the contribution of Li^6, the scattering length of lithium at zero energy equals $\frac{3}{4}(a-\gamma^2/E_r)$, or $-0.7\cdot 10^{-13}$ cm which is numerically of only qualitative significance because of the uncertainty in estimating the parameters γ^2 and E_r and the approximation involved in using the single level resonance theory.

* This work was supported by the AEC and by the Wisconsin Alumni Research Foundation.
† AEC Predoctoral Fellow.

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Nuclear Dipole Vibrations*

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July 10, 1950

RESONANCES recently observed by several authors[1] in (γ, x) processes at γ-energies $\hbar\omega \cong 20$ Mev have been interpreted by Goldhaber and Teller[1] as dipole vibrations due to collective motion of all protons relative to all neutrons inside the nucleus. Goldhaber and Teller treated in detail a very simplified model assuming periodic displacement of a rigid proton sphere relative to a rigid neutron sphere. However it seemed worth while to discuss the more plausible model, mentioned only shortly by these authors, of an interpenetrating motion of the proton fluid with density $\rho_p(r,t)$, and neutron fluid with density $\rho_n(r,t)$, under the condition of constant total density $\rho_0=\rho_p+\rho_n$, and fixed nuclear radius $R=r_0A^{\frac{1}{3}}$. This model, besides representing a case of a peculiar hydrodynamics, does not require any arbitrary parameters.

The "symmetry-energy term" in the expression for the nuclear binding energy[2] per nucleon: $K\cdot(Z-N)^2/A^2$, with $K\cong 20$ Mev, can be interpreted as $K\cdot(\rho_p-\rho_n)^2/\rho_0^2$ because of the short range of nuclear forces. By multiplication with ρ_0 we obtain an energy density:

$$\epsilon(\rho_p-\rho_n)=\rho_0\cdot K(\rho_r-\rho_n)^2/\rho_0^2=K\cdot(2\rho_p-\rho_0)^2/\rho_0, \qquad (1)$$

which is a function only of the local density-difference. This term is to be inserted into the Lagrangian L of hydrodynamics,

$$L=\int dV\{M[\rho_p\dot{\phi}_p+\rho_n\dot{\phi}_n]-\tfrac{1}{2}M\rho_p(\mathrm{grad}\phi_p)^2 +\rho_n(\mathrm{grad}\phi_n)^2]-\epsilon(\rho_p-\rho_n)\} \qquad (2)$$

with M the nucleonic mass. From $\delta\int Ldt=0$ follows Euler's equation for the relative velocity $\mathbf{v}=(\mathbf{v}_p-\mathbf{v}_n)=-(\mathrm{grad}\phi_p-\mathrm{grad}\phi_n)$:

$$Md\mathbf{v}/dt=-(\partial^2\epsilon/\partial\rho_p{}^2)\,\mathrm{grad}\rho_p=-(8K/\rho_0)\,\mathrm{grad}\rho_p, \qquad (3)$$

together with the continuity equation

$$\dot{\rho}_n=-\dot{\rho}_p=\mathrm{div}\{(\rho_p\rho_n/\rho_0)\mathbf{v}\}. \qquad (4)$$

In the customary way we have linearized the equations by omitting quadratic terms in v, $\mathrm{grad}\rho_p$, and $\dot{\rho}_p$. The velocity of propagation of relative density disturbances turns out to be

$$u=[(ZN/A^2)8K/M]^{\frac{1}{2}}\cong c/5. \qquad (5)$$

Combined with the boundary condition $v_r\equiv 0$, implying $\partial\rho_p/\partial r\equiv 0$, at $r=R$, Eqs. (3) and (4) have been solved by Lord Rayleigh.[3] Dipole vibrations of type $\rho_p=(Z/A)\rho_0+y(r)\cos\vartheta\cdot\exp(i\omega t)$ exist with lowest eigenfrequency

$$\hbar\omega_0=2.08\cdot\hbar u/R=(4ZN/A^2)^{\frac{1}{2}}\cdot 60\cdot A^{-\frac{1}{3}}\quad(\text{in Mev}). \qquad (6)$$

For comparison with experimental data see Fig. 1.

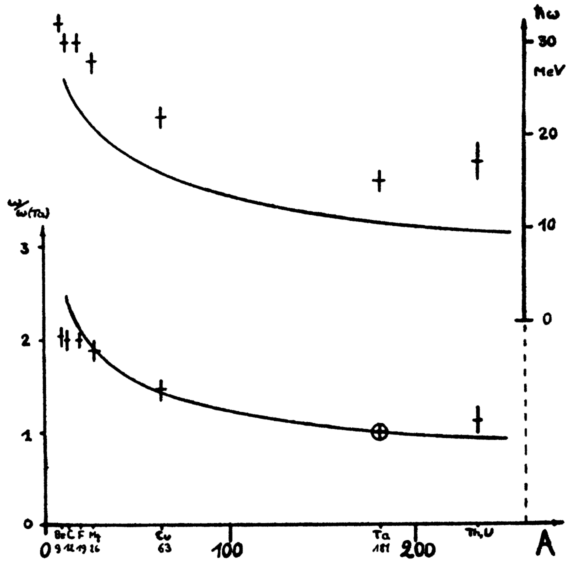

FIG. 1. Abscissa: atomic number; ordinate: resonance frequency for (γ, x). Upper curve: absolute values $\hbar\omega_0$ (in Mev); crosses experimental values. Lower curve: relative values, $\omega_0:\omega_0(\mathrm{Ta})$.

It is easy to include in the formalism the effects of radiative damping and of damping by dissipation of collective motion into disordered motion (heating by friction[1]) followed by nucleon emission. We introduce the radiation width Γ_r,

$$\hbar\Gamma_r/2mc^2=(2NZm/3AM)(e^2/\hbar c)(\hbar\omega/2mc^2)^2 \qquad (7)$$

(m = electronic mass), the total width Γ which must be taken from experiments, and the phase angle φ between incident field strength and induced nuclear dipole moment, defined by $\mathrm{tg}\varphi=\Gamma\omega/(\omega^2-\omega_0{}^2)$. In this motion the total cross section σ is

$$\sigma=4\pi(e^2/2mc^2)^2(NZm/AM)(e^2/\hbar c)(2mc^2/\hbar\Gamma)\sin^2\varphi, \qquad (8)$$

and the scattering cross section is $\sigma_r=\sigma\Gamma_r/\Gamma$. For the integrated total cross section we obtain:

$$(\hbar/2mc^2)\int\sigma d\omega=2\pi^2(e^2/2mc^2)^2\cdot(NZm/AM)(\hbar c/e^2) \qquad (9)$$

which is one-half of the corresponding value given by Goldhaber and Teller for the rigid sphere model.

* See also: J. H. D. Jensen and P. Jensen, Zeits. f. Naturforsch. **5a**, 343 (1950), and H. Steinwedel and J. H. D. Jensen, Zeits. f. Naturforsch. (to be published).

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Proton Density Variation in Nuclei

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July 10, 1950

A VERY simple estimate of the variation of proton density ρ_p inside the nucleus is afforded by the assumption that the "symmetry-energy term" in the expression for the nuclear binding energy per nucleon:

$$K\cdot(Z-N)^2/A^2,\quad\text{with}\quad K\cong 20\ \text{Mev},$$

should be understood as resulting from an energy density

$$\epsilon(\rho_p-\rho_n)=K\cdot(\rho_p-\rho_n)^2/\rho_0=K(2\rho_p-\rho_0)^2/\rho_0, \qquad (1)$$

as proposed by Steinwedel, Jensen, and Jensen in the preceding letter. This assumption should be a good approximation at least for the heavier nuclei because of the short range of nuclear forces. When combining this term with the Coulomb energy to give

$$H=\int dV\epsilon(\rho_p-\rho_n)+\tfrac{1}{2}e^2\int dVdV'\rho_p(\mathbf{r})\rho_n(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'| \quad (2)$$

we obtain from $\delta H=0$, under the condition $\rho_p+\rho_n=\rho_0=3/4\pi r_0^3$ ($r_0=e^2/2mc^2$),

$$\partial\epsilon/\partial\rho_p=(4K/\rho_0)(2\rho_p-\rho_0)=-e\Psi_p(r), \quad (3)$$

$\Psi(r)$ being the electrostatic potential due to the proton distribution. The Laplacian derivate of (3), together with Poisson's equation $\Delta\Psi_p=-4\pi e\rho_p$, yields

$$(8K/4\pi e^2\rho_0)\Delta\rho_p=\rho_p. \quad (4)$$

With $\Lambda=(8K/4\pi e^2\rho_0)^{\frac{1}{2}}=7.3\cdot r_0$ the solution of (4) is

$$\rho_p(r)=\text{const.}\sinh(r/\Lambda)/(r/\Lambda)\cong\text{const.}\{1+\tfrac{1}{6}(r/\Lambda)^2\}. \quad (5)$$

For the density ratio $\rho_p(A^{\frac{1}{3}}r_0):\rho_p(0)=1+A^{\frac{2}{3}}/320$ we find:[1] 1.12 for U^{238}, resp. 1.08 for Xe^{130}, resp. 1.04 for Ca^{48}. Variation of the nuclear density ρ_0 inside the nucleus (compressibility of nuclear matter) would slightly enlarge those ratios. However there does not seem to exist any reliable estimate of nuclear compressibility;[2] at any rate it is so small that the ratio (5) will not be altered appreciably.

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The Acceleration of Dust Grains by Supernovae

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IT has been suggested by Spitzer[1] that the pressure of radiation from supernovae might accelerate dust grains in the galaxy and thus generate heavy cosmic-ray particles. The present letter reports an investigation of several effects which much reduce the maximum grain velocities, and which make it unlikely that this proposed process can in fact produce many of the observed heavy cosmic-ray particles.

Spitzer's paper made the following simplifying assumptions. (a) Radiation pressure on a spherical grain of radius b equals $\pi b^2/c$ times the radiative flux, where c is the speed of light. (b) The effect of the grain velocity on the acceleration is negligible. (c) The supernova luminosity L is constant with time after the outburst. (d) The absorptivity of the grain for the supernova radiation equals the grain's absorptivity in the infra-red.

With assumptions (a) to (c) a potential relative to infinity can be defined, giving for the potential energy V of a grain at an initial distance r_0 from the supernova

$$V=Lb^2/4cr_0. \quad (1)$$

We now examine the validity of these assumptions.

(1) Assumption (a) should be correct to within a factor[2] of about 2 provided that $2\pi b$ is not less than the wave-length of maximum radiative flux from the supernova, and provided also that the index of refraction of the grain for supernova radiation is not close to unity. In any case, assumption (a) will not greatly underestimate the radiation pressure.

(2) The acceleration of a grain at high velocity is diminished by the reduction in the incident photon flux, the Doppler shift of the photons, and the relativistic increase of the grain's mass. When these effects are taken into account, but assumptions (a) and (c) retained, the limiting kinetic energy, T, in units of its rest mass, for a grain which started from rest at a distance r_0 from the supernova is given by

$$T+2T^2+\tfrac{4}{3}T^3+\tfrac{2}{3}(T^2+2T)^{\frac{3}{2}}=Lb^2/4c^3mr_0; \quad (2)$$

m is the rest mass of the grain. The energy found from Eq. (2) is plotted in Fig. 1; the dashed line represents Eq. (1). For an energy E of 1 Bev per nucleon the value of Lb^2/r_0 found from (2) is seven times as great as that found from (1).

(3) The radiation curve of the supernova in all frequencies may, in the absence of definite information, be taken as the observed curve[3] for photographic radiation. If r_0 is sufficiently great, v is determined by direct integration of $L(t)$, neglecting the change in r. Numerical integration of the equation of motion and graphical interpolation were used to find v in intermediate cases. Results are given in Table I for a supernova effective temperature of 10^{5}°K, about the highest value now contemplated,[4] for a corresponding[4] value of 2×10^{43} ergs/sec. for L and for a grain radius of 5×10^{-7} cm, which is about the smallest value for which assumption (a) is valid and which yields the greatest velocity. The grain mass m has been computed for a density of 4.5 g/cm³.

TABLE I. Energies attained by grains near supernova (dependence of acceleration on velocity neglected).

γ	r_0(cm)	E(Bev/nucleon)
0.01	1.35×10^{16}	1.15
1.0	1.35×10^{17}	1.66×10^{-3}
100	1.35×10^{18}	1.38×10^{-7}

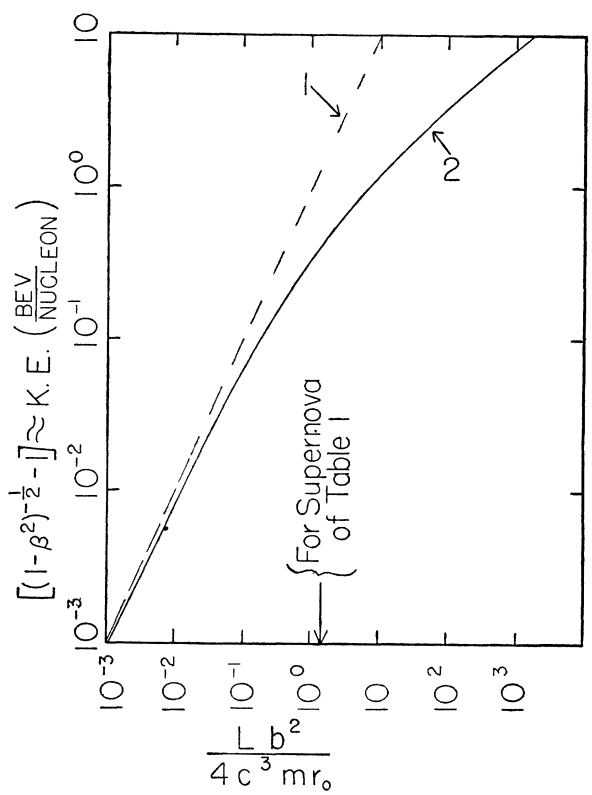

FIG. 1. Kinetic energy of the grain under assumption (c) (constant supernova radiation): curve 1 with neglect of velocity effects—see Eq. (1); curve 2 with full account of velocity effects—see Eq. (2); L is the supernova luminosity, b the radius and m the mass of the dust grain, c the velocity of light, and r_0 the initial distance of the grain from the supernova.

(4) The temperature of the grain is determined not only by the amount of radiation present but also by the absorptivities of the grain in the far ultraviolet, where the supernova radiates, and in the infra-red, where the grain radiates. Information on these absorptivities is lacking, and the results in Table I have been computed for three different values of γ, the ratio of absorptivities in the ultraviolet and infra-red, respectively. The value of r_0 is