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Magnetic Field Penetration in Superconducting Lead

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The magnetic susceptibility of small superconducting lead spheres (radius $\sim 10^{-3}$ cm) has been determined by a self-inductance method. On the basis of relations derived from the London theory for reducing the data, the penetration depth, λ, is calculated for three different lead samples at 4.22°K. Results indicate that λ is $1.3 \pm 0.3 \times 10^{-5}$ cm at this temperature. In addition, it is found that, within experimental error, λ is not a function of the size of the sphere. It is noted that there is need for using smaller sized particles and accounting for the interaction effects if future colloid investigations designed to test the London theory are to be successful.

1. INTRODUCTION

The London[1] theory describes the penetration of an applied magnetic field into a superconductor by the equation

$$\nabla^2\mathbf{H} = \mathbf{H}/\lambda^2, \tag{1}$$

where **H** is the magnetic field intensity within the superconductor and λ is the penetration depth. The measurement of the parameter λ has been the subject of many investigations[1a] since the advent of the London theory. All these experiments have indicated that $\lambda \sim 10^{-6}$ to 10^{-4} cm.

If the validity of Eq. (1) is assumed, λ can be determined from magnetic susceptibility measurements on superconducting bodies having dimensions of the order of λ. London[2] has shown that for small spheres the relation between susceptibility and λ is given by

$$\chi/\chi_0 = 1 - \frac{3\coth(R/\lambda)}{R/\lambda} + \frac{3}{(R/\lambda)^2}, \tag{2}$$

where χ is the volume susceptibility for a small sphere of radius R, and χ_0 is the susceptibility of a large sphere ($R \gg \lambda$). It follows from the "Meissner-Ochsenfeld Effect"[3] that χ_0 should be $-3/8\pi$. This value has been verified experimentally.[4]

The experiments to be described in this paper were performed to obtain a value for the penetration depth in lead at 4.22°K. The susceptibility of spherical colloidal lead specimens was determined by a self-inductance method. If Eq. (2) is correct, such measurements yield the value of λ directly if R is known.

2. EXPERIMENTAL

(A) Susceptibility Measurement

When a quantity of spherical superconducting particles suspended in a non-magnetic medium is introduced into a coil, the coil suffers a change in self-inductance ΔL given by the relation

$$\Delta L = M\chi K C_i, \tag{3}$$

where M is the mass of superconducting material in the dispersion; K is a constant determined by the coil geometry and specific gravity of the superconducting element, and C_i is a factor correcting for the magnetic interaction between particles. If we define

$$R/\lambda = \rho \tag{4}$$

and

$$\chi/\chi_0 = f(\rho) \tag{5}$$

it then follows from (3) that

$$f(\rho) = \Delta L / MK\chi_0 C_i. \tag{6}$$

Equation (6) determines λ directly from the inductance change produced by the introduction of the dispersion if R, M, K, and C_i are known.

An earlier investigation[5] has shown that although C_i is a function of the concentration of the metal in the dispersion it is apparently independent of the size of the spheres. Both K and C_i were obtained by calibrating the coil with various concentrations of large lead spheres (so that λ could be neglected, and χ was essentially χ_0).

The experimental procedure consisted of observing the change in self-inductance of the calibrated coil when the dispersion of lead in Vaseline contained in a Pyrex[5a] test tube 5 cm long and 0.6 cm inside diameter, was introduced into the coil. The coil was wound with A.W.G. 40 copper wire on a Bakelite cylinder of 1 cm diameter and 15 cm length. Its self-inductance was $6103.0\mu h$ when completely submerged in liquid helium at 4.22°K. Measurements were made with an inductance bridge which was balanced by a null method employing an amplifier and an oscilloscope. Changes of $0.1\mu h$ could be observed readily with this arrangement. The

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[1] F. London and H. London, Proc. Roy. Soc. **A149**, 71 (1935).

[1a] A summary of methods used to determine λ is given by D. Shoenberg, Intern. Conf. on Fundamental Particles and Low Temperatures, Vol. II (Physical Society), 93 (1947).

[2] F. London, Physica **3**, 450 (1936).

[3] W. Meissner, and R. Ochsenfeld, Naturwiss. **21**, 787 (1933).

[4] D. Shoenberg, Proc. Roy. Soc. **A155**, 712 (1936).

[5] M. C. Steele, Phys. Rev. **76**, 566 (1949). In this earlier paper the correction factor for the interaction effect is designated as χ/χ_0 in place of the C_i used in the present paper.

[5a] In order to account for the possible change in self-inductance due to the Pyrex tube and the Vaseline, a number of runs were made without any lead in the tubes. No change in self-inductance was observed in such cases. Even if there were some small effect, it would be very nearly canceled in determining the susceptibility of the colloids since the calibration and the final runs were made with the same sized Pyrex tubes.

bridge was supplied with 1,000 c.p.s. voltage and the a.c. magnetic field within the measuring coil was calculated to be ~ one oersted.

(B) Specimens

The specimens had to meet a number of critical requirements before any worth while observations could be made. Besides the need for particles having dimensions of the order of λ, it was desirable to have uniformity of size. In additon, they had to be as nearly spherical as possible since Eq. (6) is only applicable for spheres. Finally, there has to be enough metal in the sample to insure a sufficiently accurate determination of the susceptibility.

The technique of preparing colloidal metal particles by the chemical reduction of a suitable salt, as used by Shoenberg[6] in his study of mercury colloids, could not be successfully applied to lead or any of the other superconductors. In fact, a major portion of this investigation was the search for a method of producing small lead spheres. The procedure finally evolved consisted of stirring molten lead in a bath of silicone oil, and then allowing the entire mass to cool below the melting point of the metal. This led to the production of spherical particles having radii $\geqq 10^{-4}$ cm. Figure 1 shows a microphotograph of a typical preparation of lead before any separation into size ranges was attempted.

One advantage of this technique of colloid preparation is that the metal particles can be isolated readily from the silicone oil by using suitable solvents. Therefore an elutriation process can be used to separate the particles into rather narrow size ranges. For this experiment a Roller Air Elutriator was successfully employed[7] to separate particles having from 0 to 40 microns diameter into 5 micron ranges. Figure 2 shows a microphotograph of the 10 to 15 micron fraction after separation. It is noted that the separated particles deviate slightly from spheres when compared with the original material. Unfortunately, this distortion is unavoidable when material as soft as lead is elutriated by air. However, these slight departures from sphericity would probably not have any marked effect upon the results since it is likely that such perturbations would tend to be canceled if there is a random distribution of the particles within the dispersion.[8]

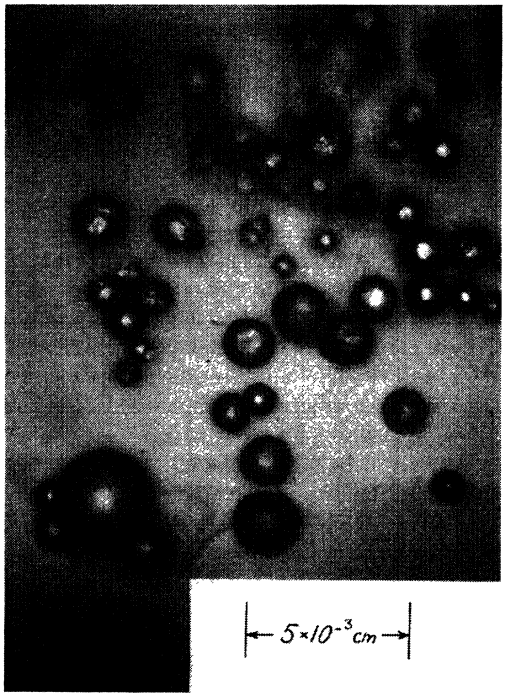

FIG. 1. Lead spheres before separation into size ranges.

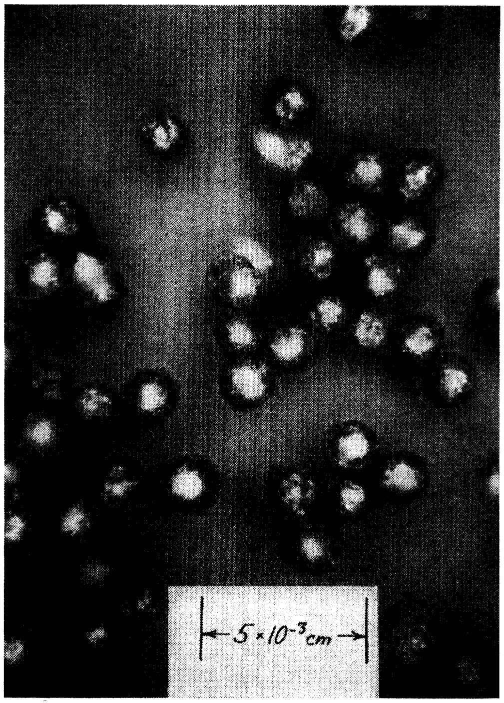

FIG. 2. Lead particles of the 10 to 15 micron fraction obtained by air elutriation.

The final specimens consisted of these separated lead particles dispersed in Vaseline and contained in the Pyrex test tubes described previously. Chemical analysis of the prepared colloidal lead showed that its purity exceeded 99.9 percent.

3. EXPERIMENTAL RESULTS AND DISCUSSION

Data were obtained for three samples of lead having size ranges 10 to 15, 15 to 20, and 30 to 35 microns

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[6] D. Shoenberg, Proc. Roy. Soc. **A175**, 49 (1940).

[7] The separations with the Roller Air Elutriator were carried out by the Metallurgy Division of the National Bureau of Standards, Washington, D. C. The author wishes to express his thanks for their willing assistance.

[8] Shoenberg (reference 6) has presented a similar argument in his paper.

TABLE I. Results for lead at 4.22°K.

$\bar{R}\times 10^4$ cm	Mass (grams)	Volume percent in dispersion	$-\Delta L(\mu h)$[a]	C_i	$f(\rho)$	ρ	$\lambda\times 10^5$ cm	$\delta\lambda\times 10^5$ cm
7.6	0.8607	3.6	15.78	0.987	0.944	53	1.4	±0.2
9.4	2.4108	13.2	43.55	0.954	0.962	78	1.2	±0.4
17.2	2.5557	9.7	47.67	0.966	0.981	157	1.1	±0.4

[a] ΔL as given is the average of five independent observations made over a period of two months.

(diameter) respectively. The average[9] radius ($\bar{R}$) of each specimen was determined from microscopic examination of several hundred particles. Table I gives a summary of the observations and calculated results for these dispersions.

From Table I the weighted mean value of λ is $1.3\pm 0.3\times 10^{-5}$ cm. This value is, of course, dependent upon the validity of Eq. (2). The only other value of $\lambda_{4.2^\circ\mathrm{K}}$ reported for lead has been given by von Laue[10] as 2.09×10^{-5} cm. Von Laue determined this best fitting value from the data of magnetic threshold experiments performed by Pontius[11] on thin lead cylinders.[12]

To date, Eq. (1) has been integrated under the assumption that λ is independent of both the magnetic field strength and the size of the body. Therefore, it is significant to note that within experimental error the results of this investigation are consistent with the latter condition on λ (i.e., λ is independent of size).

The major part of the standard deviation ($\delta\lambda$) is due to the relatively large sizes of the particles. Thus, small uncertainties in the observations are greatly magnified in the calculated value of λ. In fact, if the correction factor C_i had been neglected, the results would not only have been changed by as much as a factor of three, but also there would have been an apparent marked size dependence of λ. Measurements with smaller particles would eliminate most of this difficulty. Finally, there is the error introduced by localized clustering of the particles within the dispersion. No allowance was made for such an effect in the reduction of the data although Fig. 2 shows clearly that particles do cluster in groups. Shoenberg[6] has pointed out that it is difficult to predict the order of magnitude of such a correction.

The results of this study emphasize the need for using smaller particles and for investigating the effect of clustering if the colloid method is to be instrumental in testing the validity of the London theory.

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[9] The use of $\bar{R}$ in place of $[\Sigma_i(R_i)^3]^{\frac{1}{3}}$ does not change the values of λ given in Table I.

[10] M. von Laue, Ann. d. Physik **32**, 71 (1938).

[11] R. B. Pontius, Phil. Mag. **24**, 787 (1937).

[12] Values of $\lambda_{4.2^\circ\mathrm{K}}$ calculated from all of Pontius' data actually cover a range from 1.10×10^{-5} to 2.29×10^{-5} cm.

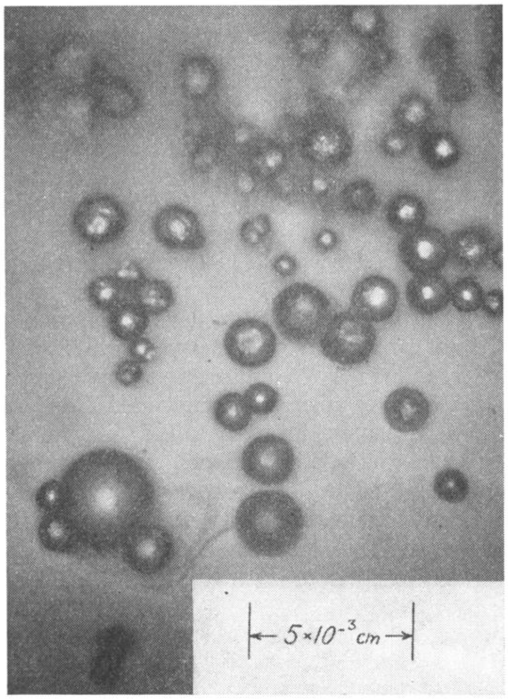

FIG. 1. Lead spheres before separation into size ranges.

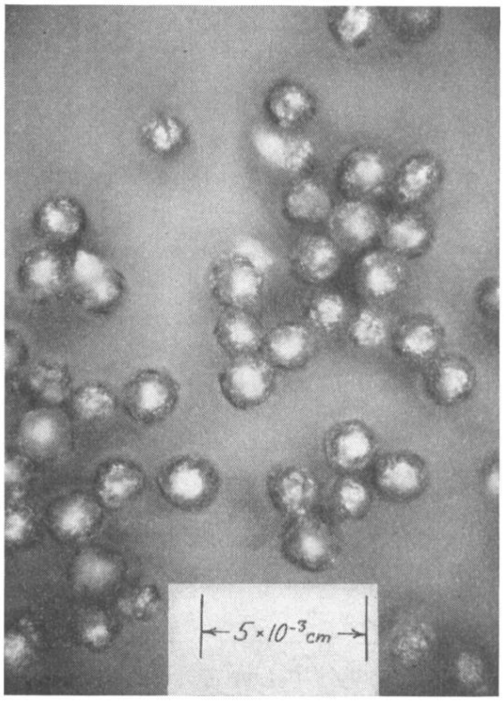

FIG. 2. Lead particles of the 10 to 15 micron fraction obtained by air elutriation.