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A Reinterpretation of the Electron Spectra of Radioactive Ytterbium and Tantalum*

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Highly purified ytterbium, on irradiation in the pile, yields radioactivities whose half-lives are found to be 32.4 days, 4.2 days, and 6.7 days, in addition to the 2.4-hour activity known to exist. The 32.4-day activity is assigned to Yb 169 which decays by *K*-capture to excited Tm 169, yielding no general beta-radiation, but about 30 electron conversion lines which are interpreted as due to 10 gamma-rays. Many of the gamma-rays fit into a simple level scheme. The 4.2-day activity is assigned to Yb 175 and in addition to a strong beta-radiation shows conversion electron lines indicative of 4 gamma-rays. It is shown that the 6.7-day activity is a daughter product in lutecium 177, growing from a radioactive ytterbium 177 which is presumably the well-known 2.4-hour activity.

A re-interpretation of the approximately 40 electron conversion lines in radioactive tantalum 182 leads to 18 gamma-rays, all of which fit well a proposed energy level scheme.

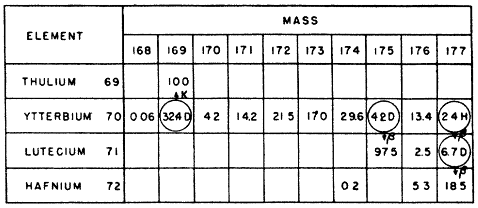
IN previous investigations, specimens of ytterbium and tantalum, irradiated in the Oak Ridge pile, were each found[1] to yield an exceedingly complicated electron spectrum. Continued study of these results has led to interpretations somewhat different from that originally reported.

YTTERBIUM

Ytterbium exists in nature as seven stable isotopes whose percentages of abundance are shown in Fig. 1. An available specimen supposedly chemically pure, gave numerous electron lines some of which showed *K–L–M* differences indicative of impurities, particularly erbium. A later specimen, of high purity, was prepared and kindly made available to us by Dr. F. H. Spedding, of Iowa State College. To check the purity of the two specimens both were examined in a Siegbahn vacuum x-ray spectrometer. The former sample showed clearly the presence of many impurities including erbium, while the Spedding sample gave no trace of neighboring elements as shown in Fig. 2.

Surprisingly, however, the pure ytterbium specimen gave most of the same electron lines that had been observed with the impure sample. Moreover, the *K–L–M* differences of certain of the groups were characteristic of thulium ($Z=69$), as would have been expected following beta-emission from a radioactive erbium. Other electron lines had *K–L–M* differences characteristic of hafnium and were recognized as exactly those due to the 6.7-day activity in radioactive lutecium. These observations may now be completely reconciled to an initially pure ytterbium. The electron lines are shown in column 1, Table I, together with their interpretation.

No long-lived activity (>100 days) as found in the impure specimen was observed in the Spedding sample. This activity has now been shown to be the 120-day emitter, thulium 170, which is formed as shown in Fig. 1. The decay curve of the pure ytterbium is shown in Fig. 3 and except for the well-known short-period activity (2.4 hour), may be resolved into three distinct activities whose half-lives are 32.4 day, 4.2 day, and 6.7 day. It had been proposed[2] by Bothe that the 33-day activity existed in Yb 169 and that it decayed by *K*-capture. Since this isotope would be made by neutron



ELEMENT		MASS									
		168	169	170	171	172	173	174	175	176	177
THULIUM	69		100								
YTTERBIUM	70	0.06	32.4D	4.2	14.2	21.5	17.0	29.6	4.2D	13.4	2.4H
LUTECIUM	71								97.5	2.5	6.7D
HAFNIUM	72							0.2		5.3	18.5

FIG. 1. The stable and radioactive isotopes of ytterbium.

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* Supported by ONR and AEC.

[1] Cork, Keller, Rutledge, Stoddard, and Sazynski, Phys. Rev. **75**, 1133 (1949); **75**, 1778 (1949).

[2] W. Bothe, Zeits. f. Naturforschung **4**, 173 (1946).

TABLE I. Electron energies from irradiated ytterbium.

Electron energy	Interpretation and energy sum Thulium	Lutecium	Hafnium	Electron energy	Interpretation and energy sum Thulium	Lutecium	Hafnium
34.0	K^2 93.5 kev			117.8	K^5 177.3 kev		
39.0	Auger L			120.9	$L_{1,2}^4$ 130.8		
or	$L_{1,2}^a$ 49.0			122.0	L_3^4 130.6		
40.5	L_3^a 49.1			128.5	M^4 130.8		
46.9	Auger M			130.0	N^4 130.5		
or	M^a 49.2			138.7	K^5 198.2		
50.2	K^3 109.7			143.5			K^2 208.9
52.8	$L_{1,2}^1$ 62.7			167.0	$K_{1,2}^5$ 176.9		
54.3	L_3^1 62.9			168.2	L_3^5 176.8		
58.5	L^b 68.4			174.6	M^5 176.8		
or	K^b 118.0			176.4	N^5 176.9		
60.7	M^1 63.0			188.2	$L_{1,2}^6$ 198.1		
62.5	N^1 63.0			195.6	M^6 197.9		
71.0	K^4 130.5			or		K^a 259.0	
74.1		K^1 137.5		197.3	N^6 197.3		
83.5	$L_{1,2}^2$ 93.4			219.2		K^2 282.6	
85.0	L_3^2 93.6			248.2	K^7 307.7		
91.3	M^2 93.6			or		$L_{1,2}^a$ 258.8	
99.8	L^3 109.7			297.8	L^7 307.7		
102.3			$L_{1,2}^1$ 113.3	304.5	M^7 306.8		
107.5	M^3 109.8			333.0		K^3 396.4	
109.5	N^3 110.0			385.5		$L_{1,2}^3$ 396.2	
110.8			M^1 113.4				

capture in the pile from Yb 168, whose abundance is only 0.06 percent the assignment might have been doubted. The present investigation, however, unquestionably confirms this interpretation indicating an enormously large cross section for neutron capture in Yb 168.

In fact, about 30 of the more than 40 observed electron lines shown in Table I decay with the 32.4-day half-life and all fit the K–L differences of thulium ($Z=69$). Since no ytterbium exists in the initial specimen, and stable thulium exists only as mass 169, then the excited states in thulium could only follow K-capture in Yb 169. The electron groups thus indicate about 10 gamma-rays for thulium 169, most of which fit well the level scheme proposed in Fig. 4.

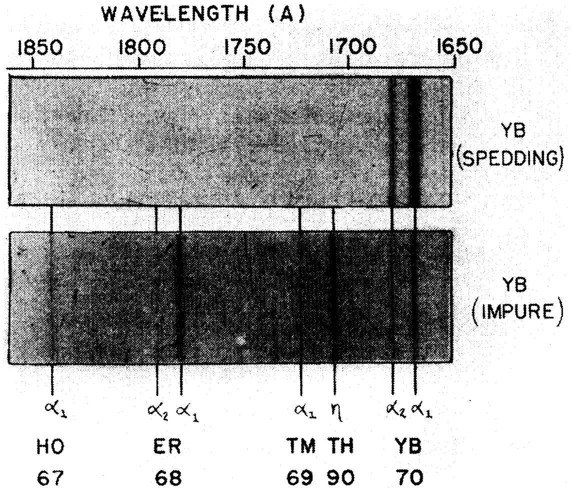

FIG. 2. Test for impurities in ytterbium by x-ray analysis using a Siegbahn vacuum spectrometer, showing the L series lines.

A relatively small number of the electron lines are satisfied by K–L differences in lutecium and are thus really due to a radioactive ytterbium decaying by beta-emission. These lines die out with the 4.2-day half-life and indicate the existence of four gamma-rays. These gamma-energies may be related by some scheme as suggested in Fig. 5, and are undoubtedly associated with the isotope of mass 175 as will be shown.

By neutron capture in the pile, Yb 176 might be expected to produce radioactive Yb 177. This could decay only by successive beta-emissions through lutecium 177 (well known),[3] to hafnium 177. A question might arise (Fig. 1) as to the assignment of the 4.2-day and the 2.4-hour activities to the isotopes of mass 175 and 177. From the decay curve it is apparent that the initial activity of the 4.2-day emitter is enormously

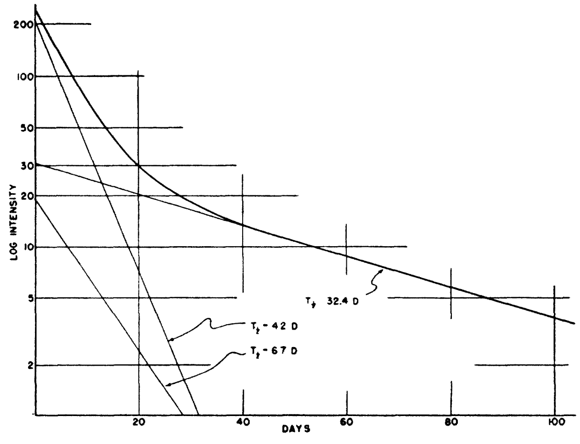

FIG. 3. Decay curve of irradiated ytterbium, resolved into its three components.

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[3] Cork, Keller, and Stoddard, Phys. Rev. 76, 986 (1949).

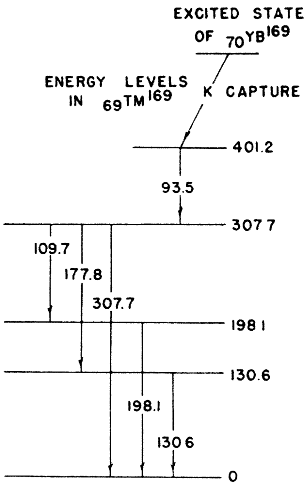

FIG. 4. Energy levels (kev) in thulium 169, following K-capture in ytterbium 169.

greater than that of the 6.7-day lutecium. Had the latter isotope grown as a daughter product from the 4.2-day activity, then it would have been relatively stronger when first examined, which was not the case. Hence, it is highly probable that the 4.2-day activity is made from Yb 174 which must also have a relatively large cross section to account for the strong yield. This would then reasonably leave the 2.4-hour emitter to be Yb 177, from which the daughter product Lu 177 is derived by beta-emission.

No continuous beta-emission for the 32.4-day activity is apparent on the photographic plates, which is as would be expected in the K-capture process, whereas a very dense general blackening accompanies the electron lines due to the 4.2-day emitter. The upper limit of this beta-radiation as determined by the Feather rule from absorption in aluminum is about 0.39 Mev. The gamma-energies from each of the three radioactive isotopes are tabulated in Table II.

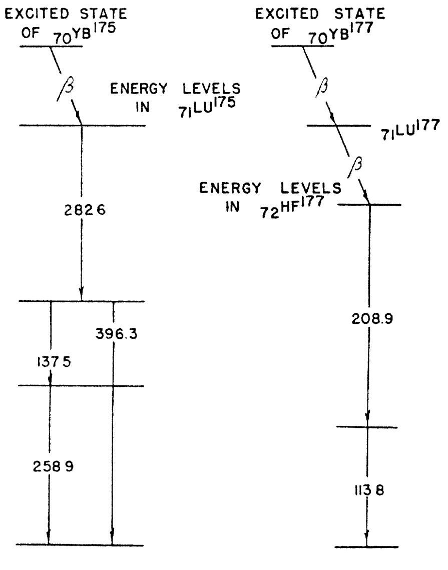

FIG. 5. Level schemes (kev) for Lu 175 following beta-emission from Yb 175 and for Hf 177 following successive beta-emissions from Yb 177.

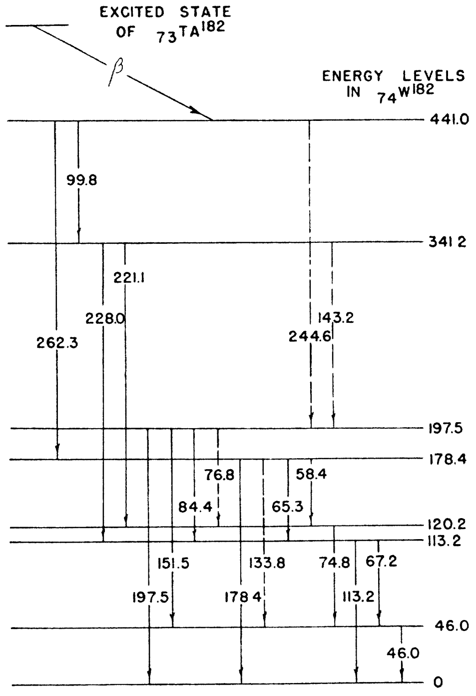

FIG. 6. Excited levels (kev) in W 182 following beta-emission from Ta 182.

TANTALUM

The electron spectrum from irradiated tantalum, consisting as it does of around 40 lines, has been rephotographed many times, leading to some slight adjustment in the reported energy of each line. It is now known that the fine structure of the L lines is in some cases resolved on the photographic plates. This process was originally thought to be too unlikely to be visible. As a result, it is now possible to re-interpret the observed electron lines shown collectively in Table III, so as to yield the gamma-energies shown in columns 4 and 8. Certain less distinct lines previously reported,

TABLE II. Summary of gamma-energies (kev) derived from ytterbium.

Arbitrary line no.	Yb 169 (32.4 day)	Yb 175 (4.2 day)	Lu 177 (6.7 day)
1	63.0	137.5	113.4
2	93.5	282.6	208.9
3	109.7	396.3	
4	130.6		
5	176.8		
6	198.1		
7	307.7		
a	49.1	258.9	
b	68.4 or 118.0		

TABLE III. The electron energies (kev) from radioactive tantalum 182. (Superscripts are arbitrary numbers assigned to the gamma-rays in the order of increasing energy.)

Electron energy	Proposed interpretation	Energy sum	Gamma-energy	Electron energy	Proposed interpretation	Energy sum	Gamma-energy
30.5	K^7	99.8		81.7	K^9	151.0	
33.9	L_1^1	46.0	46.0(1)	88.3	$L_{1,2}^7$	99.8	
43.9	K^8	113.2		89.6	L_3^7	99.8	99.8(7)
47.0	$L_{1,2}^2$	58.4		97.0	M^7	99.8	
48.2	L_3^2	58.4	58.4(2)	99.1	N^7	99.7	
53.6	$L_{1,2}^3$	65.1		101.2	$L_{1,2}^8$	113.3	
55.3	L_3^3	65.4	65.3(3)	102.8	L_3^8	113.0	113.2(8)
	$L_{1,2}^4$	66.9		109.2	K^{10}	178.5	
	M^2	58.3		110.3	M^8	113.1	
57.2	L_3^4	67.4	67.2(4)	128.2	K^{11}	197.5	
62.5	L_1^5	74.6		139.9	L_1^9	152.0	151.5(9)
	M^3	65.3		151.8	K^{12}	221.1	
64.7	L_3^5	74.9	74.8(5)	158.7	K^{13}	228.0	
	M^4	67.5		166.2	L_1^{10}	178.3	178.4(10)
	(L_1^a)	(76.8)	76.8(*a*)	175.3	M^{10}	178.1	
	(K^b)	(133.8)	133.8(*b*)		K^d	244.6	244.6(*d*)
66.7	N^4	67.3		185.5	L_1^{11}	197.6	197.5(11)
72.1	M^5	74.9		193.0	K^{14}	262.3	
	$L_{1,2}^6$	84.2		209.5	$L_{1,2}^{12}$	221.2	221.1(12)
74.0	L_3^6	84.4	84.4(6)	216.1	$L_{1,2}^{13}$	228.1	228.0(13)
	(M^a)	(76.8)		225.1	M^{13}	227.9	
	(K^c)	(143.2)	143.2(*c*)	250.6	$L_{1,2}^{14}$	262.4	262.3(14)
81.7	M^6	84.5		259.5	M^{14}	262.3	

probably exist, but since they cannot be evaluated with the same certainty, they are omitted in this report.

All fourteen of the strong gamma-rays indicated by this interpretation fit well a proposed level scheme as shown in Fig. 6. Among other possible transitions, gamma-rays of energy 76.8, 133.8, 143.2, and 243.9 kev would also be expected. On critically examining the electron energies, it may be noted that a double interpretation of certain electron lines would in fact yield these gamma-rays. These four gamma-values are shown by arbitrary letters *a*, *b*, *c*, and *d* instead of numbers in Table III and are indicated as dotted lines in Fig. 6.

Additional high energy gamma-rays are known[4] to exist in tantalum with energies of about 1.12, 1.19, and 1.234 Mev.

This investigation was made possible by the support of the ONR and the AEC.

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[4] C. H. Goddard and C. S. Cook, Phys. Rev. **76**, 1419 (1949); W. Rall and R. G. Wilkinson, Phys. Rev. **71**, 321 (1947).

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Quantum Electrodynamics: The Self-Energy Problem*

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In this paper it will be shown that the limiting processes by means of which the integrals of the Hamiltonian of the interacting electron-positron field with the electromagnetic field are defined, may be carried out so that the self-energies of the electrons and photons are zero.

I. INTRODUCTION

FIELD theories have been plagued by the appearance of divergent expressions for the self-energies of the elementary particles. The purpose of this paper is to show that by the use of appropriate limiting processes for the integrals which define the Hamiltonian of the interacting fields, one can in fact obtain the value zero for the self-energy integrals. Although we will work here only with the electron-positron field in interaction with the electromagnetic field, I believe that the technique used here will also enable one to eliminate the self-energy problem in other cases as well.

This work will be carried out in a momentum space representation. In this representation the Hamiltonian for the interacting electron-positron field and the electromagnetic field, may be written

$$
\begin{aligned}
H=&\int d\mathbf{p}d\mathbf{p}'\delta(\mathbf{p}-\mathbf{p}')e_{\mathbf{p}s}(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s})+\int d\mathbf{k}d\mathbf{k}'\delta(\mathbf{k}-\mathbf{k}')k a_{\mathbf{k}\lambda}{}^{+}a_{\lambda\mathbf{k}'}-\frac{e}{2\pi}\int d\mathbf{p}d\mathbf{p}'d\mathbf{k}\frac{\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k})}{(k)^{\frac{1}{2}}}\\
&\times\{(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s'})a_{\lambda\mathbf{k}}(\mathbf{p}s|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}'s')+(\alpha_{\mathbf{p}'s'}{}^{+}\alpha_{\mathbf{p}s})a_{\lambda\mathbf{k}}{}^{+}(\mathbf{p}'s'|\boldsymbol{\varepsilon}_{\lambda\mathbf{k}}\cdot\boldsymbol{\alpha}|\mathbf{p}s)\}+\frac{e^2}{(2\pi)^2}\int\frac{d\mathbf{p}d\mathbf{p}'d\mathbf{p}''d\mathbf{p}'''}{|p-p'|^2}\delta(\mathbf{p}-\mathbf{p}'+\mathbf{p}''-\mathbf{p}''')\\
&\times(\alpha_{\mathbf{p}s}{}^{+}\alpha_{\mathbf{p}'s'})(\alpha_{\mathbf{p}''s''}{}^{+}\alpha_{\mathbf{p}'''s'''})(\mathbf{p}s|1|\mathbf{p}'s')(\mathbf{p}''s''|1|\mathbf{p}'''s'''). \qquad (1)
\end{aligned}
$$

At this point remarks concerning the notation used in (1) are appropriate. I am using units such that $\hbar=c=1$, $e^2\simeq 1/137$. Symbols printed in bold face type are vectors, $d\mathbf{p}$ is a small volume of momentum space. $\boldsymbol{\alpha}$ is the Dirac vector with 4×4 matrix components α_1, α_2, α_3, which satisfy the relation $\alpha_i\alpha_j+\alpha_j\alpha_i=2\delta_{ij}$. The other Dirac matrix β satisfied $\beta\alpha_i+\alpha_i\beta=0$, $\beta^2=1$, with 1 the 4×4 unit matrix. By the symbol $(\mathbf{p}s|A|\mathbf{p}'s')$

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* Work performed at the Brookhaven National Laboratory, under the auspices of the AEC.

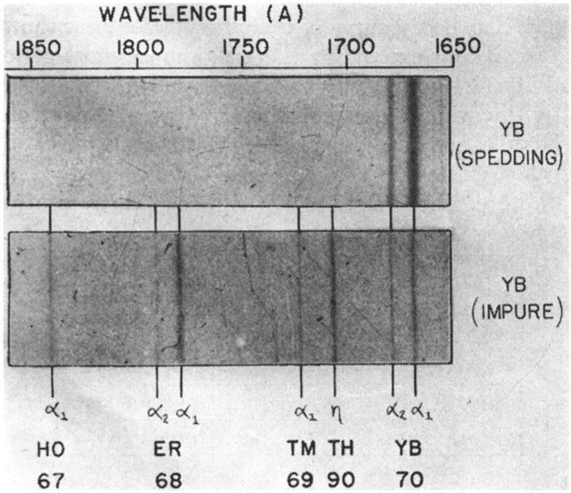

FIG. 2. Test for impurities in ytterbium by x-ray analysis using a Siegbahn vacuum spectrometer, showing the *L* series lines.