TABLE III. The electron energies (kev) from radioactive tantalum 182. (Superscripts are arbitrary numbers assigned to the gamma-rays in the order of increasing energy.)

Electron energy	Proposed interpretation	Energy sum	Gamma-energy	Electron energy	Proposed interpretation	Energy sum	Gamma-energy
30.5	K^7	99.8		81.7	K^9	151.0	
33.9	$L_1{}^1$	46.0	46.0(1)	88.3	$L_{1,2}{}^7$	99.8	
43.9	K^8	113.2		89.6	$L_3{}^7$	99.8	99.8(7)
47.0	$L_{1,2}{}^2$	58.4		97.0	M^7	99.8	
48.2	$L_3{}^2$	58.4	58.4(2)	99.1	N^7	99.7	
53.6	$L_{1,2}{}^3$	65.1		101.2	$L_{1,2}{}^8$	113.3	
55.3	$L_3{}^3$	65.4	65.3(3)	102.8	$L_3{}^8$	113.0	113.2(8)
	$L_{1,2}{}^4$	66.9		109.2	K^{10}	178.5	
	M^2	58.3		110.3	M^8	113.1	
57.2	$L_3{}^4$	67.4	67.2(4)	128.2	K^{11}	197.5	
62.5	$L_1{}^5$	74.6		139.9	$L_1{}^9$	152.0	151.5(9)
	M^3	65.3		151.8	K^{12}	221.1	
64.7	$L_3{}^5$	74.9	74.8(5)	158.7	K^{13}	228.0	
	M^4	67.5		166.2	$L_1{}^{10}$	178.3	178.4(10)
	($L_1{}^a$)	(76.8)	76.8(a)	175.3	M^{10}	178.1	
	(K^b)	(133.8)	133.8(b)		K^d	244.6	244.6(d)
66.7	N^4	67.3		185.5	$L_1{}^{11}$	197.6	197.5(11)
72.1	M^5	74.9		193.0	K^{14}	262.3	
	$L_{1,2}{}^6$	84.2		209.5	$L_{1,2}{}^{12}$	221.2	221.1(12)
74.0	$L_3{}^6$	84.4	84.4(6)	216.1	$L_{1,2}{}^{13}$	228.1	228.0(13)
	(M^a)	(76.8)		225.1	M^{13}	227.9	
	(K^c)	(143.2)	143.2(c)	250.6	$L_{1,2}{}^{14}$	262.4	262.3(14)
81.7	M^6	84.5		259.5	M^{14}	262.3	

probably exist, but since they cannot be evaluated with the same certainty, they are omitted in this report.

All fourteen of the strong gamma-rays indicated by this interpretation fit well a proposed level scheme as shown in Fig. 6. Among other possible transitions, gamma-rays of energy 76.8, 133.8, 143.2, and 243.9 kev would also be expected. On critically examining the electron energies, it may be noted that a double interpretation of certain electron lines would in fact yield these gamma-rays. These four gamma-values are shown by arbitrary letters a, b, c, and d instead of numbers in Table III and are indicated as dotted lines in Fig. 6.

Additional high energy gamma-rays are known[4] to exist in tantalum with energies of about 1.12, 1.19, and 1.234 Mev.

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[4] C. H. Goddard and C. S. Cook, Phys. Rev. **76**, 1419 (1949); W. Rall and R. G. Wilkinson, Phys. Rev. **71**, 321 (1947).

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Quantum Electrodynamics: The Self-Energy Problem*

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In this paper it will be shown that the limiting processes by means of which the integrals of the Hamiltonian of the interacting electron-positron field with the electromagnetic field are defined, may be carried out so that the self-energies of the electrons and photons are zero.

I. INTRODUCTION

FIELD theories have been plagued by the appearance of divergent expressions for the self-energies of the elementary particles. The purpose of this paper is to show that by the use of appropriate limiting processes for the integrals which define the Hamiltonian of the interacting fields, one can in fact obtain the value zero for the self-energy integrals. Although we will work here only with the electron-positron field in interaction with the electromagnetic field, I believe that the technique used here will also enable one to eliminate the self-energy problem in other cases as well.

This work will be carried out in a momentum space representation. In this representation the Hamiltonian for the interacting electron-positron field and the electromagnetic field, may be written

$$H=\int d\mathbf{p}d\mathbf{p}'\delta(\mathbf{p}-\mathbf{p}')e_{\mathbf{p}s}(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s})+\int d\mathbf{k}d\mathbf{k}'\delta(\mathbf{k}-\mathbf{k}')ka_{\mathbf{k}\lambda}{}^{+}a_{\lambda\mathbf{k}'}-\frac{e}{2\pi}\int d\mathbf{p}d\mathbf{p}'d\mathbf{k}\frac{\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})}{(k)^{\frac{1}{2}}}$$
$$\times\{(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s'})a_{\lambda\mathbf{k}}(\mathbf{p}s|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}'s')+(\alpha_{\mathbf{p}'s'}{}^{+}\alpha_{\mathbf{p}s})a_{\lambda\mathbf{k}}{}^{+}(\mathbf{p}'s'|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}s)\}+\frac{e^2}{(2\pi)^2}\int\frac{d\mathbf{p}d\mathbf{p}'d\mathbf{p}''d\mathbf{p}'''}{|p-p'|^2}\delta(\mathbf{p}-\mathbf{p}'+\mathbf{p}''-\mathbf{p}''')$$
$$\times(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s'})(\alpha_{\mathbf{p}''s''}{}^{+}\alpha_{\mathbf{p}'''s'''})(\mathbf{p}s|1|\mathbf{p}'s')(\mathbf{p}''s''|1|\mathbf{p}'''s'''). \quad (1)$$

At this point remarks concerning the notation used in (1) are appropriate. I am using units such that $\hbar=c=1$, $e^2\simeq 1/137$. Symbols printed in bold face type are vectors, $d\mathbf{p}$ is a small volume of momentum space. $\boldsymbol{\alpha}$ is the Dirac vector with 4×4 matrix components α_1, α_2, α_3, which satisfy the relation $\alpha_i\alpha_j+\alpha_j\alpha_i=2\delta_{ij}$. The other Dirac matrix β satisfied $\beta\alpha_i+\alpha_i\beta=0$, $\beta^2=1$, with 1 the 4×4 unit matrix. By the symbol $(\mathbf{p}s|A|\mathbf{p}'s')$

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is understood $\sum_{\alpha\beta} U_\alpha^*(\mathbf{p}s)A_{\alpha\beta}U_\beta(\mathbf{p}'s')$ in which $U_\beta(\mathbf{p}'s')$ is the βth component of the Dirac spin function corresponding to an electron of momentum $\mathbf{p}'$ and spin state s'. The spin function $U(\mathbf{p}s)$ satisfies $H_\mathbf{p}U(\mathbf{p}s)=e_{\mathbf{p}s}U(\mathbf{p},s)$ with $H_\mathbf{p}=\boldsymbol{\alpha}\cdot\mathbf{p}+m\beta$, with m the mass of the electron. The spin functions are normalized so that $(\mathbf{p}s|1|\mathbf{p}s')=\delta_{ss'}$. $U_\alpha^*(\mathbf{p}s)$ is the complex conjugate of $U_\alpha(\mathbf{p}s)$. $a_{\lambda\mathbf{k}}$ is the annihilation operator and $a_{\lambda\mathbf{k}}^+$ is the creation operator for a photon of momentum $\mathbf{k}$ and direction of polarization $\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}$. λ takes on the values 1 and 2, and $\boldsymbol{\varepsilon}_{1\mathbf{k}}\cdot\mathbf{k}=\boldsymbol{\varepsilon}_{2\mathbf{k}}\cdot\mathbf{k}=\boldsymbol{\varepsilon}_{1\mathbf{k}}\cdot\boldsymbol{\varepsilon}_{2\mathbf{k}}=0$. The commutation relations satisfied by the $a_{\lambda\mathbf{k}}$ and $a_{\lambda\mathbf{k}}^+$ are

$$[a_{\lambda\mathbf{k}}, a_{\lambda'\mathbf{k}'}]=0, \quad [a_{\lambda\mathbf{k}}, a_{\lambda'\mathbf{k}'}^+]=\delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}'). \tag{2}$$

By $\delta(\mathbf{p}-\mathbf{p}')$ is meant the three-dimensional Dirac δ-function, and $\delta_{\lambda\lambda'}$ is the Kronecker δ. $\alpha_{\mathbf{p}s}$ is the annihilation operator for an electron of momentum $\mathbf{p}$ if s refers to a positive energy state; if s refers to a negative energy state then $\alpha_{\mathbf{p}s}$ is the creation operator of a positron of momentum $-\mathbf{p}$. $\alpha_{\mathbf{p}s}^+$ either creates an electron or destroys a positron according as s refers to positive or negative energy states. The commutation relations satisfied by the $\alpha_{\mathbf{p}s}^+$ and $\alpha_{\mathbf{p}s}$ are

$$[\alpha_{\mathbf{p}s}, \alpha_{\mathbf{p}'s'}]_+=0, \quad [\alpha_{\mathbf{p}s}, \alpha_{\mathbf{p}'s'}^+]_+=\delta_{ss'}\delta(\mathbf{p}-\mathbf{p}'). \tag{3}$$

The plus sign is placed on the bracket to indicate an anticommutator. Whenever a parenthesis is placed around a combination of α- and α^+-operators, it is understood that all creation operators are to be written on the left of the annihilation operators whenever this makes a difference. For example, $(\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'})=\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'}$ if either or both s or s' refer to positive energy states but $(\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'})=-\alpha_{\mathbf{p}'s'}\alpha_{\mathbf{p}s}^+$ when both s and s' refer to negative energy states. Due care must be taken as to sign so as to be consistent with the anticommutation relations (3) as in the above example. The above convention as to the meaning of $(\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'})$ then insures that the Hamiltonian (1) is the Hamiltonian of electron-positron theory.

II. THE COULOMB SELF-ENERGY

The last term of (1) gives the Coulomb interaction of the electrons and positrons with themselves and each other, and includes also the Coulomb self-energy effects to order e^2. We now observe that

$$(\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'})(\alpha_{\mathbf{p}''s''}^+\alpha_{\mathbf{p}'''s'''})$$

may be written as

$$(\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'}\alpha_{\mathbf{p}''s''}^+\alpha_{\mathbf{p}'''s'''})$$

plus additional terms. It is the additional terms which give rise to the positron and electron self-energy. We also wish to point out that the integrals which define Coulomb energy are in fact divergent and that the value of the self-energy parts of the Coulomb energy depends strongly on the limiting process by means of which such integrals over infinite domains are always defined. In considering the last term of (1) we shall suppose that the integrals over $\mathbf{p}$, $\mathbf{p}'$, $\mathbf{p}''$, and $\mathbf{p}'''$ are to be integrated over a finite region. However, for different values of the spin indices s, s', s'', and s''' these regions will in general be different. Actually, of course, because of the presence of $\delta(\mathbf{p}-\mathbf{p}'+\mathbf{p}''-\mathbf{p}''')$ in the integrand these integrals are effectively over a nine-dimensional space. One condition which we impose on these regions is that the resulting operator must be Hermitian. Another condition which we impose is that the operator shall be symmetric under the interchange of positive and negative energy states. We may now abstract from (1) the self-energy of the electrons and positrons which may be written by using projection operators for positive and negative energy states as

$$H_{cs}=\frac{e^2}{(2\pi)^2}\int d\mathbf{p}d\mathbf{p}'\delta(\mathbf{p}-\mathbf{p}') \times(\alpha_{\mathbf{p}s}^+\alpha_{\mathbf{p}'s'})(\mathbf{p}s|M_{cs}(\mathbf{p})|\mathbf{p}'s') \tag{4}$$

in which

$$\begin{aligned}M_{cs}(\mathbf{p})=&\int_{I(\mathbf{p})}\frac{d\mathbf{k}}{k^2}\Big\{\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}+H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\\&-\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}-H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\Big\}\\&+\int_{II(\mathbf{p})}\frac{d\mathbf{k}}{k^2}\Big\{\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}+H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\\&-\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}-H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\Big\}\\&+\int_{III(\mathbf{p})}\frac{d\mathbf{k}}{k^2}\Big\{\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}+H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\\&+\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}+H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\\&-\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}-H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\\&-\frac{e_\mathbf{p}+H_\mathbf{p}}{2e_\mathbf{p}}\frac{e_{\mathbf{p}-\mathbf{k}}-H_{\mathbf{p}-\mathbf{k}}}{2e_{\mathbf{p}-\mathbf{k}}}\frac{e_\mathbf{p}-H_\mathbf{p}}{2e_\mathbf{p}}\Big\}.\end{aligned} \tag{5}$$

In (5) the three different integrals carry the subscripts $I(\mathbf{p})$, $II(\mathbf{p})$, and $III(\mathbf{p})$ which indicates the finite regions in the $\mathbf{k}$ space over which the integrals are to be evaluated. These three regions depend on $\mathbf{p}$ and are not necessarily the same, as one can readily see that each term is Hermitian and invarient under the interchange of positive and negative energy states. The expression (5) for the Coulomb self-energy may

be simplified to the following form

$$M_{cs}(\mathbf{p})=\frac{H_{\mathbf{p}}}{2e_{\mathbf{p}}}\int_{I(\mathbf{p})}\frac{d\mathbf{k}}{k^2}\left(1+\frac{e_{\mathbf{p}}^2-\mathbf{p}\cdot\mathbf{k}}{e_{\mathbf{p}}e_{\mathbf{p}-\mathbf{k}}}\right)$$
$$-\frac{H_{\mathbf{p}}}{2e_{\mathbf{p}}}\int_{II(\mathbf{p})}\frac{d\mathbf{k}}{k^2}\left(1-\frac{e_{\mathbf{p}}^2-\mathbf{p}\cdot\mathbf{k}}{e_{\mathbf{p}}e_{\mathbf{p}-\mathbf{k}}}\right)$$
$$+\int_{III(\mathbf{p})}\frac{d\mathbf{k}}{k^2e_{\mathbf{p}-\mathbf{k}}}\left(\frac{\mathbf{p}\cdot\mathbf{k}}{e_{\mathbf{p}}^2}H_{\mathbf{p}}-\boldsymbol{\alpha}\cdot\mathbf{k}\right). \quad (6)$$

Define the following integrals

$$A_I(\mathbf{p})=\frac{1}{2}\int_{I(\mathbf{p})}\frac{d\mathbf{k}}{k^2e_{\mathbf{p}}};\qquad A_{II}(\mathbf{p})=\frac{1}{2}\int_{II(\mathbf{p})}\frac{d\mathbf{k}}{k^2e_{\mathbf{p}}}$$
$$B_I(\mathbf{p})=\frac{1}{2}\int_{I(\mathbf{p})}\frac{d\mathbf{k}}{k^2e_{\mathbf{p}-\mathbf{k}}};\qquad B_{II}(\mathbf{p})=\frac{1}{2}\int_{II(\mathbf{p})}\frac{d\mathbf{k}}{k^2e_{\mathbf{p}-\mathbf{k}}} \quad (7)$$
$$C_I(\mathbf{p})=\frac{1}{2}\int_{I(\mathbf{p})}\frac{d\mathbf{k}(\mathbf{p}\cdot\mathbf{k})}{k^2e_{\mathbf{p}-\mathbf{k}}e_{\mathbf{p}}^2};\qquad C_{II}(\mathbf{p})=\frac{1}{2}\int_{II(\mathbf{p})}\frac{d\mathbf{k}(\mathbf{p}\cdot\mathbf{k})}{k^2e_{\mathbf{p}-\mathbf{k}}e_{\mathbf{p}}^2}$$
$$C_{III}(\mathbf{p})=\frac{1}{2}\int_{III(\mathbf{p})}\frac{d\mathbf{k}(\mathbf{p}\cdot\mathbf{k})}{k^2e_{\mathbf{p}-\mathbf{k}}e_{\mathbf{p}}^2}.$$

Then if the integral with respect to $\mathbf{k}$ is carried out with azimuthal symmetry with respect to $\mathbf{p}$ we obtain

$$M_{cs}(\mathbf{p})=H_{\mathbf{p}}\{A_I(\mathbf{p})-A_{II}(\mathbf{p})+B_I(\mathbf{p})+B_{II}(\mathbf{p})-C_I(\mathbf{p})$$
$$-C_{II}(\mathbf{p})+2C_{III}(\mathbf{p})\}-2\boldsymbol{\alpha}\cdot\mathbf{p}e_{\mathbf{p}}^2C_{III}(\mathbf{p})/\mathbf{p}^2. \quad (8)$$

By inspection of (7) one can see that $A_I(\mathbf{p})$ and $A_{II}(\mathbf{p})$ are positive and diverge linearly with respect to the integration on $\mathbf{k}$. $B_I(\mathbf{p})$ and $B_{II}(\mathbf{p})$ are positive and diverge logarithmically with respect to integration on $\mathbf{k}$. The integrals $C_I(\mathbf{p})$, $C_{II}(\mathbf{p})$, and $C_{III}(\mathbf{p})$ may be made positive or negative or zero and in general diverge linearly with the size of the region of $\mathbf{k}$ integration. In particular we may take region $III(\mathbf{p})$ so that $C_{III}(\mathbf{p})=0$. Further, the relative shape and size of regions $I(\mathbf{p})$ and $II(\mathbf{p})$ may be taken arbitrarily many ways so that $A_I(\mathbf{p})-A_{II}(\mathbf{p})+B_I(\mathbf{p})+B_{II}(\mathbf{p})-C_I(\mathbf{p})-C_{II}(\mathbf{p})=0$, because the terms $A_I(\mathbf{p})$ and $A_{II}(\mathbf{p})$ diverge linearly while $B_I(\mathbf{p})$ and $B_{II}(\mathbf{p})$ diverge only logarithmically. This shows that the Coulomb self-energy of order e^2 may be made zero when a suitable limiting process is used to define Coulomb interaction terms of the Hamiltonian.

III. THE ELECTROMAGNETIC SELF-ENERGY OF THE ELECTRON

The interaction between the electron-positron current and the transverse electromagnetic field, as given by the third term of Eqs. (1), gives rise to an electromagnetic electron self-energy. An easy way to exhibit this self-energy is to eliminate the third terms of (1) by means of a unitary transformation.

We now transform H by means of a unitary transformation of the form e^{iS_1} in which S_1 is Hermitian, $S_1^+=S_1$. The transformed Hamiltonian, H', is given by

$$H'=e^{-iS_1}(H_0+H_1+H_2)e^{iS_1}$$
$$=H_0+\{H_1-i[S_1,H_0]\}$$
$$+\{H_2-i[S_1,H_1]-\tfrac{1}{2}[S_1,[S_1,H_0]]\}+\cdots. \quad (9)$$

The second term of (9) is of order e and will be taken to be zero by an appropriate choice of S_1. The third term of (9) is then of order e^2 and contains the electrodynamic self-energy to order e^2. We now obtain

$$[S_1,H_0]=-iH_1, \quad (10)$$

then

$$H'=H_0+H_2-(i/2)[S_1,H_1]+\cdots. \quad (11)$$

A solution of (10) for S_1 is

$$S_1=-\frac{ie}{2\pi}\int\frac{dpdp'd\mathbf{k}\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})}{(k)^{\frac{1}{2}}(e_{ps}-e_{p's'}-k)}$$
$$\times\{(\alpha_{ps}{}^{+}\alpha_{p's'})a_{\lambda\mathbf{k}}(\mathbf{p}s|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}'s')$$
$$-(\alpha_{p's'}{}^{+}\alpha_{ps})a_{\lambda\mathbf{k}}{}^{+}(\mathbf{p}'s'|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}s)\}. \quad (12)$$

Using this value of S_1 we compute the electrodynamic part of H' of order e^2

$$H_{1,1}=-\frac{i}{2}[S_1,H_1]=\frac{e^2}{8\pi^2}\int\frac{dpdp'dp''dp'''d\mathbf{k}d\mathbf{k}'\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})\delta(\mathbf{p}''-\mathbf{p}'''-\mathbf{k}')}{(kk')^{\frac{1}{2}}(e_{ps}-e_{p's'}-\mathbf{k})}$$
$$\times[(\alpha_{ps}{}^{+}\alpha_{p's'})a_{\lambda\mathbf{k}}(\mathbf{p}s|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}'s')-(\alpha_{p's'}{}^{+}\alpha_{ps})a_{\lambda\mathbf{k}}{}^{+}(\mathbf{p}'s'|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}s),$$
$$(\alpha_{p''s''}{}^{+}\alpha_{p'''s'''})a_{\lambda'\mathbf{k}'}(\mathbf{p}''s''|\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}|\mathbf{p}'''s''')+(\alpha_{p'''s'''}{}^{+}\alpha_{p''s''})a_{\lambda'\mathbf{k}'}{}^{+}(\mathbf{p}'''s'''|\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}|\mathbf{p}''s'')]. \quad (13)$$

Evaluating the commutator in (13) we obtain

$$H_{1,1}=-\frac{e^2}{8\pi^2}\Big[\int\frac{dpdp'dp''dp'''\delta(\mathbf{p}-\mathbf{p}'+\mathbf{p}''-\mathbf{p}''')}{|\mathbf{p}-\mathbf{p}'|(e_{ps}-e_{p's'}-|\mathbf{p}-\mathbf{p}'|)}(\alpha_{ps}{}^{+}\alpha_{p's'})(\alpha_{p''s''}{}^{+}\alpha_{p'''s'''})$$
$$\times(\mathbf{p}s|\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}|\mathbf{p}'s')(\mathbf{p}''s''|\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}|\mathbf{p}'''s''')+\text{C.C.}\Big]$$
$$+\frac{e^2}{8\pi^2}\Big[\int\frac{dpdp'd\mathbf{k}d\mathbf{k}'}{(kk')^{\frac{1}{2}}}\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}-\mathbf{k}')\alpha_{ps}{}^{+}\alpha_{p's'}a_{\lambda\mathbf{k}}a_{\lambda'\mathbf{k}'}$$

$$\times\Big(\mathbf{p}s\Big|\frac{\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}-\mathbf{k}}+H_{\mathbf{p}-\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}-\mathbf{k}}(e_{\mathbf{p}s}-e_{\mathbf{p}-\mathbf{k}}-k)}+\frac{\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}-\mathbf{k}}-H_{\mathbf{p}-\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}-\mathbf{k}}(e_{\mathbf{p}s}+e_{\mathbf{p}-\mathbf{k}}-k)}$$

$$-\frac{\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}'+\mathbf{k}}+H_{\mathbf{p}'+\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}'+\mathbf{k}}(e_{\mathbf{p}'+\mathbf{k}}-e_{\mathbf{p}'s'}-k)}-\frac{\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}'+\mathbf{k}}-H_{\mathbf{p}'+\mathbf{k}})\boldsymbol{\epsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}'+\mathbf{k}}(-e_{\mathbf{p}'+\mathbf{k}}-e_{\mathbf{p}'s'}-k)}\Big|\mathbf{p}'s'\Big)+\text{C.C.}\Big]$$

$$+\frac{e^2}{8\pi^2}\Big[\int\frac{d\mathbf{p}d\mathbf{p}'d\mathbf{k}d\mathbf{k}'}{(kk')^{\frac{1}{2}}}\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}'-\mathbf{k})\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s'}a_{\lambda'\mathbf{k}'}{}^{+}a_{\lambda\mathbf{k}}$$

$$\times\Big(\mathbf{p}s\Big|\frac{\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}-\mathbf{k}}+H_{\mathbf{p}-\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}-\mathbf{k}}(e_{\mathbf{p}s}-e_{\mathbf{p}-\mathbf{k}}-k)}+\frac{\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}-\mathbf{k}}-H_{\mathbf{p}-\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}-\mathbf{k}}(e_{\mathbf{p}s}+e_{\mathbf{p}-\mathbf{k}}-k)}$$

$$-\frac{\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}'+\mathbf{k}}+H_{\mathbf{p}'+\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}}{e_{\mathbf{p}'+\mathbf{k}}(e_{\mathbf{p}'+\mathbf{k}}-e_{\mathbf{p}'s'}-k)}-\frac{\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'}\cdot\boldsymbol{\alpha}(e_{\mathbf{p}'+\mathbf{k}}-H_{\mathbf{p}'+\mathbf{k}})\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}}{2e_{\mathbf{p}'+\mathbf{k}}(-e_{\mathbf{p}'+\mathbf{k}}-e_{\mathbf{p}'s'}-k)}\Big|\mathbf{p}'s'\Big)+\text{C.C.}\Big]. \quad (14)$$

As was the case in my earlier discussion of the Coulomb self-energy, we consider the integral defining H_1 to be integrated over a certain finite region in $(\mathbf{p}, \mathbf{p}')$ space which region will ultimately be taken in the limit to include all the $(\mathbf{p}, \mathbf{p}')$ space. The particular way in which this limit is taken, strongly influences the value which $H_{1.1}$ approaches. There is no loss in supposing that if the point $(\mathbf{p}, \mathbf{p}')$ is in this finite region then the point $(-\mathbf{p}, -\mathbf{p}')$ is also in this region. To preserve the charge symmetry of the theory we may now take the region for $(e_{\mathbf{p}s}>0;\ e_{\mathbf{p}'s'}>0)$ to be the same as the region $(e_{\mathbf{p}s}<0;\ e_{\mathbf{p}'s'}<0)$ and label this region, $I(\mathbf{p}, \mathbf{p}')$; we must also take the region for $(e_{\mathbf{p}s}<0;\ e_{\mathbf{p}'s'}>0)$ to be the same as the region for $(e_{\mathbf{p}s}>0;\ e_{\mathbf{p}'s'}<0)$ and label this region, $II(\mathbf{p}, \mathbf{p}')$.

The electromagnetic self-energy may now be obtained from (14) and is

$$H_{ems}=\frac{e^2}{2(2\pi)^2}\int d\mathbf{p}d\mathbf{p}'(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s'})(\mathbf{p}s|M_{ems}(\mathbf{p})|\mathbf{p}'s')\delta(\mathbf{p}-\mathbf{p}'), \quad (15)$$

in which

$$M_{ems}(p)=\int_{A(\mathbf{p})}\frac{d\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|}\Big\{\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}+H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}$$

$$\frac{-e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}-H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}\Big\}\frac{1}{e_{\mathbf{p}}-e_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|}$$

$$+\int_{B(\mathbf{p})}\frac{d\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|}\Big\{\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}+H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}$$

$$-\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}-H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}\Big\}\frac{1}{-e_{\mathbf{p}}-e_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|}$$

$$+\int_{C(\mathbf{p})}\frac{d\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|}\Big\{\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}+H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}$$

$$-\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}-H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}\Big\}\frac{1}{e_{\mathbf{p}}-e_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|}$$

$$+\int_{D(\mathbf{p})}\frac{d\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|}\Big\{\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}+H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}$$

$$-\frac{e_{\mathbf{p}}-H_{\mathbf{p}}}{2e_{\mathbf{p}}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}'}-H_{\mathbf{p}'}}{2e_{\mathbf{p}'}}\boldsymbol{\varepsilon}_{\lambda\mathbf{p}-\mathbf{p}'}\cdot\boldsymbol{\alpha}\frac{e_{\mathbf{p}}+H_{\mathbf{p}}}{2e_{\mathbf{p}}}\Big\}\frac{1}{-e_{\mathbf{p}}-e_{\mathbf{p}'}-|\mathbf{p}-\mathbf{p}'|}+\text{C.C.} \quad (16)$$

In (16) the region $A(\mathbf{p})$ over which $\mathbf{p}'$ is integrated is the region common to $I(\mathbf{p}, \mathbf{p}')$ and $I(\mathbf{p}', \mathbf{p})$ for fixed $\mathbf{p}$. The integration region, $B(\mathbf{p})$ for $\mathbf{p}'$ is the region common to $II(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}', \mathbf{p})$ with fixed $\mathbf{p}$. The integration region, $C(\mathbf{p})$, for $\mathbf{p}'$ is the region common to $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}', \mathbf{p})$ with fixed $\mathbf{p}$. The integration region, $D(\mathbf{p})$, for $\mathbf{p}'$ is the region common to $II(\mathbf{p}, \mathbf{p}')$ and $I(\mathbf{p}', \mathbf{p})$ with fixed $\mathbf{p}$. As was the case for the Coulomb self-energy we may write here

$$M_{ems}(\mathbf{p}) = A(\mathbf{p}) H_{\mathbf{p}} + \mathbf{B}(\mathbf{p}) \cdot \boldsymbol{\alpha}. \tag{17}$$

Here $A(\mathbf{p})$ is composed of the sum of two logarithmically divergent integrals plus the difference between two linearly divergent integrals, plus some indefinite linearly divergent integrals. The vector $\mathbf{B}(\mathbf{p})$ of (17) is an indefinite linearly divergent integral. Thus, we see that by an appropriate choice of the regions $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ both $A(\mathbf{p})$ and $\mathbf{B}(\mathbf{p})$ may be made to vanish.

IV. THE PHOTON SELF-ENERGY

We have already computed the expression (14) from which the self-energy of the photon may be obtained. To make the self-energy vanish it is not necessary to have the two regions $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ different. Since making these two regions the same results in considerable simplification in the expression for the photon self-energy, and does not alter the character of divergent integrals in terms of which it is expressed, we make this simplification even though it is not consistent with (16). The self-energy of the photons may now be written:

$$\begin{aligned} H_{phs} = \frac{e^2}{2\pi^2}\Bigg[\int \frac{d\mathbf{k} d\mathbf{k}'}{k} a_{\lambda\mathbf{k}} a_{\lambda'\mathbf{k}'} \delta(\mathbf{k}+\mathbf{k}') \Bigg\{ \int_{E(\mathbf{k})} d\mathbf{p}\ \mathrm{spur} \Bigg[\boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\alpha} \frac{1}{-e_{\mathbf{p}} - H_{\mathbf{p}-\mathbf{k}} - k} \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \cdot \boldsymbol{\alpha} \frac{e_{\mathbf{p}} - H_{\mathbf{p}}}{2e_{\mathbf{p}}} \Bigg] \\ - \int_{F(\mathbf{k})} d\mathbf{p}\ \mathrm{spur} \Bigg[\boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \cdot \boldsymbol{\alpha} \frac{1}{H_{\mathbf{p}+\mathbf{k}} + e_{\mathbf{p}} - k} \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\alpha} \frac{e_{\mathbf{p}} - H_{\mathbf{p}}}{2e_{\mathbf{p}}} \Bigg] \Bigg\} + \mathrm{C.C.} \Bigg] + \frac{e^2}{8\pi^2} \int \frac{d\mathbf{k} d\mathbf{k}'}{k} a_{\lambda'\mathbf{k}'}^{+} a_{\lambda\mathbf{k}} \delta(\mathbf{k}'-\mathbf{k}) \\ \times \Bigg[\int_{E(\mathbf{k})} d\mathbf{p}\ \mathrm{spur} \Bigg\{ \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\alpha} \frac{1}{-e_{\mathbf{p}} - H_{\mathbf{p}-\mathbf{k}} - k} \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \cdot \boldsymbol{\alpha} \frac{e_{\mathbf{p}} - H_{\mathbf{p}}}{2e_{\mathbf{p}}} + \frac{e_{\mathbf{p}} - H_{\mathbf{p}}}{2e_{\mathbf{p}}} \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \cdot \boldsymbol{\alpha} \frac{1}{-e_{\mathbf{p}} - H_{\mathbf{p}-\mathbf{k}} - k} \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\alpha} \Bigg\} \\ - \int_{F(\mathbf{k})} d\mathbf{p}\ \mathrm{spur} \Bigg\{ \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \cdot \boldsymbol{\alpha} \frac{1}{H_{\mathbf{p}+\mathbf{k}} + e_{\mathbf{p}} - k} \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\alpha} \frac{e_{\mathbf{p}} - H_{\mathbf{p}}}{2e_{\mathbf{p}}} + \frac{e_{\mathbf{p}} - H_{\mathbf{p}}}{2e_{\mathbf{p}}} \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\alpha} \frac{1}{H_{\mathbf{p}+\mathbf{k}} + e_{\mathbf{p}} - k} \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \cdot \boldsymbol{\alpha} \Bigg\} \Bigg]. \end{aligned} \tag{18}$$

In (18) the region $E(\mathbf{k})$ is composed of those values of $\mathbf{p}$ for which the points $\mathbf{p}$ and $\mathbf{p}-\mathbf{k}$ lie in $I(\mathbf{p}, \mathbf{p}-\mathbf{k})$ for fixed $\mathbf{k}$. The $F(\mathbf{k})$ is comprised of those values of $\mathbf{p}$ for which the points $\mathbf{p}$ and $\mathbf{p}+\mathbf{k}$ lie in $I(\mathbf{p}, \mathbf{p}+\mathbf{k})$ for fixed $\mathbf{k}$. The spurs in (18) are readily evaluated and we obtain

$$\begin{aligned} H_{phs} = \frac{e^2}{8\pi^2}\Bigg[\int \frac{d\mathbf{k} d\mathbf{k}'}{k} a_{\lambda\mathbf{k}} a_{\lambda'\mathbf{k}'} \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \delta(k+k') \Bigg\{ - \int_{E(\mathbf{k})} \frac{d\mathbf{p}}{e_{\mathbf{p}}} \left(1 + \frac{2p_\lambda^2}{e_{\mathbf{p}} k + \mathbf{p} \cdot \mathbf{k}} \right) - \int_{F(\mathbf{k})} \frac{d\mathbf{p}}{e_{\mathbf{p}}} \left(1 - \frac{2p_\lambda^2}{e_{\mathbf{p}} k + \mathbf{p} \cdot \mathbf{k}} \right) \Bigg\} + \mathrm{C.C.} \Bigg] \\ + \frac{e^2}{(2\pi)^2} \int \frac{d\mathbf{k} d\mathbf{k}'}{k} a_{\lambda'\mathbf{k}'}^{+} a_{\lambda\mathbf{k}} \boldsymbol{\varepsilon}_{\lambda\mathbf{k}} \cdot \boldsymbol{\varepsilon}_{\lambda'\mathbf{k}'} \delta(\mathbf{k}-\mathbf{k}') \Bigg\{ - \int_{E(\mathbf{k})} \frac{d\mathbf{p}}{e_{\mathbf{p}}} \left(1 + \frac{2p_\lambda^2}{e_{\mathbf{p}} k + \mathbf{p} \cdot \mathbf{k}} \right) - \int_{F(\mathbf{k})} \frac{d\mathbf{p}}{e_{\mathbf{p}}} \left(1 - \frac{2p_\lambda^2}{e_{\mathbf{p}} k + \mathbf{p} \cdot \mathbf{k}} \right) \Bigg\}. \end{aligned} \tag{19}$$

In (19), p_λ is the component of $\mathbf{p}$ in the direction of the polarization vector $\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}$. The expression in (14) determining the photon self-energy is the sum of two negative quadratically diverging integrals, one positive cubically diverging integral and one negative cubically diverging integral. This combination can be made to vanish by an appropriate choice of the region $I(\mathbf{p}, \mathbf{p}')$.

Is it possible to choose the regions $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ so that both the photon and electron self-energy vanish simultaneously? This question can be answered with certainty in the affirmative provided each of the regions $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ are split into two parts which may be different depending on whether the plane of polarization of the photon lies in the plane of the two vectors, $\mathbf{p}$ and $\mathbf{p}'$, or is perpendicular to this plane. If we had made this separation of $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ then (16) would have been composed of the sum of eight integrals. The vanishing of the coefficient $A(\mathbf{p})$ of (17) depends essentially on making the two regions $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ of different total volume. The vector coefficient $\mathbf{B}(\mathbf{p})$ of (17) is shape dependent and depends essentially on the component of $\mathbf{p}'$ which is perpendicular to the polarization vectors of the photons. On the other hand, the cubically divergent terms of (19) depend on the component of $\mathbf{p}$ which is parallel to the polarization. If we had properly treated the photon self-energy, the expression for the photon self-energy would then have been the sum of eight integrals. The leading divergences in these integrals are the same as those in (19). It is this correct expression for the photon self-energy which one must make zero rather than (19). We now see that the photon and electron self-energies can both be made to vanish.

V. COMMENTS ON RELATIVISTIC INVARIANCE

The condition that a theory be relativistically invariant[1] may be based on the system of commutation relations

$$\begin{aligned}
&[P_x, P_y]=0, \text{ etc.}\\
&[H, P_x]=0, \text{ etc.}\\
&[L_x, L_y]=[M_y, M_x]=iL_z, \text{ etc.}\\
&[M_x, L_y]=[L_x, M_y]=iM_z, \text{ etc.}\\
&[L_x, M_x]=0, \text{ etc.}\\
&[L_x, P_x]=0, \text{ etc.}\\
&[L_x, H]=0, \text{ etc.}\\
&[L_x, P_y]=iP_x, \text{ etc.}\\
&[M_x, H]=iP_x, \text{ etc.}\\
&[M_x, P_x]=iH, \text{ etc.}\\
&[M_x, P_y]=0, \text{ etc.}
\end{aligned} \tag{20}$$

In these equations H is the energy of the system and P_x is the x-component of the momentum, L_z is the z-component of the angular momentum, and the M_x, M_y, M_z are the generators of infinitesimal Lorentz transformations. If we use for H the expression as given by Eq. (1) omitting the Coulomb self-energy parts, and if we use for the P_x, P_y, P_z the sum of the momenta of the photons, electrons and positrons, we may then solve equations $[M_x, P_x]=iH$, $[M_x, P_y]=0$, and $[M_x, P_z]=0$ for M_x. We can also solve for M_y and M_z. The structure of the terms M_x, M_y and M_z is similar to that of H. Simple forms for L_x, L_y and L_z are obtainable which are similar to those of P_x, P_y and P_z. One can then show that, provided certain indefinite divergent integrals are taken to be zero, all the commutation rules relating to (20) are satisfied. These integrals arise, for example, when one computes $[M_x, H]$ and must be taken to be zero if we are to have $[M_x, H]=iP_x$. This means, of course, that the limiting processes by means of which H is defined must be carried out so as to satisfy (20).

In Eq. (9) we introduced a unitary transformation to eliminate the interaction between the positron-electron current and the transverse electromagnetic field. We may look upon this transformation in a manner which is different from the way we have used it thus far. We may suppose that the Coulomb self-energy of the electron has been eliminated from (1) and that the integrals defining H_1 have been carried out in such a manner as to insure the satisfying of relations (20). S_1, a finite Hermitian operator, is then taken to eliminate those parts of H_1 contained in the regions $I(\mathbf{p}, \mathbf{p}')$, $II(\mathbf{p}, \mathbf{p}')$. Since S_1 is now a finite Hermitian operator, e^{iS_1} is, in fact, a unitary operator and $H'=e^{-iS_1}He^{iS_1}$, $M_x'=e^{-iS_1}M_xe^{iS_1}$, etc. will also satisfy relations (20). We now note that all the matrix elements of e^{iS_1} approach zero as the regions $I(\mathbf{p}, \mathbf{p}')$ and $II(\mathbf{p}, \mathbf{p}')$ become infinitely large. In view of this behavior of e^{iS_1} it is not surprising that differing limiting processes can lead to very different results for the first few terms of the expansion of H' as given by (9). We wish to note here that if this above procedure is adopted, then $i[S_1, H_1]$ is no longer the same as $-[S_1, [S_1, H_0]]$ so that (9) can no longer be reduced to (11). It suffices to say that this modification does not alter the possibility of obtaining a vanishing photon and electron self-energy to the order e^2.

[1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).