thus giving the fraction of secondaries emerging at angles less than θ. Experimental information on the directional distribution is very limited, but indicates that the emission per unit solid angle is approximately proportional to $\cos\theta$, which corresponds to $\sin^2\theta$ for the fraction plotted in the figure. The deviations between our result and $\sin^2\theta$, which appear in the figure, are probably too small to be significant.

The angular dependence implied by the theory is best seen analytically by expanding the integral (35) in powers of $\sin^2\theta$, rather than by considering its exact value. When this is done, one finds that for $\mu_0^2=1.6$, the emission per unit solid angle is proportional to:

$$\cos\theta(1+0.28\sin^2\theta+0.14\sin^4\theta+\cdots) \quad (36)$$

and that the coefficients vary slowly with μ_0 in the range of interest.

7. CONCLUDING REMARKS

It appears here, as in the earlier work of Kadyschewitsch, that calculations based on the free electron model of Sommerfeld can lead to a considerably improved understanding of secondary electron emission from metals, so that this phenomenon should have its place along with the many others which have been illuminated by this simple picture.

The author wishes to acknowledge the interest and encouragement of colleagues at Battelle, especially Drs. H. R. Nelson and F. C. Todd.

PHYSICAL REVIEW VOLUME 78, NUMBER 6 JUNE 15, 1950

Measurement of the Proton Moment in Absolute Units*

H. A. THOMAS, R. L. DRISCOLL, AND J. A. HIPPLE
National Bureau of Standards, Washington, D. C.
(Received January 23, 1950)

By measuring the absolute value of the magnetic field and the frequency required for nuclear resonance absorption in a water sample, the gyromagnetic ratio of the proton has been determined to be $\gamma_P=(2.67528\pm0.00006)\times10^4$ sec.$^{-1}$ gauss^{-1}. With this value and Planck's constant the value of the magnetic moment of the proton in absolute units becomes $\mu_P=(1.4100\pm0.0002)\times10^{-23}$ dyne cm/gauss.

A combination of our result with recent measurements of the proton moment in Bohr magnetons by Gardner and Purcell results in a value of $e/m=(1.75890\pm0.00005)\times10^7$ e.m.u. gram^{-1}.

I. INTRODUCTION

SINCE the development of molecular beam,[1] nuclear induction,[2] and nuclear resonance absorption[3,4] techniques, considerable work has been done on the determination of nuclear gyromagnetic ratios. From this ratio and the nuclear spin, the magnetic moment of the nucleus can be calculated.

The measurement of a gyromagnetic ratio γ involves the measurement of the frequency ν and magnetic field of induction B_0 required for resonance as indicated by the condition for resonance, $\omega=2\pi\nu=\gamma B_0$. The comparison of either gyromagnetic ratios or magnetic moments requires only frequency determinations and for this reason much of the data on magnetic moments now available is of this type. A few direct measurements of gyromagnetic ratios in absolute units have been made with accuracies of the order of 0.5 percent, which is about the best that can be done with the ordinary techniques of measuring magnetic fields. In the experiment reported here the proton gyromagnetic ratio has been determined with much greater accuracy by using more elaborate methods of measuring the magnetic field and frequency. This precise measurement will allow previous relative determinations to be recalculated in absolute units and will also provide a convenient standard of magnetic field for the measurement of other atomic constants.

The nuclear absorption method of Purcell, Torrey, and Pound[3,4] was used for detecting resonance because the field involved lends itself more readily to precise measurement than that used in the molecular beam method and the apparatus appeared somewhat simpler to construct than that employed by Bloch in the nuclear induction experiment.

The use of one of the Bureau of Standards precision solenoids would provide the most accurately known magnetic field but this possibility was initially discarded because the maximum field available was only of the order of 20 gauss. The nuclear resonance signal-to-noise ratio becomes very low in such a weak field and at the time this experiment was planned no attempt had yet been made to work in this range. The recent success of Brown and Purcell[5] in working in fields as low as 11 gauss now makes the solenoid method more at-

* Further details of this experiment will be published in J. Research Nat. Bur. Stand.
[1] Rabi, Millman, Kusch, and Zacharias, Phys. Rev. **55**, 526 (1939).
[2] F. Bloch, Phys. Rev. **70**, 460 (1946).
[3] Purcell, Torrey, and Pound, Phys. Rev. **69**, 37 (1946).
[4] Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).
[5] L. M. Brown and E. M. Purcell, Phys. Rev. **75**. 1262 (1949).

tractive than when the present work was undertaken; as a means of eliminating systematic errors and possibly improving the accuracy, it is important that the measurements described here be repeated. This is now being done.

In the experiment here described an electromagnet with relatively strong field was used which reduced the signal-to-noise problem but which required auxiliary apparatus for measuring the field. The method of measuring the magnetic field is one similar to that previously described in the literature.[6–8] In this method the magnetic field is determined by measuring the force on a known length of conductor carrying a known current. A long rectangular coil is suspended from an analytical balance with the lower end of the coil in the gap of the magnet. The vertical sides of the coil act as connecting leads to the "force conductors" which are formed by the lower horizontal portion of the coil. The fringing field of the magnet at the upper end of the coil is reduced to a negligible value by means of Helmholtz coils.

Since the force f is measured in the vertical or y direction and the current i is everywhere the same in the circuit the force may be given by

$$f=\int_{\text{coil}} iB(x, y)dx. \tag{1}$$

The integration can be carried out numerically from measurements of the width of the coil at various vertical positions together with determinations of the field at these vertical positions. If we use as a reference point the average value of the field over the bottom wires $\bar{B}$, then the effective width X of this coil when used as described with a particular magnet and Helmholtz coil arrangement may be defined by the relationship

$$X=(1/\bar{B})\int_{\text{coil}} B(x, y)dx. \tag{2}$$

The average field in terms of the observed force f

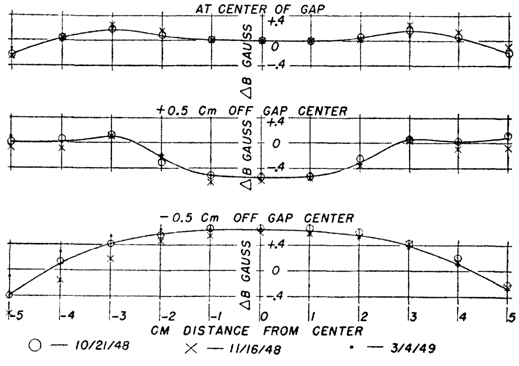

FIG. 1. Field distribution.

(dynes), X (cm), and I (amp.) in a coil of n turns is

$$\bar{B}=10f/nIX. \tag{3}$$

Since it is the average field over the region occupied by the force conductors that is measured by the force method, it is essential that the field distribution be precisely determined and that the field in this region be reasonably uniform. Then from the value of the average field and the field distribution the value of the field at the proton sample, located just below the coil, may be determined.

II. APPARATUS AND EXPERIMENTAL PROCEDURE

A nuclear resonance detector employing a radio-frequency bridge will show both the absorption and dispersion depending on the amount and kind of unbalance in the bridge.[4] Any dispersion signal present would introduce some error in determining the true center of the resonance line. For this reason and also to eliminate the microphonic problem, an "amplitude bridge" detector was developed[9] which is insensitive to phase changes and hence will not indicate the dispersion signal.

A 0.4-cc sample of water adjusted with a ferric salt was used. The amount of ferric salt added (for adjustment of relaxation time[4] T_1) was just enough to give maximum signal with no appreciable line broadening. Under these conditions the line width, which was determined by the inhomogeneity of the field, was 0.25 gauss at the half-power points. Substitution of an oil sample (Nujol) for the water sample indicated no observable shift in the line due to the susceptibility of the ferric salt. The glass sample holder and r-f coil were mounted in the end of a $\frac{3}{4}$-in. brass tube which served as the coaxial line feeding the coil. Small Helmholtz coils were mounted on either side of the sample for modulating the magnetic field. This whole resonance probe assembly was checked with a magnetometer for magnetic impurities.

The magnetic field of about 5000 gauss was produced by a water-cooled magnet which had auxiliary coils in addition to the main exciting windings. The poles of the magnet were 8.5×12.5 in. and the gap separation was 2 in. The value of the field was adjusted so that the resonance frequency would be 20 Mc which can be heterodyned directly with WWV for the frequency measurement.

The field distribution in the gap was obtained by using two resonance probes: one to plot the field; the other capable of regulating it to two parts per million (ppm) for a change of one percent in the main exciting current by means of a phase detector and a power amplifier supplying the auxiliary coils. The first probe was mounted on a cross feed that would allow accurate positioning in the gap. As the probe is moved, a varia-

[6] A. Cotton and G. Dupouy, Congres Internat. d'Elec. Section 3, **2**, 208 (1932).
[7] F. A. Scott, Phys. Rev. **46**, 633 (1934).
[8] G. H. Briggs and A. F. A. Harper, J. Sci. Inst. **13**, 119 (1936).
[9] H. A. Thomas and R. D. Huntoon, Rev. Sci. Inst. **20**, 516 (1949).

TABLE I. Typical results of a series of measurements. Platinum-iridium mass $m=8777.29$ mg; current$=0.1017274$ amp.; sensitivity of the balance $S=1.59$ mg/cm.

Date	Temp. of coil °C	$\delta \cdot s$ (mg)	Temp. corr. (mg)	Corrected mass (mg)	$\bar{B}$ gauss	$(B_0-\bar{B})$ gauss	B_0 gauss	dev. from mean
May 6, 1949	26.9	+0.30	−0.16	8777.43	4697.56	+0.02	4697.58	0.03
May 9, 1949	26.2	−0.08	−0.10	8777.11	4697.38	+0.11	4697.49	0.06
May 12, 1949	26.4	+0.09	−0.12	8777.26	4697.46	+0.07	4697.53	0.02
May 16, 1949	25.6	+0.08	−0.05	8777.32	4697.50	+0.09	4697.59	0.04
							Mean 4697.55	±0.04

Note.—$\nu=20{,}001{,}075\pm20$ cycles per sec. γ_P = gyromagnetic ratio of proton $=2\pi\nu/B_0=2.67523\times10^4$ sec.$^{-1}$ gauss^{-1}.

tion in field is indicated by a shift in the resonance line on the oscilloscope. A second pair of small Helmholtz coils mounted on this probe was used to bring the resonance line back to its original position on the oscilloscope screen. This Helmholtz pair had previously been calibrated by measuring the shift in resonance frequency produced by a given change in current in the coils. To provide reference lines on the oscilloscope, the audiofrequency voltage applied to the field modulating coils was also applied to a peak clipping and differentiating circuit that would produce sharp pulses. With this arrangement, variations in field as a function of position could be plotted with an error of less than 0.02 gauss.

After shimming with thin nickel shims spatial variations in the field were held to a quite low value as indicated by the typical distribution curves shown in Fig. 1. It was not assumed that the field distribution would remain constant and so field distribution data were taken just preceding and following each run of force measurement data. In addition to that shown in Fig. 1 the field distribution up the sides of the coil was taken for the purpose of determining the effective width X of the coil. These data were taken with the resonance probe as far as possible and then with a rotating coil fluxmeter (accuracy 0.2 percent) the remainder of the distance to the top of the coil.

The precision coil for measuring the magnetic field was wound on the edge of a glass plate 10×70 cm and 0.7 cm thick. The coil has nine turns of wire that lie in grooves formed around the edge of the plate.[10] The wires pass around the sides and bottom of the plate parallel to the edge of the glass and cross over to the next groove at the top. Oxygen-free copper wire was drawn through a special die and wound directly on the plate under constant tension.

Measurements on the coil width were made relative to Johannson gage blocks in a temperature controlled cabinet by means of a motor driven micrometer devised by Moon.[11] Measurements of the temperature and resistance of the coil made simultaneously with the width measurements gave a means of correcting later for possible coil expansion. The diameter of the wire was measured by a similar method using samples of wire taken initially from the ends of the coil. After completion of the experiment the turns of the coil were cut from the plate and samples from each side of each turn were measured under proper loading at various vertical positions and angles. By this means an initial uncertainty in the effective width of the coil of 40 ppm was reduced to 10 ppm.

The current through the coil was measured by the conventional method of comparing directly the drop in a standard resistor to the e.m.f. of a standard cell. The errors associated with this measurement are listed later (Table II).

The force produced on the current-carrying coil was evaluated by comparison with the action of gravity on a known mass that was placed on the scale pan of the balance when the current was reversed. In case the comparison was not exact, a small correction was made to the mass in terms of the sensitivity of the balance, s, and the change in its rest point, δ. Remembering the reversal of current and denoting the mass of the removable weight by m, and the acceleration of gravity by g, a working formula for the field $\bar{B}$ is given by

$$\bar{B}=\frac{5(m+\delta\cdot s)g}{nIX[1+a(t-25)]}, \tag{4}$$

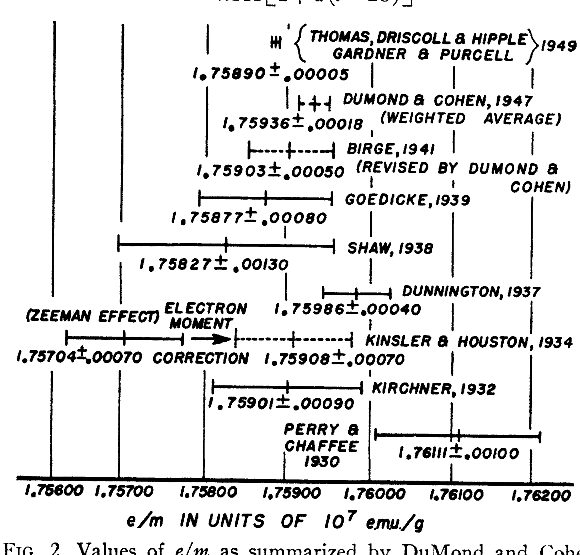

FIG. 2. Values of e/m as summarized by DuMond and Cohen compared with the value reported in this paper.

[10] Curtis, Moon, and Sparks, J. Research Nat. Bur. Stand. **21**, 375 (1938), RP1137.

[11] C. Moon, Bur. Stand. J. Research **10**, 249 (1933), RP528.

TABLE II. Tabulation of errors.

Contributing errors	Parts per million
Platinum-iridium mass—m	1
Precision of balance used in this experiment—$\delta\cdot s$	9
Gravity	3
Electrical standards	10
Comparison of current with standards	3
Width of coil	10
Length standard	5
Field distribution	10
Neutralization of stray field at upper wires	5
Adjustment of resonance pip to reference point on oscilloscope	4
Calibration of Helmholtz coils used for field distribution	4
Resonance frequency	1
Effect of ferric ions in sample	4

where a is the coefficient of thermal expansion for the coil and t its temperature. In practice the current I had some fixed value according to the standards selected and the magnetic field had some value consistent with the frequency required for proton resonance; therefore, the mass m was adjusted by trial to a point where the correction $(\delta\cdot s)$ was so small that the uncertainty in the sensitivity of the balance did not produce more than a tolerable error in the force. The mass was then standardized by the mass section of the Bureau.

III. RESULTS

Two measurements of γ_P were made over a period of about six months with slight improvements in technique being made in the second series. The results of the two series are practically identical. Considering all runs the average deviation is approximately 10 ppm. Typical results of a series of measurements are shown in Table I.

The known contributing errors listed in Table II were estimated conservatively. The square root of the sum of their squares results in an uncertainty of 22 ppm in the gyromagnetic ratio of the proton. The result can be stated as $\gamma_P=(2.67523\pm0.00006)\times10^4$ sec.$^{-1}$ gauss^{-1} uncorrected for the diamagnetic effect. The stated error is thought to be several times the corresponding probable error. If a diamagnetic correction,[12] which was not included in the preliminary result,[13] is applied to the above value one obtains $\gamma_P=(2.67528\pm0.00006)\times10^4$ sec.$^{-1}$ gauss^{-1}.

The magnetic moment of the proton is known less accurately than the gyromagnetic ratio because of the uncertainty in the value of Planck's constant h. Using a value[14] of

$$h=(6.6234\pm0.0011)\times10^{-27}\text{ erg sec.},$$

the magnetic moments of the proton is

$$\mu_P=(1.4100\pm0.0002)\times10^{-23}\text{ dyne cm/gauss.}$$

IV. COMPARISON WITH OTHER MEASUREMENTS

Gardner and Purcell[15] have just completed a measurement of the ratio of the precession frequency of the proton $\omega=\gamma_pB_0$ to the cyclotron frequency $\omega_e=eB_0/m$ of a free electron in the same magnetic field. After making the same diamagnetic correction they give for the ratio $\omega/\omega_e=(1.52100\pm0.00002)\times10^{-3}$. This ratio is the magnetic moment of the proton in Bohr magnetons, and agrees very well with the value $\mu_P=(1.52106\pm0.00007)\times10^{-3}$ obtained by Taub and Kusch.[16] These values of μ_p may be combined with the value of the gyromagnetic ratio of the proton to give a value of e/m. In this calculation it is preferable to use the value of Gardner and Purcell because of its greater accuracy and because the electron moment correction is not involved. The value of e/m becomes $e/m=\gamma_p\omega_e/\omega=(1.75890\pm0.00005)\times10^7$ e.m.u. gram^{-1}. This differs only slightly from the previously published value[17] which was obtained by combining our preliminary value of γ_p with the measurement of Taub and Kusch. A comparison of the measured values of e/m is shown on Fig. 2.

The value of the gyromagnetic ratio of the proton and hence the above value of e/m has recently received further confirmation since it has been used in determining the faraday[18] in which excellent agreement was obtained.

Contributions of other staff members are gratefully acknowledged and in particular the authors wish to thank Dr. Charles Moon for advice on the construction and measurement of the coil and Dr. R. D. Huntoon for his many helpful suggestions.

[12] W. E. Lamb, Jr., Phys. Rev. **60**, 817 (1941). This correction (1.8×10^{-5}) is for the hydrogen atom. N. F. Ramsey [Phys. Rev. **77**, 567 (1950)] has calculated the correction for the H_2 molecule to be 2.7×10^{-5}. If the frequency shift between H_2 and our sample were determined, the result given in this paper would be improved.

[13] Thomas, Driscoll, and Hipple, Phys. Rev. **75**, 902 (1949).

[14] J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. **20**, 82 (1948).

[15] J. H. Gardner and E. M. Purcell, Phys. Rev. **76**, 1262 (1949).

[16] H. Taub and P. Kusch, Phys. Rev. **75**, 1481 (1949); P. Kusch and H. Taub, Phys. Rev. **75**, 1477 (1949).

[17] Thomas, Driscoll, and Hipple, Phys. Rev. **75**, 992 (1949).

[18] Hipple, Sommer, and Thomas, Phys. Rev. **76**, 1877 (1949).