\partial'\bar\psi}{\partial x_{(\nu}}\gamma^{\mu)}\psi\right)-\tfrac{1}{2}ic(\bar\psi\gamma^{(\mu}\psi\cdot A_{\nu)}), \quad (3b)$$

$$\theta_\mu=\tfrac{1}{4}ic\bar\psi(x)(\gamma^\mu\gamma^4\gamma^\lambda+\gamma^\lambda\gamma^4\gamma^\mu)\psi(x)\cdot A_\lambda(x)$$
$$-\tfrac{1}{32}ie^2\int d^4x'(\bar\psi(x)\gamma^\mu\gamma^4M_F(x-x')\psi(x')+(\leftarrow))$$
$$-\tfrac{1}{32}e^2\int\int d^4x'd^4x''\delta(t-t')$$
$$\times(\bar\psi(x')\gamma^\mu S(x'-x'')M_F(x''-x)\psi(x)+(\leftarrow)). \quad (3c)$$

$(\leftarrow)$ indicates the reversed expression:

$$\bar\psi(x)M_F(x-x'')S(x''-x')\gamma^\mu\psi(x)$$

The e^2—self-energy density $h_s{}^{(2)}(x)$ in the above expression has been written as

$$-\tfrac{1}{8}ie^2\int d^4x'(\bar\psi(x)M_F(x-x')\psi(x')$$
$$+\bar\psi(x')M_F(x'-x)\psi(x)), \quad (4)$$
$$M_F=\gamma^\lambda S_F\gamma^\lambda\cdot D_F,$$

that is, in the form which follows directly from its definition (see below).

The variation in time of the state vector Ψ in the representation chosen above obeys the Schroedinger equation

$$i(\partial\Psi/\partial t)=\mathcal{H}'\Psi.$$

Write $\Psi(t)=S(t)\Psi_0$. Then

$$S(t)=P\left[\exp\left(-i\int^t dt'\mathcal{H}(t')\right)\right],$$

where P is the operator of ordering in time introduced by Dyson.[10]

The self-energy operator is defined by the requirement that a one-particle state must be an eigenstate of the system, which implies that

$$\langle S(\infty)\rangle_1=1. \quad (5)$$

This furnishes a recursive definition of the self-energy corrections of different orders. In first order in $\alpha=e^2/4\pi$ Eq. (5) yields

$$\tfrac{1}{2}(-i)^2\int\int dx_1dx_2\langle P(h_I(x_1),h_I(x_2))\rangle_1$$
$$-(-i)\int d^4xh_s{}^{(2)}(x)=0$$

from which (4) can be extracted as the operator representing the self-energy density.

If we adopt the definition $S_F=S^{(1)}-2i\bar S$, $D_F=D^{(1)}-2i\bar D$, it follows that

$$\epsilon(t-t')\langle P(\psi(x),\bar\psi(x'))\rangle_0=-\frac{1}{2}S_F(x-x'), \quad (6a)$$

but

$$\epsilon(t-t')\left\langle P\left(\frac{\partial\psi(x)}{\partial x_\nu}\bar\psi(x')\right)\right\rangle_0$$
$$=-\frac{1}{2}\frac{\partial S_F(x-x')}{\partial x_\nu}+i\delta_{\nu 4}\gamma^4\delta(x-x'). \quad (6b)$$

The energy-momentum tensor representing the physical electron: $\tilde T_{\mu\nu}(x)$, may be written as

$$\tilde T_{\mu\nu}(x)=\left\langle P\left[T_{\mu\nu}(x),\exp\left(-i\int_{-\infty}^{+\infty}\mathcal{H}(t')dt'\right)\right]\right\rangle_1. \quad (7)$$

(This expression actually represents $S(\infty)\cdot\bar T_{\mu\nu}$ (where $\bar T_{\mu\nu}$ is the transformed operator in the sense[11] of Schwinger). This however reduces to $\tilde T_{\mu\nu}$ in view of the mass renormalization. See Eq. (5).)

In the following we confine ourselves to the first-order radiative correction: $\tilde T_{\mu\nu}{}^{(1)}$. This expression includes non-covariant parts by θ_μ, (3), and through (6b), but these are easily shown to cancel.

Let us write $\tilde T_{\mu\nu}{}^{(1)}=\tilde T_{\mu\nu}{}^P+\tilde T_{\mu\nu}{}^E$, representing the parts that arise from the photon field and electron field

[10] F. J. Dyson, Phys. Rev. **75**, 486 (1949).
[11] J. Schwinger, Phys. Rev. **76**, 790 (1949).

tensors, respectively. From (7) it follows that

$$\tilde{T}_{\mu\nu}{}^{P}(x) = -\frac{1}{2}\int\int dx_1 dx_2 \langle P(T_{\mu\nu}{}^{P}(x), h_I(x_1), h_I(x_2))\rangle_1.$$

Introducing

$$I_{\alpha\beta;\epsilon\zeta} = \int\int dx_1 dx_2 \bar{\psi}(x_1)\gamma^{\alpha} S_F(x_1-x_2)\gamma^{\beta}\psi(x_2) \times(\partial D_F(x_1-x)/\partial x_\epsilon)(\partial D_F(x_2-x)/\partial x_\zeta),$$

we can write this as

$$\tilde{T}_{\mu\nu}{}^{P} = -\tfrac{1}{8}e^2(I_{\lambda\lambda,(\mu\nu)} + I_{(\mu\nu),\lambda\lambda} - I_{\lambda(\mu,\nu)\lambda} - I_{(\mu\lambda,\lambda\nu)} - \delta_{\mu\nu}(I_{\lambda\lambda,\sigma\sigma} - I_{\lambda\sigma,\sigma\lambda})). \quad (8)$$

In a similar manner we get

$$\tilde{T}_{\mu\nu}{}^{E} = \frac{e^2}{32}\int\int dx_1 dx_2 \Bigg(\bar{\psi}(x_1)\gamma^{\lambda}\Bigg[S_F(x_1-x)\gamma^{(\mu}\frac{\partial S_F(x-x_2)}{\partial x_{\nu)}} - \frac{\partial S_F(x_1-x)}{\partial x_{(\nu}}\gamma^{\mu)}S_F(x-x_2)\Bigg]\gamma^{\lambda}\psi(x_2)\Bigg) D_F(x_1-x_2)$$
$$+\frac{e^2}{32}\int\int dx_1 dx_2 \Bigg(\bar{\psi}(x_1) M_F(x_1-x_2) \Bigg[S_F(x_2-x)\gamma^{(\mu}\frac{\partial\psi}{\partial x_{\nu)}} - \frac{\partial S_F(x_2-x)}{\partial x_{(\nu}}\gamma^{\mu)}\psi(x)\Bigg] + \text{h.c.}\Bigg)$$
$$-\frac{ie^2}{8}\int dx_1 (\bar{\psi}(x_1)\gamma^{(\mu} S_F(x_1-x) \times D_F(x_1-x)\gamma^{\nu)}\psi(x) + \text{h.c.}). \quad (9)$$

In this last formula the second line is not quite correct insofar as it represents the expression omitting the terms appearing through the mass renormalization:

$$\frac{1}{4}\int dx_1 \Bigg\langle P\Bigg(\bar{\psi}\gamma^{(\mu}\frac{\partial\psi}{\partial x_{\nu)}} - \frac{\partial\bar{\psi}}{\partial x_{(\nu}}\gamma^{\mu)}\psi,\; h_s{}^{(2)}(x_1)\Bigg)\Bigg\rangle_1$$

and the second and third line of (3c). If we include these terms, then in the second line of (9), $S_F(x_2-x)$ and $\partial S_F(x_2-x)/\partial x_\nu$ are to be replaced by

$$-i(\bar{S}(x_2-x) + \tfrac{1}{2}\epsilon(x_1-x)S(x_2-x)) \quad (10)$$

and its derivative

$$-i(\partial/\partial x_\nu)(\bar{S}(x_2-x) + \tfrac{1}{2}\epsilon(x_1-x)S(x_2-x)).$$

III. THE CONDITIONS FOR VANISHING DIVERGENCE OF THE REGULARIZED TENSOR

Let us introduce the two "force densities" f_ν, defined by

$$f_\nu{}^{P} = \partial\tilde{T}_{\mu\nu}{}^{P}/\partial x_\nu, \quad f_\nu{}^{E} = (\partial\tilde{T}_{\mu\nu}{}^{E}/\partial x_\nu). \quad (11)$$

These f_ν can be determined by a straightforward calculation, making use only of the equations

$$\Box^2 D_F(x) = 2i\delta(x), \quad (\gamma^{\lambda}(\partial/\partial x^{\lambda}) + m)S_F(x) = 2i\delta(x) \quad (12)$$

and some integrations by parts. The result is[12]

$$f_\nu{}^{E} = -f_\nu{}^{P} = \tfrac{1}{4}i\int dx_1 \Bigg[(\bar{\psi}(x_1)\gamma^{\lambda}S_F(x_1-x) \times\gamma^{\lambda}\psi(x) + (\leftarrow))\frac{\partial D_F(x_1-x)}{\partial x_\nu} - (\bar{\psi}(x_1)\gamma^{\nu} \times S_F(x_1-x)\gamma^{\lambda}\psi(x) + (\leftarrow))\frac{\partial D_F(x_1-x)}{\partial x_\lambda}\Bigg]. \quad (13)$$

Thus the divergence of the complete tensor apparently vanishes. It should be observed, however, that f_ν is represented by a quadratically divergent integral in momentum space. A direct computation in momentum space will therefore in general not give zero for $f_\nu{}^{E} + f_\nu{}^{P}$, since all we can expect for $f_\nu{}^{E}$ and $f_\nu{}^{P}$ are two integrals that differ by a translation of the integration variables.[3]

From this situation it does not follow that any invariant regularization process that makes the expressions for $f_\nu{}^{P}$ and $f_\nu{}^{E}$ finite would yield automatically zero for the divergence of $\tilde{T}_{\mu\nu}$. The result (13) is based essentially on the wave equations (12), and any regularization of the D_F or the S_F function destroys the validity of the corresponding equation. If for instance we introduced individually regularized D-functions

$$(D_F(x))_R = D_F(x) - D_F{}^{(M)}(x) = \frac{-2i}{(2\pi)^4}\int_0 d^4k e^{ikx}\int_0^1 du \frac{M^2}{(k^2+M^2u)^2},$$

$f_\nu{}^{E}$ would go over into the corresponding regularized expression, but this is not the case for $f_\nu{}^{P}$ which takes the form

$$-\frac{M^2}{8}\int\int dx_1 dx_2 (\lambda,\lambda)[D_F{}^{(M)}(x_1-x)(\partial D_F(x_2-x)/\partial x_\nu)_R + (\partial D_F(x_1-x)/\partial x_\nu)_R D_F{}^{(M)}(x_2-x)] + a\ similar\ term.$$

$$(\lambda_1,\lambda) = \bar{\psi}(x_1)\gamma^{\lambda}S_F(x_i-x_2)\gamma^{\lambda}\psi(x_2)$$

Another still purely formal device would consist in regularizing $\tilde{T}_{\mu\nu}{}^{(1)}$ itself and defining the regularized expression by

$$(\tilde{T}_{\mu\nu}{}^{(1)})_R = \tilde{T}_{\mu\nu}{}^{(1)}(S_F, D_F) - \tilde{T}_{\mu\nu}{}^{(1)}(S_F, D_F{}^{(M)}). \quad (14)$$

Again $f_\nu{}^{E}$ is then replaced by the corresponding regularized expression. In the calculation of $(f_\nu{}^{P})_R$ the wave equation for $D_F{}^{(M)}$ comes into play which intro-

[12] To derive expression for $f_\nu{}^{E}$, Eq. (9) has been used without corrections (10). It is easy to verify that the term affected by (10) gives no contribution to $f_\nu{}^{E}$.

duces an additional term $\sim M^2$; in this way $(f_\nu{}^P)$ goes over into

$$(f_\nu{}^P)_R - M^2\frac{1}{8}\int\int dx_1dx_2\{(\bar{\psi}(x_1)\gamma^\lambda S_F(x_1-x_2)$$
$$\times\gamma^\nu\psi(x_2)+(\leftarrow))D_F{}^{(M)}(x_1-x)[\partial D_F{}^{(M)}(x_2-x)/\partial x_\lambda]$$
$$-\text{a term with } (\lambda,\nu) \text{ interchanged}\}. \quad (15)$$

To get rid of this additional term we are guided by the statement that this regularization has already some features of an "auxiliary field theory," but with the "unrealistic" feature of an imaginary coupling, *i.e.*, of this auxiliary (neutral vector-meson) field to the electron field.[5] This fact suggests the inclusion of the complete energy-momentum tensor of this auxiliary field into the system considered, and the discarding for the moment of the non-hermitian character of the interaction with the electron field. This procedure changes the situation in two respects: (a) The components ϕ_ν of the auxiliary field can no longer be quantized as independent scalars[12] and instead of $\delta_{\mu\nu}D_F{}^{(M)}(x)$ we have to put $(\delta_{\mu\nu}-M^{-2}\partial_\mu\partial_\nu)D_F{}^{(M)}(x)$. It is easy to show however that the terms arising from $M^{-2}\partial_\mu\partial_\nu$ never contribute to $\tilde{T}_{\mu\nu}$, which is just an expression of the gauge invariance of this operator.[13] (b) The energy-momentum tensor of the vector meson field differs from $T_{\mu\nu}{}^P$ by a term

$$M^2(\phi_\mu\phi_\nu-\tfrac{1}{2}\delta_{\mu\nu}\phi_\lambda\phi_\lambda),$$

which contributes

$$\frac{1}{8}M^2\int\int dx_1dx_2(\bar{\psi}(x_1)\gamma^{(\mu}S_F(x_1-x_2)\gamma^{\nu)}\psi(x_2)$$
$$-\delta_{\mu\nu}\bar{\psi}(x_1)\gamma^\lambda S_F(x_1-x_2)\gamma^\lambda\psi(x_2))$$
$$\cdot D_F{}^{(M)}(x_1-x)\cdot D_F{}^{(M)}(x_2-x). \quad (16)$$

to the first-order radiative correction $\tilde{T}_{\mu\nu}{}^{(1)}$. One immediately sees that the divergence of (16) and the additional term in (15) cancel.

Such a device can no longer be called a purely formal prescription. The aim of such a formal method would have consisted in providing a generally applicable rule which should be formulated in such a way as to be in accordance with the conservation laws and gauge-invariance, which are both formal properties of the non-regularized theory. Apparently this program has not yet been carried through in a satisfactory way, and the actual situation seems to indicate that only a realistic theory (f-field)[7] will be able to re-establish a satisfactory situation.

IV. CALCULATION OF $\tilde{T}_{\mu\nu}(p', p)$

Let $\bar{\psi}(p')\exp(-ip'\cdot x)$ and $\psi(p)\cdot\exp(+ip\cdot x)$ be Fourier-components of $\bar{\psi}(x)$ and $\psi(x)$, respectively. $\tilde{T}_{\mu\nu}{}^{(1)}(x)$ may then be written as an integral over terms of the form

$$\bar{\psi}(p')e^{-ip'x}\tilde{T}_{\mu\nu}(p'p)e^{ipx}\psi(p). \quad (17)$$

The operator $\tilde{T}_{\mu\nu}(p'p)$ thus introduced has the general form

$$\Delta p_\mu\Delta p_\nu f(\Delta p^2)+\delta_{\mu\nu}g(\Delta p^2)$$
$$+(i\gamma^{(\mu}P_{\nu)})h(\Delta p^2)+P_\mu P_\nu k(\Delta p^2), \quad (18)$$
$$(\Delta p=p'-p;\quad P=p'+p).$$

The symmetry of $\tilde{T}_{\mu\nu}(p'p)$ in p and p' is a necessary condition for the validity of the conservation laws

$$\Delta p_\mu\tilde{T}_{\mu\nu}(p'p)=0. \quad (19)$$

(Non-symmetrical terms

$$\sim i(\gamma^{(\mu}\Delta p_{\nu)})H(\Delta p^2)+\Delta p_{(\mu}P_{\nu)}K(\Delta p^2)$$

would contribute

$$i\gamma^\nu\Delta p^2H(\Delta p^2)+P_\nu\Delta p^2K(\Delta p^2)$$

to the divergence (19), which is $\neq 0$ unless $H=K\equiv 0$.)

In the symmetrical expression (18), $g(\Delta p^2)$ will be written as

$$S+\Delta p^2\cdot\bar{g}(\Delta p^2).$$

The conservation laws then require that

$$S=0,\quad \bar{g}(\Delta p^2)=-f(\Delta p^2).$$

It is the purpose of this paragraph to show anew how the regularization device which proved to be successful in Section III leads to a result satisfying (19). But furthermore it will be explicitly shown that the final result is now finite and independent of the auxiliary mass M up to terms $O(M^{-2}\log M)$. Finally the actual form of the functions $f(\Delta p^2)$, $h(\Delta p^2)$ and $k(\Delta p^2)$ may be of some interest.

Contribution of $\tilde{T}_{\mu\nu}{}^P(x)$

This part is expressed in terms of

$$I_{\alpha\beta;\epsilon\zeta}(x)=\int\int d\xi d\eta\bar{\psi}(x+\xi)\gamma^\alpha S_F(\xi-\eta)\gamma^\beta\psi(x+\eta)$$
$$\times\partial D_F(\xi)/\partial\xi_\epsilon\cdot\partial D_F(\eta)/\partial\eta_\zeta.$$

The corresponding $I_{\alpha\beta;\epsilon\zeta}(p'p)$, defined through (17), reads

$$I_{\alpha\beta;\epsilon\zeta}(p'p)=\frac{8i}{(2\pi)^4}\int d^4k\gamma^\alpha(i\gamma k-m)\gamma^\beta$$
$$\times\frac{(k_\epsilon-p_\epsilon')(k_\zeta-p_\zeta)}{(k^2+m^2)(k-p')^2(k-p)^2}. \quad (20)$$

(It should be noticed that in the terms $(\mu\lambda, \lambda\nu)$, $(\lambda\mu, \nu\lambda)$ and $(\sigma\lambda, \lambda\sigma)$ a part containing (k^2+m^2) in the numerator may be isolated. These parts are easily shown to give a vanishing contribution, their removal

[13] This argument is of purely formal character, since the expressions involved are divergent. A proof which avoids the divergences must make use of Stueckelberg's B-field formalism (see W. Pauli, Rev. Mod. Phys. **13**, 203 (1941), Part II/2) to describe the neutral vector meson.

in this stage of the calculation however is an essential simplification of the results.)

To regularize these expressions in the manner outlined, we write

$$\frac{1}{a\cdot b\cdot c}-\frac{1}{a(b+M^2)(c+M^2)}=\frac{1}{a}\left\{\frac{M^2}{b\cdot c\cdot(c+M^2)}+\frac{M^2}{b(b+M^2)(c+M^2)}\right\},$$

$$a=k^2+m^2;\quad b=(k-p')^2;\quad c=(k-p)^2.$$

On introducing properly chosen Feynman parameters, this can be written as

$$3!M^2\int_0^1 du\int_0^u dv\int_0^u dw[(k-p(u-v)-p'v)^2+m^2(1-u)^2+\Delta p^2(u-v)v+M^2w]^{-4}.$$

If $k'=k-p(u-v)-p'v$ is introduced as new variable the numerator in (20) takes the form

$$k'^2\Gamma_1+\Gamma_2$$

and a straightforward integration yields

$$I_{\alpha\beta;\epsilon\zeta}(p'p)=\frac{-2}{(2\pi)^2}\int_0^1 du\int_0^u dv\left[2\Gamma_1\log\frac{C+M^2u}{C}+\Gamma_2\left(\frac{1}{C}-\frac{1}{C+M^2u}\right)\right],$$

$$C=m^2(1-u)^2+\Delta p^2(u-v)v.$$

For the purpose of the following discussion it is convenient to introduce the variables

$$z=u,\quad zt=2v-u.$$

With (8) it follows now that

$$\tilde{T}_{\mu\nu}{}^P(p'p)=\frac{-e^2}{8}\sum I(p'p)=\frac{e^2}{4(2\pi)^2}\int_0^1 dt\int_0^1 zdz\times\left[I\cdot\log\frac{C+M^2z}{C}+II\left(\frac{1}{C}-\frac{1}{C+M^2z}\right)\right],\quad(21)$$

where

$$I=2\sum\Gamma_1=2z[m\delta_{\mu\nu}+(i\gamma^{(\mu}P_{\nu)})],$$

$$II=\sum\Gamma^2=m\Delta p_\mu\Delta p_\nu[(z-2)z^2t^2+z]-2m^3\delta_{\mu\nu}[(z-1)^2+\epsilon^2z^2(1-t^2)]+mP_\mu P_\nu(z-1)^2z-m^2(i\gamma^{(\mu}P_{\nu)})[(z-1)^2(z+2)+\epsilon^2z^3(1-t^2)],\quad(21a)$$

$$C=m^2[(z-1)^2+\epsilon^2z^2(1-t^2)],\quad\epsilon^2=\Delta p^2/4m^2.$$

It is easy to verify that the traces of I and II vanish separately. In the following we shall omit the contributions from $II.\ (C+M^2\cdot z)^{-1}$, which are of the order $M^{-2}\cdot\log M$. A partial integration with respect to t splits the logarithmic term into a part independent of Δp^2 and a part proportional to Δp^2. Thus

$$\tilde{T}_{\mu\nu}{}^P(p'p)=\frac{e^2}{2(2\pi)^2}\int_0^1 dz\left[z^2(m\delta_{\mu\nu}+(i\gamma^{(\mu}P_{\nu)}))\times\log\frac{C_0+M^2z}{C_0}-mz\delta_{\mu\nu}\right]+\frac{e^2}{4(2\pi)^2}\int_0^1 dt\int_0^1 zdz$$
$$\times\{m^2(\Delta p_\mu\Delta p_\nu-\Delta p^2\delta_{\mu\nu})z^3t^2-m^2(i\gamma^{(\mu}P_{\nu)})[(z-1)^2(z+2)+\epsilon^2z^3(1+3t^2)]+mP_\mu P_\nu(z-1)^2z+m\Delta p_\mu\Delta p_\nu(z-2z^2t^2)\}\cdot 1/C,\quad(22)$$
$$(C_0=C(\epsilon=0)).$$

An explicit calculation shows that the last term $\sim\Delta p_\mu\Delta p_\nu$ vanishes. The constant (that is, Δp^2-independent) terms are easily shown to give ($\mu=M/m$):

$$\frac{+e^2}{2(2\pi)^2}\left[m\delta_{\mu\nu}\left(\frac{1}{3}\log(1+\mu^2)+\frac{11}{18}\right)+(i\gamma^{(\mu}P_{\nu)})\left(\frac{1}{3}\log(1+\mu^2)+\frac{10}{9}\right)\right].\quad(23)$$

Contribution to $\tilde{T}_{\mu\nu}(p'p)$ from $\tilde{T}_{\mu\nu}{}^E(x)$

From (9) it follows that

$$T_{\mu\nu}{}^E(p'p)=\frac{e^2}{32(2\pi)^4}\left[\int d^4k\gamma^\lambda S_F(p'-k)\times(i\gamma^{(\mu}\cdot P_{\nu)}-2k_{\nu)})S_F(p-k)\gamma^\lambda\cdot D_F(k)+\int d^4q\int d^4\xi\left(e^{-i(p'-q)\xi}M_F(q)\left(-i\bar{S}(q)-\frac{i}{2}\epsilon(\xi_0)S(q)\right)\times\left(\gamma^{(\mu}\overleftarrow{\frac{\partial}{\partial\xi_{\nu)}}}+i\gamma^{(\mu}p_{\nu)}\right)+(p'\rightleftarrows p)\right)\right]$$
$$-\frac{ie^2}{8}(\gamma^{(\mu}(S_FD_F)(p)\gamma^{\nu)}+\gamma^{(\mu}(S_FD_F)(p')\gamma^{\nu)}).\quad(24)$$

Introducing

$$M_F(q)=(i\gamma q+m)\cdot F(\rho)+G(\rho),\quad\rho=q^2+m^2,$$

the second line becomes

$$\int d^4q\int d^4\xi\left(e^{-i(p'-q)\xi}\left(-F(\rho)+(i\gamma q-m)\frac{G(\rho)-G(0)}{\rho}\right)\times(i\gamma^{(\mu}p_{\nu)})+(p'\rightleftarrows p)\right)$$
$$=-2(2\pi)^4(F(0)+2mG'(0))(i\gamma^{(\mu}P_{\nu)}).$$

Before performing the k space integrations we substitute $D_F - D_F^{(M)}$ for D_F. We have then

$$M_F(q)=\frac{2i}{(2\pi)^2}\int_0^1 dz(i\gamma p(1-z)+2m) \times\log\frac{m^2z^2+M^2(1-z)+\rho(z-z^2)}{m^2z^2+\lambda^2(1-z)+\rho(z-z^2)}, \quad (25a)$$

and

$$(S_FD_F)(q)=\frac{-i}{(2\pi)^2}\int_0^1 dz(i\gamma p(1-z)-m) \times\log\frac{m^2z^2+M^2(1-z)+\rho(z-z^2)}{m^2z+\lambda^2(1-z)+\rho(z-z^2)}. \quad (25b)$$

For the first line of (24) the technique already outlined yields

$$\frac{-e^2}{8(2\pi)^2}\int_0^1 dt\int_0^1 zdz\left[I'\cdot\log\frac{C'+M^2(1-z)}{C'} +II'\left(\frac{1}{C'}-\frac{1}{C'+M^2(1-z)}\right)\right]. \quad (26)$$

Here a small "photon mass" λ has been introduced to avoid difficulties at the lower limit of the k-integration. We have also

$$I'=4m(z+1)\delta_{\mu\nu}+(z-1)(i\gamma^{(\mu}P_{\nu)}),$$

$$II'=(1-z)[(2-4z+z^2)m^2 +(1-z+\tfrac{1}{4}z^2(1-t^2))\Delta p^2](i\gamma^{(\mu}P_{\nu)}) +2m(z-1)^2zP_\mu P_\nu+2m(z+1)z^2t^2\Delta p_\mu\Delta p_\nu, \quad (26a)$$

$$C'=m^2[z^2(1+\epsilon^2(1-t^2))+\eta^2(1-z)],$$

$$\epsilon^2=\Delta p^2/4m^2, \quad \eta=\lambda/m.$$

The second and third lines of (24) are, by (25)

$$\frac{-e^2}{8(2\pi)^2}\int_0^1 dz\left[(3(1-z)(i\gamma^{(\mu}P_{\nu)})-4mz\delta_{\mu\nu}) \times\log\frac{C_0'+M^2(1-z)}{C_0'}-2m^2z(1-z^2)(i\gamma^{(\mu}P_{\nu)}) \left(\frac{1}{C_0'}-\frac{1}{C_0'+M^2(1-z)}\right)\right], \quad (27)$$

In (27) and also in the factor multiplying II', the terms $\sim(C'+M^2(1-z))^{-1}$ will be dropped again. By a partial integration with respect to t we get:

$$T_{\mu\nu}{}^E(p'p)=\frac{-e^2}{8(2\pi)^2}\int_0^1 dt\int_0^1 zdz\{2m(z+1)z^2t^2 \times[\Delta p_\mu\Delta p_\nu-\Delta p^2\delta_{\mu\nu}]+2m(z-1)^2zP_\mu P_\nu +(1-z)[(2-4z+z^2)m^2+(1-z+\tfrac{1}{4}z^2(1-t^2))\Delta p^2] \times(i\gamma^{(\mu}P_{\nu)})\}\cdot\frac{1}{C'}-\frac{e^2}{8(2\pi)^2}\int_0^1 dz\left[(4mz^2\delta_{\mu\nu} +(3-z)(1-z)(i\gamma^{(\mu}P_{\nu)}))\log\frac{C_0'+M^2(1-z)}{C_0'} -2m^2z(1-z^2)(i\gamma^{(\mu}P_{\nu)})\cdot\frac{1}{C_0'}\right]. \quad (28)$$

The constant terms contribute the quantity

$$\frac{-e^2}{2(2\pi)^2}\left[m\delta_{\mu\nu}\left(\frac{1}{3}\log(1+\mu^2)-\frac{7}{18}\right) +(i\gamma^{(\mu}P_{\nu)})\left(\frac{1}{3}\log(1+\mu^2)+\frac{65}{72} -\frac{1}{2}m^2\int_0^1 dz(1-z^2)/C_0'\right)\right]. \quad (29)$$

By comparison of this result with (23) it is seen that all contributions proportional to $\log(1+\mu^2)$ cancel. There remains, however, a finite part containing $\delta_{\mu\nu}$:

$$S=\frac{e^2}{2(2\pi)^2}m\delta_{\mu\nu}\frac{11+7}{18}=\frac{\alpha}{2\pi}m\delta_{\mu\nu}. \quad (30)$$

This corresponds just to the result of Pais and Epstein,[8] where it is deduced by calculating the so-called stress that

$$S=\tilde{T}_{11}{}^{(1)}(pp)=\tfrac{1}{3}(\tilde{T}_{\mu\mu}{}^{(1)}-\tilde{T}_{44}{}^{(1)}).$$

But $\tilde{T}_{44}{}^{(1)}=0$ (this result is due to our representation, compare Section II, in contradistinction to the situation in references 2 and 9). Since $\bar{T}_{\mu\mu}{}^P=0$, we should have

$$S=\tfrac{1}{3}\cdot\tilde{T}_{\mu\mu}{}^E(pp) \quad (31)$$

and this can indeed be verified from (28) and (29):

$$\tilde{T}_{\mu\mu}{}^E(pp)=\frac{e^2m}{2(2\pi)^2}\left\{\int_0^1 dz(-4+3z+z^2)+4\left(\frac{7}{18}+\frac{65}{72}\right)\right\} =\frac{\alpha}{2\pi}m\left(-4+\frac{3}{2}+\frac{1}{3}+\frac{31}{6}\right)=3\cdot\frac{\alpha}{2\pi}m.$$

The stress vanishes, as was shown in Section III, Eq. (16), if one includes the additional term arising through the energy-momentum tensor of the auxiliary field

$$M^2(\phi_\mu\phi_\nu - \tfrac{1}{2}\delta_{\mu\nu}\phi_\lambda^2). \quad (32)$$

Its contribution to $\tilde{T}_{\mu\nu}^{(1)}(p'p)$ is

$$\frac{-e^2M^2}{4(2\pi)^2}\int_0^1 dt\int_0^1 zdz[2m\delta_{\mu\nu}+z(i\gamma^{(\mu}P_{\nu)})]\cdot\frac{1}{C+M^2Z} \quad (33)$$

The minus sign in (33) is due to the choice of an imaginary coupling *ie* of this auxiliary field.

Neglecting again terms of the order $M^{-2}\cdot\log M$, Eq. (33) gives

$$-(e^2/2(2\pi)^2)[m\delta_{\mu\nu}+\tfrac{1}{4}(i\gamma^{(\mu}P_{\nu)})]. \quad (34)$$

Thus the formerly remaining stress-part (30) is now cancelled and a divergence free energy-momentum tensor is obtained. There remains, however, an infra-red term in the third line of (28). Because of the factor Δp^2 this term plays no role in the energy-momentum four-vector $P_\mu = \int T_{\mu 4}d^3x$. It indicates, however, that as soon as we try to localize energy and momentum and to build up wave packets, the well-known failure of the power series approach in constructing of the physical electron becomes manifest. Expressed in terms of the operators of the present approximation, the physical electron carries still a cloud of virtual low energy photons with itself, the effect of which is discarded on going to the one-particle, no-photon part of $\tilde{T}_{\mu\nu}$. Thus, to investigate the energy-momentum density distribution, the above calculation would have to be completed by a sort of Bloch-Nordsieck treatment, but which is not within the scope of the present investigation.

The discussion of the energy-momentum four vector P_μ displays again the necessity for the inclusion of (32) into the system, in a way which, of course, is connected with the self-stress difficulty. Since the change in the physical properties of the electron due to its interaction with the radiation field vacuum consists only in a change of its mass, and since this change has already been included in the representation of the electron field, $\tilde{T}_{\mu\nu}{}^P + \tilde{T}_{\mu\nu}{}^E$ should give no contribution at all to P_μ. From this statement it follows that the coefficients of $(i\gamma^{(\mu}P_{\nu)})$ and $P_\mu P_\nu$ should vanish too for $\Delta p^2 = 0$. This fact is easily verified from (22), (23), (28), (29), and (34). The inclusion of the contribution $\sim(i\gamma^{(\mu}P_{\nu)})$ in (34) is essential and exhibits again the failure of a purely formal mass regularization.

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Microwave Collision Diameters I. Experimental*

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The diameters of fifteen polar and non-polar gases for collisions with ammonia are obtained from measurements of the pressure broadening of the ammonia 3–3 inversion line in the mixed gases at low pressures. The design of the spectroscope used and the experimental technique involved are discussed.

I. INTRODUCTION

PRESSURE broadening of spectral lines has been studied for many years in the optical and infra-red regions. In 1936 a review of work done up to that time was given by Margenau and Watson.[1] The difficulties encountered in this region are great. A large Doppler breadth and insufficient resolution hamper measurements at low pressures while at high pressures multiple collisions complicate the process. Because of these and other difficulties the problem suffered from neglect and never was brought to satisfactory completion.

More recently, the expansion of microwave spectroscopy has caused new interest to arise and a number of theoretical and experimental papers have appeared.[2] In the microwave region, resolution is high and Doppler breadth is only about 70 kc. At breadths of the order of a megacycle, the more intense lines, especially the ammonia 3–3 inversion line, may easily be displayed on an oscilloscope for observation.

In the case of collision broadening in a pure gas, the

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