the contribution of Li^6, the scattering length of lithium at zero energy equals $\frac{2}{3}(a-\gamma^2/E_r)$, or $-0.7\cdot10^{-13}$ cm which is numerically of only qualitative significance because of the uncertainty in estimating the parameters γ^2 and E_r and the approximation involved in using the single level resonance theory.

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Nuclear Dipole Vibrations*

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RESONANCES recently observed by several authors[1] in (γ, x) processes at γ-energies $\hbar\omega\cong20$ Mev have been interpreted by Goldhaber and Teller[1] as dipole vibrations due to collective motion of all protons relative to all neutrons inside the nucleus. Goldhaber and Teller treated in detail a very simplified model assuming periodic displacement of a rigid proton sphere relative to a rigid neutron sphere. However it seemed worth while to discuss the more plausible model, mentioned only shortly by these authors, of an interpenetrating motion of the proton fluid with density $\rho_p(r, t)$, and neutron fluid with density $\rho_n(r, t)$, under the condition of constant total density $\rho_0=\rho_p+\rho_n$, and fixed nuclear radius $R=r_0A^{\frac{1}{3}}$. This model, besides representing a case of a peculiar hydrodynamics, does not require any arbitrary parameters.

The "symmetry-energy term" in the expression for the nuclear binding energy[2] per nucleon: $K\cdot(Z-N)^2/A^2$, with $K\cong20$ Mev, can be interpreted as $K\cdot(\rho_p-\rho_n)^2/\rho_0^2$ because of the short range of nuclear forces. By multiplication with ρ_0 we obtain an energy density:

$$\epsilon(\rho_p-\rho_n)=\rho_0\cdot K(\rho_r-\rho_n)^2/\rho_0^2=K\cdot(2\rho_p-\rho_0)^2/\rho_0, \quad (1)$$

which is a function only of the local density-difference. This term is to be inserted into the Lagrangian L of hydrodynamics,

$$L=\int dV\{M[\rho_p\dot{\phi}_p+\rho_n\dot{\phi}_n]-\tfrac{1}{2}M\rho_p(\mathrm{grad}\phi_p)^2 +\rho_n(\mathrm{grad}\phi_n)^2]-\epsilon(\rho_p-\rho_n)\} \quad (2)$$

with M the nucleonic mass. From $\delta\int Ldt=0$ follows Euler's equation for the relative velocity $\mathbf{v}=(\mathbf{v}_p-\mathbf{v}_n)=-(\mathrm{grad}\phi_p-\mathrm{grad}\phi_n)$:

$$Md\mathbf{v}/dt=-(\partial^2\epsilon/\partial\rho_p^2)\,\mathrm{grad}\rho_p=-(8K/\rho_0)\,\mathrm{grad}\rho_p, \quad (3)$$

together with the continuity equation

$$\dot{\rho}_n=-\dot{\rho}_p=\mathrm{div}\{(\rho_p\rho_n/\rho_0)\mathbf{v}\}. \quad (4)$$

In the customary way we have linearized the equations by omitting quadratic terms in v, $\mathrm{grad}\rho_p$, and $\dot{\rho}_p$. The velocity of propagation of relative density disturbances turns out to be

$$u=[(ZN/A^2)8K/M]^{\frac{1}{2}}\cong c/5. \quad (5)$$

Combined with the boundary condition $v_r\equiv0$, implying $\partial\rho_p/\partial r\equiv0$, at $r=R$, Eqs. (3) and (4) have been solved by Lord Rayleigh.[3] Dipole vibrations of type $\rho_p=(Z/A)\rho_0+y(r)\cos\vartheta\cdot\exp(i\omega t)$ exist with lowest eigenfrequency

$$\hbar\omega_0=2.08\cdot\hbar u/R=(4ZN/A^2)^{\frac{1}{2}}\cdot60\cdot A^{-\frac{1}{3}}\quad\text{(in Mev)}. \quad (6)$$

For comparison with experimental data see Fig. 1.

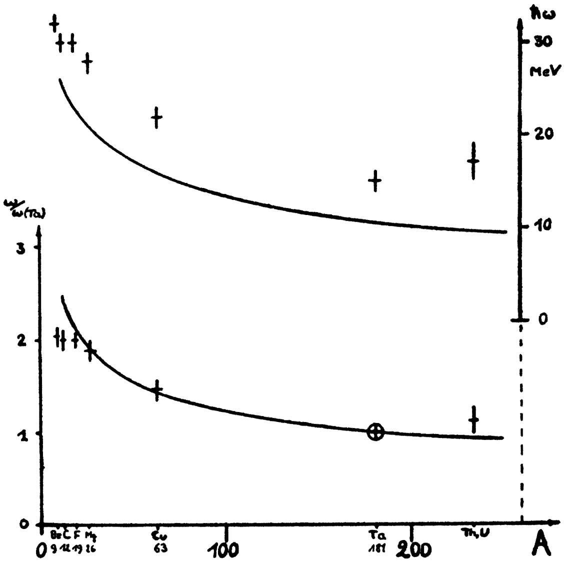

FIG. 1. Abscissa: atomic number; ordinate: resonance frequency for (γ, x). Upper curve: absolute values $\hbar\omega_0$ (in Mev); crosses experimental values. Lower curve: relative values, $\omega_0 : \omega_0(\mathrm{Ta})$.

It is easy to include in the formalism the effects of radiative damping and of damping by dissipation of collective motion into disordered motion (heating by friction[1]) followed by nucleon emission. We introduce the radiation width Γ_r,

$$\hbar\Gamma_r/2mc^2=(2NZm/3AM)(e^2/\hbar c)(\hbar\omega/2mc^2)^2 \quad (7)$$

(m = electronic mass), the total width Γ which must be taken from experiments, and the phase angle φ between incident field strength and induced nuclear dipole moment, defined by $\mathrm{tg}\varphi=\Gamma\omega/(\omega^2-\omega_0^2)$. In this motion the total cross section σ is

$$\sigma=4\pi(e^2/2mc^2)^2(NZm/AM)(e^2/\hbar c)(2mc^2/\hbar\Gamma)\sin^2\varphi, \quad (8)$$

and the scattering cross section is $\sigma_r=\sigma\Gamma_r/\Gamma$. For the integrated total cross section we obtain:

$$(\hbar/2mc^2)\int\sigma d\omega=2\pi^2(e^2/2mc^2)^2\cdot(NZm/AM)(\hbar c/e^2) \quad (9)$$

which is one-half of the corresponding value given by Goldhaber and Teller for the rigid sphere model.

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Proton Density Variation in Nuclei

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A VERY simple estimate of the variation of proton density ρ_p inside the nucleus is afforded by the assumption that the "symmetry-energy term" in the expression for the nuclear binding energy per nucleon:

$$K\cdot(Z-N)^2/A^2,\quad\text{with}\quad K\cong20\text{ Mev},$$

should be understood as resulting from an energy density

$$\epsilon(\rho_p-\rho_n)=K\cdot(\rho_p-\rho_n)^2/\rho_0=K(2\rho_p-\rho_0)^2/\rho_0, \quad (1)$$

as proposed by Steinwedel, Jensen, and Jensen in the preceding letter. This assumption should be a good approximation at least