

TABLE I. Temperature dependence of the energy gap.

| | C_c (ev) | C_f (ev) | β (degree ⁻¹) | $\partial E_G/\partial T$ (ev/degree) | | $\partial E_G/\partial T$ (ev/degree) experimental |
|-----------|------------|------------|---------------------------------|---------------------------------------|-----------------------|--|
| | | | | Eq. (6) | Eq. (4)+Eq. (6) | |
| Silicon | 9.75 | 16.9 | 10×10^{-6} | -1.78×10^{-4} | -5.3×10^{-4} | -4.7×10^{-4a} -3.0×10^{-4b} -4.6×10^{-4c} |
| Germanium | 2.55 | 3.6 | 23×10^{-6} | -0.95×10^{-4} | -1.2×10^{-4} | |

^a See reference 1.
^b See reference 4.
^c See reference 2.

gap. The effect of volume dilation has been recently considered by Shockley and Bardeen.⁵ We shall consider the effect of lattice vibrations, following the method used by Fröhlich and his co-workers⁶ for polar crystals. The second-order perturbation gives

$$E(K) = E_0(K) + \sum_q \left[\frac{|H'_{K-q,K}|^2}{E_0(K) - E_0(K-q, n_q+1)} + \frac{|H'_{K+q,K}|^2}{E_0(K) - E_0(K+q, n_q-1)} \right], \quad (1)$$

where K is the electron wave number, n_q is the quantum number of the lattice vibration mode with wave number q , the subscript 0 refers to the unperturbed solution. The matrix elements have the usual expression.

$$H'_{K\pm q,K} = \mp i \frac{1}{N^{1/2}} \frac{2}{3} Cq \left(\frac{\hbar}{2M\omega_q} \right)^{1/2} \left\{ \begin{matrix} (n_q) \\ (n_q+1) \end{matrix} \right\}^{1/2}, \quad (2)$$

where N is the total number of unit cells, M is the atomic mass, and C is approximately constant for longitudinal modes and zero for transverse modes. For $\hbar kT > \hbar\omega_q$, the longitudinal acoustic waves give

$$E(0) - E_0(0) = -0.175 C^2 \Omega^2 (m\hbar kT / \hbar^2 M c_l^2), \quad (3)$$

where Ω is the volume of the unit cell and c_l is the wave velocity. The value of C can be estimated from the mean free path due to lattice scattering as was done by Lark-Horovitz and Johnson.⁷ For the conduction band C_c is to be estimated from electron mobility and m_e is the effective mass of the electrons. For the filled band C_f should be estimated from hole mobility and m_f is the effective mass of the holes with negative sign.

The more general variation method⁶ gives an expression similar to (1) except that in the denominators $E_0(K)$ is replaced by $E(K)$. The result reduces (3) by approximately 20 percent. On the other hand, materials with a diamond structure, such as silicon and germanium, have optical branches of vibration. The frequencies of these vibrations are difficult to estimate, but it can be shown that their effect is less than one-third of (3). We shall take (3) as the first approximation. The reduction of the energy gap is then

$$\partial E_G / \partial T = -0.175 (\Omega^{2/3} k / \hbar^2 M c_l^2) [m_e C_c^2 + |m_f| C_f^2]. \quad (4)$$

The change in energy gap with lattice dilation as given by Shockley and Bardeen⁵ is

$$E_{1G} \equiv \partial E_G / \partial (\Delta V / V) = E_{1c} - E_{1f}, \quad (5)$$

where

$$E_{1c} = \frac{2}{3} C_c, \quad E_{1f} = \frac{2}{3} C_f.$$

This effect alone should give

$$\partial E_G / \partial T = \frac{2}{3} (C_c - C_f) \beta, \quad (6)$$

where β is the volume coefficient of expansion. The net result should be the sum of (4) and (6).

The results of the calculations are presented in Table I. To facilitate comparison, the same values for C_c , C_f , β , and c_l are adopted as used by Shockley and Bardeen. The value of β_{oe} is taken from the tables of Landolt and Börnstein. It is seen that for silicon the lattice vibrations have larger effect in producing the temperature variation of the energy gap than the lattice dilation. For germanium it is smaller but not negligible. The calculated value for silicon agrees well with the experimental result

of optical measurements. For germanium the calculated value seems to be too low. The optical measurements are now being repeated with greater precision. It must be pointed out that the values of C_c and C_f were calculated by taking the free electron mass for both m_e and m_f . More accurate knowledge of C 's and m 's is necessary for better estimation of $\partial E_G / \partial T$.

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² Measurement by M. Becker.

³ M. Becker and H. Y. Fan, Phys. Rev. **78**, 301 (1950). Similar results have been reported by F. S. Goucher of Bell Telephone Laboratories.

⁴ G. L. Pearson and J. Bardeen, Phys. Rev. **75**, 865 (1949).

⁵ W. Shockley and J. Bardeen, Phys. Rev. **77**, 407 (1950).

⁶ Fröhlich, Pelzer, and Zienau, Phil. Mag. **41**, 221 (1950). The author is indebted to Professor Fröhlich for discussions.

⁷ K. Lark-Horovitz and V. Johnson (private communication).

On the Origin of the Cosmic Radiation from the Sun

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IT has been shown by Forbush,¹ Ehmert² and others that from certain solar flares in suitable heliographic positions there originates cosmic radiation which can be observed on the earth's surface. The intensity is of the order of 1 particle/(cm²-sec.), the energy of the particles may be as large as 5×10^9 ev. The maximum of cosmic-ray intensity is often reached $\frac{1}{2}$ to 2 hr. after the intensity maximum of the flare itself (H_α).³ Several explanations have been proposed: Swann⁴ thinks that potential differences of the order of 10^9 to 10^{10} volts can be induced by the growth of magnetic fields in sunspots, and Bagge and Biermann⁵ think that they can be induced by the observed relative movement of the spots with their fields. But we must not forget that the solar atmosphere in which these fields have to be set up shows an electric conductivity of the order of 10^{15} e.s.u. so that the acceleration path is practically short-circuited. It is generally assumed that the conductivity depends strongly on the presence of a magnetic field. The effect of this field is practically canceled by an electric field which is set up by the movement of charges perpendicular to the magnetic field, as has been pointed out by Schlüter.⁶ We shall therefore always use the conductivity in the absence of a magnetic field.

Menzel and Salisbury⁷ suggest that a solar flare could be the source of electromagnetic radiation of extremely long wave-length ($\lambda \gtrsim 3 \times 10^{10}$ cm) the field of which is decreasing as $1/r$ at the earth's distance. This field would then be able to accelerate elementary particles to an energy of 10^9 ev or more. But here too it can be said that this radiation would be unable to escape the solar corona, or even to propagate in the interplanetary space with its conductivity of $\sigma \approx 10^{12}$ e.s.u. This same objection holds for a proposal of Alfvén⁸ which supposes the existence of an electric field inside a solar corpuscular beam producing geomagnetic storms and aurora. His mechanism also cannot explain why the cosmic-ray outburst comes more or less simultaneously with the solar flare.

It seems essential that the acceleration mechanism should affect only a limited number of charged particles. It seems to us that this limitation is merely the result of the high conductivity

of an ionized gas. Every magnetic perturbation produces in its vicinity a shielding system of induction currents which flow, even in a compressible medium, in a layer of the thickness of the order of $c(\tau/\sigma)^{1/2}$ where τ is the period (e.g., in the case of periodic variations) of the field, and σ is the electric conductivity which is practically independent of the density, n , of electrons and ions. Consequently the thickness of the current bearing layer is independent of n , and the number of charged particles forming the shielding current is therefore proportional to n . The accelerations of charges that can be acquired in this way in the chromosphere ($n \approx 10^{10} \text{ cm}^{-3}$, $\sigma \approx 10^{14} \text{ e.s.u.}$) and in the corona ($n = 10^8$ to 10^9 cm^{-3} , $\sigma \approx 5 \times 10^{15} \text{ e.s.u.}$) are negligible but may be of some importance for the interpretation of some deviations from thermodynamic equilibrium observed in these layers. In interplanetary space ($n \approx 10^3 \text{ cm}^{-3}$, $\sigma \approx 10^{12} \text{ e.s.u.}$), however, very small magnetic field perturbations will be able to accelerate the few effective charges to speeds approaching c . The acquired energies, of course, will never exceed the kinetic energy which a single charge can get when falling through the potential difference which follows from Faraday's law of induction.

After careful consideration of all known phenomena of the solar atmosphere, it seems that the required conditions are best fulfilled in the vicinity of those corpuscular beams which originate from solar flares, and reach the earth 26 hr. later if the heliographic position of the flare is approximately central. The speed of the beam is about 10^8 cm/sec . Measurements based on the absorption of the K line of Ca II or on the absorption of radio waves⁹ ($\lambda = 4.1 \text{ m}$) give almost the same value of the density of $n \approx 10^8$ electrons and of Ca^+ ions, the latter of which may include an unknown small number of protons. The diameter, D , of the beam may be of the order of 10^{10} cm in the neighborhood of the sun, and its length, L , equal to $4 \times 10^{11} \text{ cm}$; this corresponds to an emission period of one hour. The flares, being observed mostly on the edges of single spots or between spots of opposite polarity, seem to occur in regions where the component of the magnetic field is mostly tangential. Therefore, the beam originating here should be magnetized perpendicular to its movement and should carry an induced current system flowing lengthwise along the beam and should conserve inside it the initial magnetic field.

The lifetime of this current system is practically unlimited because the transformation of electromagnetic energy into Joule heat takes a time of the order of $\sigma a^2/c^2$, where $\sigma \approx 10^{13} \text{ e.s.u.}$ and $a \approx 10^{11} \text{ cm}$ is the dimension of the current system. Only the thermal expansion, which we can ignore near the sun, dilutes the field sensibly on its way to the earth. Let us make the modest assumption that the energy density of the field, $H^2/8\pi$, does not exceed the beam's kinetic energy density, $\rho v^2/2$; which means that $H \lesssim 1$ gauss. The beam leaving the solar corona will then be surrounded by an additional thin sheet of shielding current formed by ionized interplanetary matter ($n \approx 10^3 \text{ cm}^{-3}$, $\sigma \approx 10^{12} \text{ e.s.u.}$). The induced potential difference, Φ , causing this current will be of the order of $10^{-8}LD(dH/dt) = 10^{-8}DHv \approx 10^{10}$ to 10^{11} volts. The number of electrons and protons contributing to the shielding current is so small as a result of the high conductivity that they can acquire along the beam an energy of nearly $\Phi/2 \text{ ev}$. Protons and electrons, and also heavier ions, therefore, are ejected along the axis of the beam in both directions, that is, toward the sun and toward the earth. The total kinetic energy of the corpuscular beam, $LD^2\rho v^2/2$ amounts to 10^{29} ergs; the total energy of the current system may be of the same order. The total energy involved in a solar cosmic-ray burst is of the order of 6×10^{22} ergs which corresponds to an additional intensity of 1 particle/($\text{cm}^2\text{-sec.}$) of $5 \times 10^9 \text{ ev}$ during 20 min. radiated through a surface of 10^{23} cm^2 . It follows that only one out of every 10^7 charges of the induced current system has to become a cosmic-ray particle to fit the observed intensities.

The observed delay between maxima of the flare and cosmic-ray intensities of about one hour is readily explained on this model by the times of formation and traveling of the corpuscular beam from the chromosphere to the outer region of the corona. It is to be understood that charged particles of all masses acquire the same

energy by the given acceleration method, hence this appears to provide a satisfactory stellar injection mechanism required in Fermi's¹⁰ theory for interstellar cosmic-ray production.

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Hyperfine Structure of $4^1,^3F - nG$ of the Spectrum of Al II

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IN order to clarify the anomalous hyperfine structure (h.f.s.) of atomic spectra, which appears when the mutual distances of terms of a multiplet are comparable with the hyperfine splitting due to the nuclear spin, the h.f.s. of the lines $4^3P - 4^3D$ of Al II was studied.¹ The extreme case, in which the triplet-singlet distance as well as the triplet separation of the upper term vanishes, will present a quite extraordinary h.f.s. As has been pointed out by Paschen² and by Goudsmit and Bacher³ the $4^1,^3F - nG$ series of Al II is a very typical example of such a case, but these experimental and theoretical investigations seem to require further study.

The spectrum of Al II was excited in a water-cooled aluminum hollow cathode-discharge tube, which was filled with helium. The helium pressure was adjusted until the $4^1,^3F - nG$ series was emitted with such intensity that it could be photographed through an interferometer of high resolving power. The fine structure was examined by a quartz Lummer-plate (thickness = 4.4 mm) and a Fabry-Pérot etalon. The observed structure of $4^1,^3F - nG$ was found to be identical for $n = 6$ and 7 within experimental error, the mean of $n = 6$ and 7 being shown in Fig. 1(a).

$4^1F - nG$ consists of two sharp components whose mutual distance could be measured with the accuracy of 0.001 cm^{-1} .

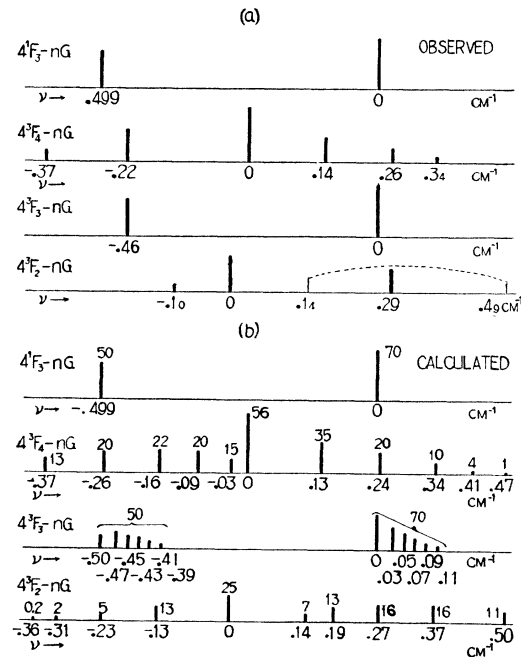


FIG. 1. Hyperfine structure of $4^1,^3F - nG$ of Al II. (a) Observed, (b) calculated.