A Theory of Secondary Electron Emission from Metals*

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A theory of secondary electron emission from metals is formulated on the basis of the Sommerfeld free-electron model, momentum transfer between electrons and lattice being included by introducing a finite mean free path for elastic scattering. The approach to the problem is similar to that of Kadyschewitsch, but the development is simpler and comparison with experiment is made in more detail. An understanding is reached of the influence of work function and the width of the conduction band, making it clear why, on the average, metals with large work function might be expected to be the best emitters. The observed effect of changing the work function of a given metal by surface layers of foreign atoms is interpreted, an inverse relationship between emission and work function being obtained which is in qualitative agreement with experiment. The theory also accounts for the velocity distribution of the secondaries, giving the general shape of the curve and determining the approximate position of the maximum, and is consistent with the observed angular distribution of the secondaries. The investigation is not carried far enough to give new theoretical information concerning the variation of secondary emission with primary energy. However, the relation of the present theory to some work of Bruining is indicated, and attention directed to an important empirical relationship.

1. INTRODUCTION

IT is well appreciated that a tremendous clarification of phenomena occurring in metals followed Sommerfeld's application of Fermi-Dirac statistics to the free electrons. The simplicity of the Sommerfeld model gave it lasting value, even in cases where more complicated models were employed with considerable success in later work. It is therefore remarkable that until a few years ago almost no effort was made to provide a simple treatment of secondary electron emission in terms of this model.

The theoretical papers most often quoted in this field are those of Fröhlich[1] and Wooldridge,[2] in which the problem is handled quantum mechanically, the electrons of the metal being described by wave functions of the Bloch type. In a way, this degree of complexity is necessary since, as was emphasized by Fröhlich, momentum transfer between electrons and lattice plays an essential part in secondary emission. In particular, without such transfer no secondaries could leave a surface bombarded by normally incident primaries. When Bloch wave functions are used in the manner of Fröhlich and Wooldridge, this transfer enters automatically, since the conservation of momentum equation for the interaction producing a secondary includes a term representing momentum change of the lattice. This type of treatment also brings to light certain interesting quantum effects associated with the discrete momentum changes which are permitted the lattice.

There would seem to be qualitative value, however, in a much simpler theory in which the primary is considered to interact in a classical manner with a free electron gas, the possibility of momentum transfer to the lattice being introduced by assuming a finite mean free path for elastic collision. Several years ago such a theory was developed by Kadyschewitsch in a series of three papers.[3] In the first of the papers, the author formulates a theory and applies it to metals, giving some attention to most secondary emission phenomena which have been studied experimentally. In the later papers he extends the theory to insulators and semiconductors. The calculations are almost entirely classical and are intended to give insight into the mechanism of emission and to emphasize properly the influence of electron scattering and absorption. Although the use of classical principles greatly simplifies the calculations, the problem remains complex and it is not clear that all the mathematical difficulties have been adequately met. The final results are therefore probably more limited in significance than would have been judged from the starting assumptions. It is partly because of this circumstance, and partly because it is felt that more attention should be given the Sommerfeld model, that the present paper is offered. Here an effort is made to develop an alternative treatment which is less elaborate than that of Kadyschewitsch, but still is capable of giving some results of about the same accuracy. Attention is limited to a few aspects of emission from metals, and certain substantial simplifications are introduced from the beginning. Although the development gives some insight into a variety of secondary emission phenomena, it is most successful in showing how the Sommerfeld model can clarify the role of work function, and account in a qualitative way for the velocity distribution of the secondaries.

2. THE VELOCITIES OF INTERNAL SECONDARIES

Since no attention is to be given to the small temperature dependence of secondary emission from metals, the electrons will be treated as a completely degenerate Fermi-Dirac gas. In momentum space all states lying within a sphere of radius p_0 about the origin are filled, while states of greater momentum are empty. The momentum p_0 corresponds to the maximum Fermi

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[1] H. Fröhlich, Ann. d. Physik **5**, 13, 229 (1932).

[2] D. E. Wooldridge, Phys. Rev. **56**, 562 (1939).

[3] A. E. Kadyschewitsch, J. Phys. U.S.S.R. **2**, 115 (1940); **4**, 341 (1941); **9**, 431 (1945).

energy, $E_0=p_0^2/2m$, and is related to the space density of free electrons in the familiar manner.[4]

Primary electrons of about one hundred electron-volts or greater are considered. The collisions of greatest interest are then those in which the fast primary transfers a small part of its energy to an electron of the metal. In calculating the momentum transfer for such a collision one may consider that the primary moves with constant speed along a straight line and interacts with a metal electron whose motion during the collision is negligible. In this approximation, the momentum transferred is perpendicular to the primary path and is given by the integral:

$$\Delta p=\int_{-\infty}^{\infty}\frac{e^2\rho dt}{(\rho^2+V^2t^2)^{\frac{3}{2}}}=\frac{2e^2}{V\rho}. \tag{1}$$

Here, e is the electronic charge, ρ, the distance between the metal electron and the primary path, and V, the velocity of the primary. The time, t, is measured from the instant of closest approach.

Thus, for the metal electrons at distance ρ, the effect of the passing primary is a shift of the center of the occupied momentum-space sphere from the origin by the amount Δp. The number of secondaries *per unit volume* at this distance having momentum greater than μp_0, is the volume of the displaced sphere which lies outside a sphere of radius μp_0 about the origin, multiplied by the density in phase space, $2/h^3$. Denoting this number by G, and calculating for $\mu>1$, which is the only case of interest here, one finds:

$$G(s)=\frac{\pi p_0^3}{6h^3 s}\{3(\mu^2-1)^2-8(\mu^3-1)s+6(\mu^2+1)s^2-s^4\} \tag{2}$$

$$\mu-1\leqslant s\leqslant\mu+1$$

$$G(s)=8\pi p_0^3/3h^3 \quad s\geqslant\mu+1, \tag{3}$$

where h is Planck's constant and $s=\Delta p/p_0$.

By carrying out an integration over ρ, one can now calculate $N(\mu)$, the production of secondaries *per unit primary path length* with momentum exceeding μp_0. According to (1), the element of area is

$$2\pi\rho d\rho=-(8\pi e^4/V^2p_0^2)ds/s^3 \tag{4}$$

and $N(\mu)$ is given by the integral:[5]

$$N(\mu)=(8\pi e^4/V^2p_0^2)\int_{\mu-1}^{mV/p_0}G(s)s^{-3}ds. \tag{5}$$

When the integral is evaluated for high primary velocity, that is $mV\gg(\mu+1)p_0$, one obtains:

$$N(\mu)=\frac{32\pi^2e^4p_0}{3h^3V^2(\mu^2-1)}. \tag{6}$$

The derivative, $-dN/d\mu$, gives the momentum distribution of the internal secondaries, μ being momentum measured in terms of the maximum Fermi gas momentum, p_0. Introducing the maximum Fermi energy, E_0, and the kinetic energy of the primary electron, W, (6) becomes:

$$N(\mu)=BE_0^{\frac{1}{2}}/W(\mu^2-1). \tag{7}$$

When the energies are in electron volts

$$B=\frac{160\pi^2(6)^{\frac{1}{2}}m^{3/2}e^{7/2}}{3h^3}=2.95\times10^8\ \text{ev}^{\frac{1}{2}}-\text{cm}^{-1}. \tag{8}$$

It is of interest to consider whether the calculated rate of production is at all reasonable. If μ_0 is the minimum value of μ needed for escape from the metal, one finds for typical metals, and for $W\sim10^2$ electron volts:

$$N(\mu_0)\sim10^7\ \text{cm}^{-1}.$$

This result does not seem unreasonable. It means that if there were no secondary absorption, so that all secondaries having sufficient energy ultimately escaped, a primary range of about 10A would account for the one secondary per primary ordinarily emitted when the primary energy is several hundred electron volts.

3. THE SECONDARY EMISSION COEFFICIENT

In this paper, attention is limited to perpendicular incidence, and the elastic scattering of the primaries by the metal lattice is neglected. All the secondaries leaving the metal must then have undergone at least one elastic collision. The relative sizes of groups having undergone various numbers of such collisions will depend on the ratio of the mean free path for secondary scattering, l, to that for absorption,[6] λ. In a reasonably complete theory which was to be valid for all values of this ratio, it would be in order to formulate expressions for the contributions to the emission from the various groups, and to obtain the total emission by summing these contributions. The calculations of Kadyschewitsch are organized in this manner. However, in the present discussion, it is not proposed to enter into such relatively complex calculations. Rather, two special calculations will be made. First, the emission of singly scattered secondaries will be considered, which would approximate the total emission only if l were much larger than λ. Then, the total emission will be obtained for the opposite extreme, $l\ll\lambda$. Comparison of results in the two cases will indicate that some fea-

[4] See, for example, the outline of the free electron theory of metals given by F. Seitz, *The Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), Chapter IV.

[5] The upper limit corresponds to a momentum transfer equal to the total momentum of the primary. In this calculation, certain errors are made in the small ρ-range. However, this should not lead to a significant final error, since the collisions in this range are relatively few.

[6] For simplicity, it is pretended that the secondaries can only undergo elastic scattering or absorption; and the energy dependence of secondary free paths is neglected. These two simplifications should partially compensate each other.

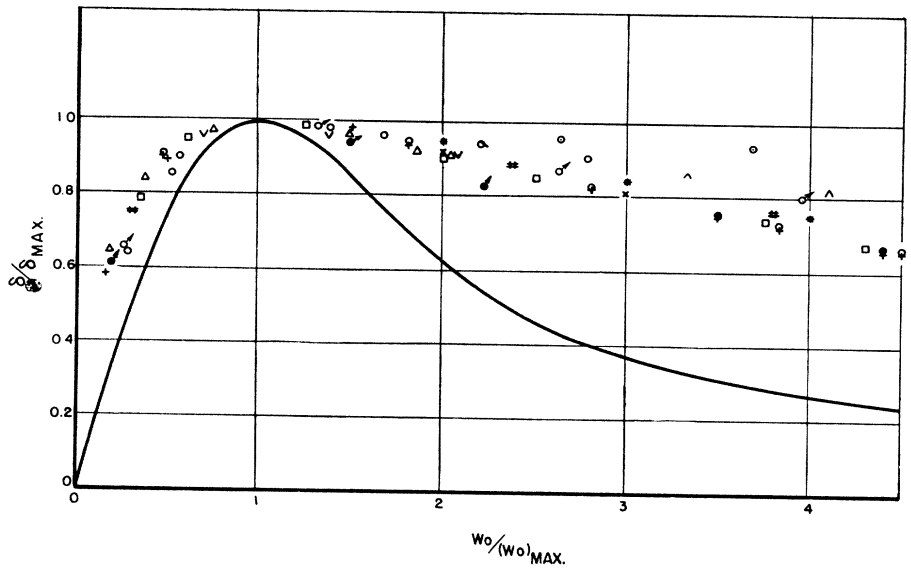

FIG. 1. Variation of secondary emission with primary energy. δ/δ_{max} is the secondary emission coefficient divided by its maximum value, and $W_0/(W_0)_{max}$ is the primary energy divided by the energy for maximum emission. The points show experimental results for various materials, taken from small-scale published curves: + Cu, × Ni, ♀ Li, △ Ti, □ Mo, ○ Pt, ∨ Al, ♂ Mg, # Fe, ♂ Cb, ⊙ Rb, ∧ B, ✳ Ge, ♀ Si. The points approximate a single curve. The curve, based on Eq. (24), is qualitatively similar to the experimental curve.

tures of secondary emission are rather well covered by the calculations, in spite of their simplicity.

Singly Scattered Secondaries

An expression will be obtained for δ_1, the number of secondary electrons which leave the metal surface per incident primary after having undergone a single elastic collision. The probability that this collision takes place at a distance between r and $r+dr$ from the point of interaction with the primary, can be expressed in the form:

$$\exp(-\sigma r)\cdot dr/l, \tag{9}$$

where σ is the inverse mean free path $(l+\lambda)/l\lambda$. Integrating over r, and assuming isotropic scattering, one has for the scattering probability per unit solid angle, $(4\pi\sigma l)^{-1}$. Letting q denote the cosine of the angle with the outwardly directed normal to the surface, the probability of scattering into any range dq is

$$dq/2l\sigma. \tag{10}$$

The typical secondary will be considered to move perpendicularly to the primary path before elastic collision with the lattice, so that a once-scattered secondary, initially produced at depth z and moving in a direction specified by q, must travel a distance z/q to reach the surface. For the numerous weak collisions which account for most of the secondaries from a fast primary, the momentum transferred is, in fact, approximately perpendicular to the primary path. Since the initial velocity distribution of the electrons is isotropic, the behavior taken as typical does correspond in a rough way to the average process.

Letting $n(q, z)$ be the number of secondaries produced per unit path length at depth z with sufficient energy to escape at the angle $\cos^{-1}q$, and taking account of secondary scattering and absorption during traversal of the layer of thickness z, one has:

$$\delta_1=(2l\sigma)^{-1}\int_0^1\int_0^\infty n(q, z)\exp(-\sigma z/q)dzdq. \tag{11}$$

According to Eq. (7) of the previous section

$$n(q, z)=N\left(\frac{\mu_0}{q}, z\right)=\frac{BE_0^{\frac{1}{2}}q^2}{W(\mu_0^2-q^2)}, \tag{12}$$

where $\mu_0^2=(E_0+\Phi)/E_0$, Φ being the work function of the metal. For the typical monovalent metals, to which the present theory should best apply, μ_0^2 may be computed from a calculated E_0 and the observed work function. Values obtained in this way range from about 1.5 to 2.0.

The dependence of production on z enters through the slowing down of the primary as it penetrates into the metal. The simplest approximation is that the space rate of energy loss, $-dW/dz$ is inversely proportional to W, which implies:

$$W^2=W_0^2-az. \tag{13}$$

Here W_0 is the initial energy of the primary, and a is a constant which is characteristic of the metal. With these assumptions (11) takes the form:

$$\delta_1=\frac{BE_0^{\frac{1}{2}}}{2l\sigma}\int_0^1\int_0^{W_0^2/a}\frac{q^2\exp(-\sigma z/q)dzdq}{(W_0^2-az)^{\frac{1}{2}}(\mu_0^2-q^2)}. \tag{14}$$

The integrand in (14) has a singularity at the upper limit of the integration which results from using the expression for the production of secondaries by a *fast* primary to the very end of the primary range. However, the integral converges in spite of the singularity, and, in fact, does not differ significantly from a more carefully calculated value based on a more reasonable variation of productivity in the low energy range.

A simple change of variable[7] makes it possible to express the z-integral in terms of tabulated functions. If, in addition, $\omega=q^{-\frac{1}{2}}$ is introduced, one is led finally to the following expression for δ_1:

$$\delta_1=\frac{2BE_0^{\frac{1}{2}}}{l\sigma^{\frac{3}{2}}a^{\frac{1}{2}}}\int_1^\infty \frac{F(H\omega)d\omega}{(\mu_0^2\omega^4-1)\omega^4}, \tag{15}$$

where

$$F(x)=e^{-x^2}\int_0^x \exp(t^2)dt \tag{16}$$

and

$$H=(W_0^2\sigma/a)^{\frac{1}{2}}. \tag{17}$$

The function $F(H\omega)$ has a maximum at $H\omega=0.92$, and varies slowly in this neighborhood. On the other hand, the remaining factors in the integrand appearing in Eq. (15) are largest at the lower limit $\omega=1$, and decrease rapidly with increasing ω. Thus for H near unity (15) can be approximated by the expression:

$$\delta_1=\frac{2BE_0^{\frac{1}{2}}}{l\sigma^{\frac{3}{2}}a^{\frac{1}{2}}}F(H)\int_1^\infty \frac{d\omega}{(\mu_0^2\omega^4-1)\omega^4}. \tag{18}$$

To this approximation, the variation of δ_1 with primary energy is given simply by $F(H)$. A close examination of (15) with μ_0^2 given the reasonable value 1.6 shows that (18) is, in fact, a very good approximation. The chief difference is that, according to the more exact expression, the maximum of δ_1 occurs at $H=0.82$, instead of $H=0.92$. On the other hand, the value of the maximum is changed only very slightly by the approximation, being increased by about 1 percent.

Multiply Scattered Secondaries

When l is much smaller than λ, most electrons leaving the metal have undergone a large number of elastic collisions and have moved to the surface by a diffusion process. Considering a primary current of unit density (one electron/cm²-sec.), and using Eqs. (7) and (13), we may write for the production of secondaries with μ in the differential range $d\mu$ per unit volume per unit time:

$$\frac{2BE_0^{\frac{1}{2}}\mu d\mu}{(W_0^2-az)^{\frac{1}{2}}(\mu^2-1)^2}. \tag{19}$$

If all secondaries with $\mu>\mu_0$ could freely leave the metal on arrival at the surface, elementary diffusion theory[8] would give for the secondary current density the integral of the following over the primary range and over all $\mu>\mu_0$:

$$\frac{2BE_0^{\frac{1}{2}}\exp(-z/L)\mu d\mu dz}{(W_0^2-az)^{\frac{1}{2}}(\mu^2-1)^2}, \tag{20}$$

where L is the diffusion length $(\lambda l/3)^{\frac{1}{2}}$. Actually an electron approaching the surface with momentum μp_0 will escape only if the cosine of the angle with the normal exceeds μ_0/μ. Expression (20) must accordingly be multiplied by the corresponding fraction of the forward solid angle, $(\mu-\mu_0)/\mu$, before integration. In this way, one obtains:

$$\delta=\frac{2BE_0^{\frac{1}{2}}L^{\frac{1}{2}}}{a^{\frac{1}{2}}}F[(W_0^2/aL)^{\frac{1}{2}}]\int_{\mu_0}^\infty \frac{(\mu-\mu_0)d\mu}{(\mu^2-1)^2}, \tag{21}$$

where $F(x)$ is again the function defined in Eq. (16). The dependence of the emission on the primary energy

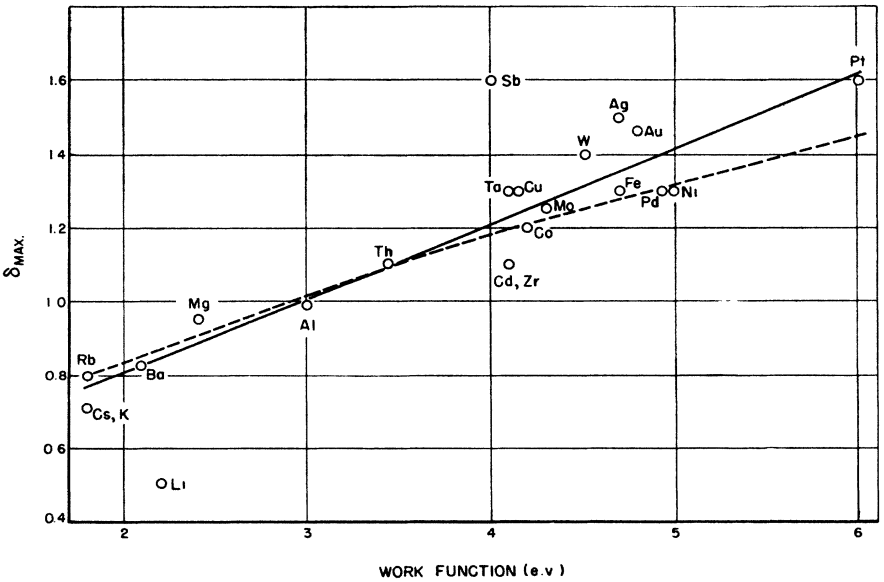

FIG. 2. Correlation between the maximum secondary emission, δ_{max}, and work function, Φ, for various metals. The solid line is drawn to show the trend of the experimental points, while the dashed line is a plot of $\delta_{max}=(0.35\Phi)^{\frac{1}{2}}$. The agreement shows that the influence of work function suggested by expression (26) is in accord with experiment.

[7] H. Bruining, *Die Sekundär-Elektronen-Emission fester Körper* (Verlag Julius Springer, Berlin, 1942; Edwards Brothers, Inc., Ann Arbor, 1944), p. 61.

[8] See, for example, P. R. Wallace, Nucleonics 4, 30 (1949).

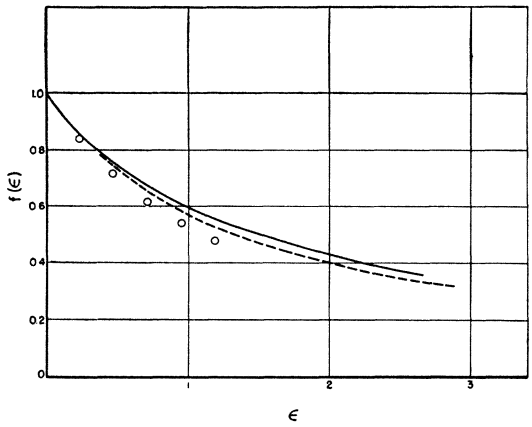

FIG. 3. Normal energy distribution of the secondaries. The ordinate, $f(\epsilon)$, is the fraction of the secondaries emitted with normal energy greater than $\epsilon\Phi$, where Φ is the work function. The curves follow from (27) with $\mu_0^2=1.6$ (solid) and 2.0 (dashed). The points are based on experimental data for Mo.

is similar to that found in Eq. (18) but the diffusion length, L, has taken over the position of σ^{-1}. Our principal interest in Eq. (21) lies in the function of μ_0 which appears. It will be seen later that comparison with the corresponding function in Eq. (18) indicates that the dependence of the emission on μ_0 is not sensitive to assumptions concerning multiple scattering.

4. THE VARIATION OF SECONDARY EMISSION WITH PRIMARY ENERGY

Our development of the theory does not go far enough to permit us to draw any new conclusions about the dependence of secondary emission on primary energy. However, we wish to interpolate a few paragraphs on this question in order to indicate the relationship of the present work to earlier considerations given by Bruining.[9] Also, the opportunity will be taken to point out an important fact about the experimental data which has apparently been ignored in previous publications.

The assumptions made by Bruining, as well as by other authors mentioned by him, are that the production of secondaries per unit primary path is proportional to the rate of primary energy loss, and that secondaries are absorbed exponentially. These lead to the following:

$$\delta=K\int(-dW/dz)e^{-\alpha z}dz, \tag{22}$$

where K is a proportionality constant characteristic of the metal, and α is the absorption coefficient. Bruining then uses the energy loss relationship, which we have also adopted in Eq. (13), and obtains the result

$$\delta=K(a/\alpha)^{\frac{1}{2}}F[(W_0^2\alpha/a)^{\frac{1}{2}}]. \tag{23}$$

The dependence of δ on primary energy so deduced is the same as would appear in the present work if we assumed $l\gg\lambda$, so that δ_1, as given by Eq. (18) represented the total emission.

Bruining showed that $F(x)$ has a maximum at $x=0.92$ so that the primary energy for maximum emission is $(W_0)_{\max}=0.92(a/\alpha)^{\frac{1}{2}}$. Also, using values of a and α which he regarded as reasonable, he showed that this leads to optimum energies which are of the correct order of magnitude. However, he does not report a complete comparison of the form of the experimental curve of emission *versus* primary energy with that implied by Eq. (23). This is most simply done by plotting $\delta/\delta_{\max}$ as a function of $W_0/(W_0)_{\max}$. If the emission were given by (23), the equation relating these quantities would be:

$$\delta/\delta_{\max}=1.85F\{0.92W_0/(W_0)_{\max}\}. \tag{24}$$

This relation is plotted in Fig. 1, together with experimental data for a number of materials. The substantial difference between the experimental and theoretical curves is hardly surprising. The interesting thing is the striking degree to which the experimental points approximate a single curve. As far as the writer knows, the existence of such a universal curve has not been pointed out previously, although a related regularity was studied by Copeland[10] and Warnecke.[11]

5. THE INFLUENCE OF WORK FUNCTION ON THE TOTAL NUMBER OF SECONDARIES

There was a natural tendency among early workers to expect that metals with low work function would be characterized by relatively large values of the secondary emission coefficient, but measurements have shown that this is by no means true. The most significant reason for this lack of correlation is the fact that the bulk of the electrons in the conduction band of the metal contribute to the secondary emission. This is in marked contrast to thermionic emission where only the electrons near the top of the band are important and the width of the band is of no significance.

The importance of the width of the conduction band appears in the present investigation when, in each of the alternative treatments, δ is found to be proportional to $E_0^{\frac{1}{2}}$ multiplied by a function of the parameter $\mu_0=(1+\Phi/E_0)^{\frac{1}{2}}$. In each case, this function appears as an integral. On evaluation, one obtains from Eq. (18)

$$(\mu_0^{\frac{3}{2}}/2)(\text{ctnh}^{-1}\mu_0^{\frac{1}{2}}-\text{ctn}^{-1}\mu_0^{\frac{1}{2}})-\tfrac{1}{3},$$

and from Eq. (21)

$$\tfrac{1}{2}(\mu_0\,\text{ctnh}^{-1}\mu_0-1).$$

For the range of interest, there is little difference between these functions, as may be seen by comparing the series expansions appropriate for $\mu_0>1$. Letting $x=\mu_0^{-2}$, these are respectively

$$x/7+x^2/11+x^3/15$$

[9] Reference 7, Chapter VI.

[10] P. L. Copeland, Phys. Rev. **46**, 167 (1934).

[11] R. Warnecke, L'Onde Electr. **16**, 509 (1937).

and

$$x/6+x^2/10+x^3/14+\cdots.$$

The fact that consideration of the two extreme cases leads to very similar functions is a strong indication that these are valid approximations for the actual case.

Since the two functions of μ_0 can be regarded as equally good approximations, the second will be used in examining the implications of the theory, because it is the simpler. The emission is then proportional to

$$E_0^{\frac{1}{2}}(\mu_0 \operatorname{ctnh}^{-1}\mu_0-1). \tag{25}$$

It is often an advantage to express E_0 in terms of the work function, Φ, so that only one parameter appears which is not readily measured. With this change (25) takes the form:

$$[\Phi^{\frac{1}{2}}/(\mu_0^2-1)^{\frac{1}{2}}](\mu_0 \operatorname{ctnh}^{-1}\mu_0-1). \tag{26}$$

If the reasonable assumption is made that the ratio μ_0^2, and such quantities as the secondary mean free paths, do not vary in a systematic way with work function one concludes from (26) that the tendency should be for δ to increase as the square root of the work function. Now McKay, in his recent review article,[12] has plotted $\delta_{\max}$ against Φ for a large number of metals and has drawn a line among the scattered points which shows the general trend. A similar plot is given in Fig. 2, where the solid line is that drawn by McKay. The dashed line is a plot of $(0.35\Phi)^{\frac{1}{2}}$, where the constant 0.35 has been chosen to give agreement with McKay's line at intermediate work functions. It is seen that the theory is correct in predicting an increase of $\delta_{\max}$ with Φ, and (over a reasonable range) is about right concerning the rate of increase.

Other experimental data which can be used as a check of the theory are those concerning the effect of surface layers of foreign atoms which leave E_0 unaltered, while changing μ_0 through changes of the work function. Bruining[13] gives such data, originally obtained by Treloar and by Sixtus, for oxygen and thorium on tungsten. Treloar's data, taken at a primary energy of 300 ev (maximum emission around 600 ev) are: for clean tungsten ($\delta=1.31$ $\Phi=4.52$ ev) and for tungsten with oxygen covering ($\delta=1.06$ $\Phi=6.30$ ev).

Expression (25), based on a simple spherical momentum-space distribution, would not be expected to apply too well to a complex metal like tungsten. However, it should give a qualitatively correct result if applied with μ_0 computed from the measured Φ and some arbitrary but reasonable value of E_0. Choosing $E_0=5$ ev, one gets for the ratio of emission coefficients after and before covering

$$\frac{1.50 \operatorname{ctnh}^{-1}1.50-1}{1.38 \operatorname{ctnh}^{-1}1.38-1}=0.78$$

in excellent but partly fortuitous agreement with the observed 0.81.

In the work of Sixtus, the increased emission resulting from decreasing the work function from 4.52 ev, to 3.30 ev and to 2.63 ev, was observed. The primary energy was varied from 100 ev to 1100 ev, which is well past the point of maximum emission. The fractional change of δ was found to be a function of the primary energy, decreasing by about a factor two between 100 ev and 600 ev, and remaining approximately constant thereafter. Constant fractional change is, of course, predicted by the expressions for δ developed in the present paper. Numerical estimates, made in the manner outlined above, lead to a fractional change about twice as large as that observed in the large energy range. It is seen from these comparisons, that the theory leads in a straightforward manner to a work function dependence of about the right magnitude.

6. VELOCITY DISTRIBUTION OF THE EMITTED SECONDARIES

(a) Normal Energy Distribution

Expression (25) may also be used to calculate the fraction of the emitted secondaries for which the "normal energy" (that is, $\frac{1}{2}mv_z^2$) exceeds the value, E. This is just that fraction which could still escape if the work function were greater by the amount E,[14] namely:

$$\frac{\mu_1 \operatorname{ctnh}^{-1}\mu_1-1}{\mu_0 \operatorname{ctnh}^{-1}\mu_0-1}, \tag{27}$$

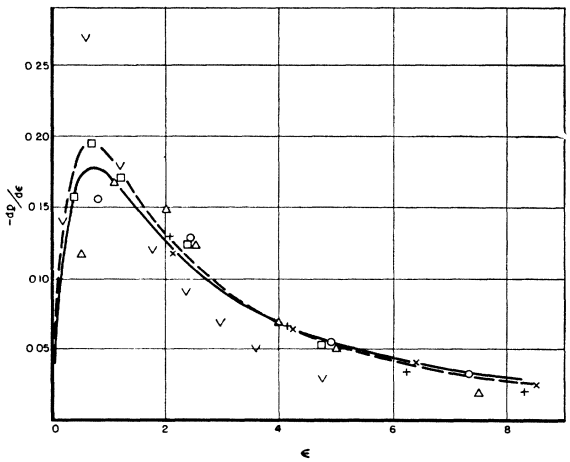

FIG. 4. Total energy distribution of the secondaries. $P(\epsilon)$ denotes the fraction of secondaries emitted with kinetic energy greater than $\epsilon\Phi$. Here $-dP/d\epsilon$, the fraction of secondaries per unit of ϵ, is plotted against ϵ. The curves are from Eq. (30) with $\mu_0^2=1.6$ (solid) and 2.0 (dashed). The points show experimental results for various metals: + Au, × Ag, ○ Cu (Rudberg, 1936); □ Mo, △ Cb (Haworth, 1935 and 1936); ∨ Mc (Kollath, 1941).

[12] K. G. McKay, contribution to *Advances in Electronics* (Academic Press, Inc., New York, 1948), L. Marton, editor, Vol. I.

[13] Reference 7, p. 56.

[14] [L. R. G. Treloar, Proc. Phys. Soc. London **49**, 392 (1937).] The simple relationship between the normal velocity distribution and the effect of work function change has been recognized a long time. Treloar used it in showing that his experimental results on work function change were consistent with observed velocity distributions.

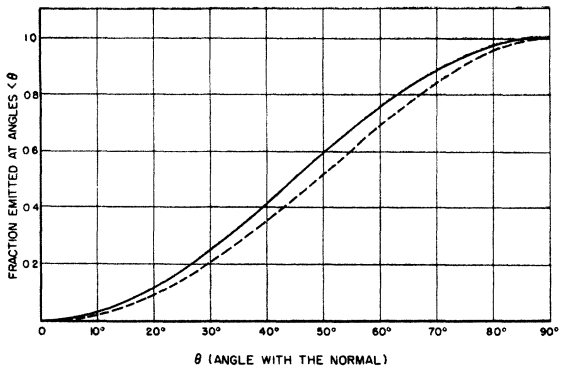

FIG. 5. Angular distribution of the secondaries. The fraction emitted at angles with the normal less than θ is plotted against θ. The solid line, $\sin^2\theta$, is the approximate result of experiment. The dashed line follows from expression (35) with $\mu_0^2=1.6$.

where

$$\mu_1^2=1+(\Phi+E)/E_0=\mu_0^2+(\mu_0^2-1)E/\Phi. \quad (28)$$

The curves in Fig. 3 show the fraction (27) as a function of $\epsilon=E/\Phi$, for $\mu_0^2=1.6$ and 2.0. It appears that the fraction is practically independent of μ_0. The points shown are based on numbers from Treloar's paper,[14] which were computed from experimental data of Haworth on the distribution in total energy of the secondaries from molybdenum ($\Phi=4.2$ ev). In translating from total energy to normal energy, Treloar assumed, in approximate agreement with experiment, that the intensity of emitted secondaries is proportional to the cosine of the angle between the direction of emission and the normal to the surface.

(b) Total Energy Distribution

According to the considerations leading to Eq. (21), the number of electrons leaving the metal with kinetic energy in excess of $\epsilon\Phi$ is proportional to

$$Q(\mu_1)=\int_{\mu_1}^{\infty}(\mu-\mu_0)(\mu^2-1)^{-2}d\mu, \quad (29)$$

where $\mu_1^2=\mu_0^2+(\mu_0^2-1)\epsilon$. The corresponding fraction of the secondaries is $P(\epsilon)=Q(\mu_1)/Q(\mu_0)$, and the differential distribution is:

$$-\frac{dP}{d\epsilon}=\frac{(\mu_0^2-1)(\mu_1-\mu_0)}{2\mu_1(\mu_1^2-1)^2Q(\mu_0)}, \quad (30)$$

In Fig. 4 this function is plotted against ϵ for $\mu_0^2=1.6$ and 2.0. Since these values cover a rather large range of this parameter, one may draw the conclusion that it is actually not of great importance in determining the velocity distribution.

Experimental points for a number of metals are shown in Fig. 4, the data having been taken from small-scale graphs in original papers by Haworth, Kollath, and Rudberg.[15] An adjustment of ordinates was made to make the experimental data correspond roughly to the normalization of the theoretical curves of unit total area. In general, the agreement is actually better than would have been expected. Possibly the data of Kollath, showing more slow secondaries than given by the theory, are more representative of what might reasonably be expected. It may be added that over a reasonable range of μ_0, Eq. (30) gives 0.7 Φ as the energy of the greatest number of secondaries. This agrees well with observation for a number of metals, and seems to be within a factor two of the measured value in essentially all cases.

(c) The Angular Distribution of the Secondaries

It is an almost immediate consequence of the simplifying assumptions made here that the secondary electrons arriving at the metal surface are moving approximately isotropically. It is therefore important to examine explicitly what this implies concerning the angular distribution of the secondaries after leaving the metal. According to the familiar refraction law, an electron incident from the inside at an angle β with the normal, and having momentum μp_0, will emerge at an angle θ given by:

$$\sin\theta=\mu\sin\beta/(\mu^2-\mu_0^2)^{\frac{1}{2}}. \quad (31)$$

For a selected value of β, the electrons emerging at angles less than θ are therefore those for which

$$\mu/(\mu^2-\mu_0^2)^{\frac{1}{2}}<\sin\theta/\sin\beta \quad (32)$$

or

$$\mu^2>\frac{\mu_0^2\sin^2\theta}{\sin^2\theta-\sin^2\beta}. \quad (33)$$

According to Eq. (7), the number of such electrons is proportional to

$$\frac{\sin^2\theta-\sin^2\beta}{(\mu_0^2-1)\sin^2\theta+\sin^2\beta} \quad (34)$$

for $\beta\leqslant\theta$, and zero outside this range. Introducing the assumption that the number of electrons incident in the range $d\beta$ is proportional to the corresponding solid angle, the number of electrons emerging at angles less than θ becomes proportional to the integral:

$$\int_0^{\theta}\frac{(\sin^2\theta-\sin^2\beta)\sin\beta}{(\mu_0^2-1)\sin^2\theta+\sin^2\beta}d\beta. \quad (35)$$

This is readily expressed in terms of elementary functions. The integral, divided by its value at $\theta=\pi/2$, is plotted in Fig. 5 against θ for the typical case, $\mu_0^2=1.6$,

[15] L. J. Haworth, Phys. Rev. **50**, 216 (1936); E. Rudberg, Phys. Rev. **50**, 138 (1936); R. Kollath, Ann. Phys. Lpz. **39**, 59 (1941). Kollath reports that annealing produces substantial changes in the distribution curves. This effect is outside the scope of the present investigation.

thus giving the fraction of secondaries emerging at angles less than θ. Experimental information on the directional distribution is very limited, but indicates that the emission per unit solid angle is approximately proportional to $\cos\theta$, which corresponds to $\sin^2\theta$ for the fraction plotted in the figure. The deviations between our result and $\sin^2\theta$, which appear in the figure, are probably too small to be significant.

The angular dependence implied by the theory is best seen analytically by expanding the integral (35) in powers of $\sin^2\theta$, rather than by considering its exact value. When this is done, one finds that for $\mu_0^2=1.6$, the emission per unit solid angle is proportional to:

$$\cos\theta(1+0.28\sin^2\theta+0.14\sin^4\theta+\cdots) \quad (36)$$

and that the coefficients vary slowly with μ_0 in the range of interest.

7. CONCLUDING REMARKS

It appears here, as in the earlier work of Kadyschewitsch, that calculations based on the free electron model of Sommerfeld can lead to a considerably improved understanding of secondary electron emission from metals, so that this phenomenon should have its place along with the many others which have been illuminated by this simple picture.

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Measurement of the Proton Moment in Absolute Units*

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By measuring the absolute value of the magnetic field and the frequency required for nuclear resonance absorption in a water sample, the gyromagnetic ratio of the proton has been determined to be $\gamma_P=(2.67528\pm0.00006)\times10^4$ sec.$^{-1}$ gauss^{-1}. With this value and Planck's constant the value of the magnetic moment of the proton in absolute units becomes $\mu_P=(1.4100\pm0.0002)\times10^{-23}$ dyne cm/gauss.

A combination of our result with recent measurements of the proton moment in Bohr magnetons by Gardner and Purcell results in a value of $e/m=(1.75890\pm0.00005)\times10^7$ e.m.u. gram^{-1}.

I. INTRODUCTION

SINCE the development of molecular beam,[1] nuclear induction,[2] and nuclear resonance absorption[3,4] techniques, considerable work has been done on the determination of nuclear gyromagnetic ratios. From this ratio and the nuclear spin, the magnetic moment of the nucleus can be calculated.

The measurement of a gyromagnetic ratio γ involves the measurement of the frequency ν and magnetic field of induction B_0 required for resonance as indicated by the condition for resonance, $\omega=2\pi\nu=\gamma B_0$. The comparison of either gyromagnetic ratios or magnetic moments requires only frequency determinations and for this reason much of the data on magnetic moments now available is of this type. A few direct measurements of gyromagnetic ratios in absolute units have been made with accuracies of the order of 0.5 percent, which is about the best that can be done with the ordinary techniques of measuring magnetic fields. In the experiment reported here the proton gyromagnetic ratio has been determined with much greater accuracy by using more elaborate methods of measuring the magnetic field and frequency. This precise measurement will allow previous relative determinations to be recalculated in absolute units and will also provide a convenient standard of magnetic field for the measurement of other atomic constants.

The nuclear absorption method of Purcell, Torrey, and Pound[3,4] was used for detecting resonance because the field involved lends itself more readily to precise measurement than that used in the molecular beam method and the apparatus appeared somewhat simpler to construct than that employed by Bloch in the nuclear induction experiment.

The use of one of the Bureau of Standards precision solenoids would provide the most accurately known magnetic field but this possibility was initially discarded because the maximum field available was only of the order of 20 gauss. The nuclear resonance signal-to-noise ratio becomes very low in such a weak field and at the time this experiment was planned no attempt had yet been made to work in this range. The recent success of Brown and Purcell[5] in working in fields as low as 11 gauss now makes the solenoid method more at-

* Further details of this experiment will be published in J. Research Nat. Bur. Stand.

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[5] L. M. Brown and E. M. Purcell, Phys. Rev. **75**. 1262 (1949).