Photo-Disintegration of the Deuteron at High Energies*

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(Received March 13, 1950)

Total cross sections and angular distributions for the photo-disintegration of the deuteron for energies between 10 and 150 Mev are computed for various types of central neutron-proton interactions. Most of the explicit computations are carried out for the case of 50 percent ordinary and 50 percent exchange interaction. Neutron-proton interactions considered are of the square well, Yukawa, and exponential type with two effective ranges. Theoretical analysis of the experimental data on high energy neutron-proton scattering excludes the square well, but is compatible with both Yukawa and exponential types of potentials. The Yukawa potential is replaced by an equivalent Hulthén potential. For the square-well and exponential potentials explicit analytic solutions were obtained. Our treatment of the case of the exponential potential is rigorous and at the same time almost as simple as that of the Hulthén potential. A qualitative discussion of the photoelectric dipole transition is given on the basis of the f-sum rule. This shows how exchange forces greatly increase the cross section at high energies. The discussion can be carried out for any exchange ratio. It is pointed out that only the photoelectric dipole and quadrupole cross sections can be obtained independently of specific meson theories of nuclear forces. The results for photo-disintegration show a pattern similar to those on neutron-proton scattering: very little difference between Yukawa and exponential potentials, somewhat larger difference between these two and the square-well potential. There is a pronounced dependence on effective range. In the Appendix a simple derivation of the dipole cross section for the ordinary square well potential is given, based on a transformation involving the equations of motion.

I. INTRODUCTION

OUR present knowledge of the neutron-proton interaction has been obtained from neutron-proton scattering data and from studies of the ground state of the deuteron. From low energy neutron-proton scattering one can obtain the effective range of the neutron-proton interaction,[1,2] but no information about the shape and exchange character of the potential. The values of the quadrupole and magnetic dipole moments of the deuteron indicate the existence of a tensor force between neutron and proton. High energy n–p scattering should yield, in addition, information concerning the shape of the potential, its exchange character, and further information about the nature of tensor forces. Theoretical interpretation of the data available at present[3] indicates that the potential is of the "long-tailed" type, but it is not possible to distinguish between the Yukawa and exponential potentials. The percentage of charge exchange seems to be somewhat higher than 50 percent but the experimental evidence is not complete enough to assign a definite value. The effect of tensor forces is not negligible at high energies though the present data do not yield conclusive evidence on the strength of the tensor forces.

Similar information can be obtained from a study of the photo-disintegration of the deuteron. At low energies the photo-disintegration is independent of the shape of the potential and depends in a simple way on the effective range of the neutron-proton interaction. At high energies total cross sections and angular distributions depend on the shape of the potential and the percentage of charge exchange, and will be influenced by the type and range of tensor forces assumed.

In the present paper the photo-effect is discussed for three possible potentials: square well, exponential, and Hulthén[4] (which is a good approximation to the Yukawa potential)[5] in the energy range 10 to 150 Mev. In the case of the exponential and Hulthén potentials all calculations are made on the assumption of purely central interaction and an equal mixture of ordinary and charge exchange forces, while for the square well an estimate is made of the influence of different mixtures.

Section II contains the constants for the three n–p potentials used. Section III presents a general discussion of the photoelectric dipole cross section based on the f-sum rule generalized for the case of exchange forces. Section IV presents a summary of the calculations in the form of graphs. Section V contains the principal formulas used in the detailed calculations, together with a brief discussion of the limitations of our present theory of the deuteron photo-effect from the point of view of meson theory.

SYMBOLS USED IN THE TEXT

ψ_i: Eigenfunction for the ground state of the system.
u_i: Radial eigenfunction for the ground state.
ψ_l: Eigenfunction for state of angular momentum l.
u_l: Radial eigenfunction for state of angular momentum l.
M: Mean mass of neutron and proton.
μ_n: Magnetic moment of neutron.
μ_p: Magnetic moment of proton.
ϵ: Binding energy of the deuteron.
E_γ: Energy of the incident γ-ray in the center-of-mass system.

* Assisted in part by ONR. Some of our results using a square well have been published. [J. F. Marshall and E. Guth, Phys. Rev. 76, 1879, 1880 (1949)].
† Part of this work was done while the author was an AEC Predoctoral Fellow.
[1] J. M. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949).
[2] H. A. Bethe, Phys. Rev. 76, 38 (1949).
[3] R. S. Christian and E. W. Hart, Phys. Rev. 77, 441 (1950). We are indebted to Drs. Christian and Hart for informing us of their results prior to publication. Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. 75, 351 (1949).
[4] J. S. Levinger, Phys. Rev. 76, 699 (1949).
[5] L. I. Schiff has kindly informed us of independent computations, using Hulthén and exponential potentials. His conclusions and ours are in general agreement.

$j_l(x)$: Spherical Bessel function of order l

$$j_l(x)=(\pi/2x)^{\frac{1}{2}}J_{l+\frac{1}{2}}(x).$$

x: Fraction of exchange forces in $n-p$ interaction.
σ_d: Total electric dipole cross section.
σ_q: Total electric quadrupole cross section.
σ_{mq}: Total magnetic quadrupole cross section

$$\alpha^2=(M/\hbar^2)\epsilon,\quad k^2=(M/\hbar^2)(E_\gamma-\epsilon).$$

II. CONSTANTS FOR THE $n-p$ INTERACTION

The constants for Yukawa and exponential potentials were obtained from the graphs of Blatt and Jackson[1] using the latest value of the effective range ($r_0=1.74\times10^{-13}$ cm) of the neutron-proton interaction.[6] The corresponding potentials are then

$$V_Y(r)=Ae^{-\mu r}/r;\quad A=1.53\times10^{13}\text{ cm}^{-1},\quad \mu=0.614\times10^{13}\text{ cm}^{-1},\quad (1)$$

$$V_E(r)=B^2e^{-2\beta r};\quad B^2=4.27\times10^{26}\text{ cm}^{-2},\quad \beta=0.717\times10^{13}\text{ cm}^{-1}.\quad (2)$$

The range of the Hulthén potential is chosen to give the same low energy electric dipole cross section as the Yukawa potential (see footnote 21), yielding:

$$V_H(r)=(\beta^2-\alpha^2)\frac{e^{-\beta r}}{e^{-\alpha r}-e^{-\beta r}};\quad \beta=1.41\times10^{13}\text{ cm}^{-1}.\quad (3)$$

The binding energy of the deuteron is taken equal to the value used by Blatt and Jackson, $\epsilon=2.208$ Mev.

Calculations for the square well were made at an earlier date and the constants for it correspond to an effective range, $r_0=1.54\times10^{-13}$ cm, and a binding energy, 2.237 Mev.

III. GENERAL DISCUSSION OF DEUTERON PHOTO-EFFECT EMPLOYING SUM RULES

Evaluation of the photo cross sections involves computation of integrals of the type

$$\int_0^\infty \psi_i z^l \psi_l d\tau.\quad (4)$$

For low energies ($E_\gamma<10$ Mev) it is sufficient to take only $l=1$; i.e., consider the dipole cross section only. It is also permissible to use for ψ_1 the wave function of a free nucleon

$$\psi_1\rightarrow\psi_{1\text{ free}}.\quad (5)$$

At low energies the main contribution to (4) comes from the region "outside" the n-p interaction. For intermediate and high energies the interactions of the nucleons must be taken into account, and the contribution from the "inside" increases with energy.

For an equal mixture of ordinary and exchange forces (5) is exact and the computation is quite simple. The dipole cross section in this case depends on the range and shape of the n-p interaction only through the ground wave function. For the case of an arbitrary exchange ratio x ($x=0$, ordinary, $x=1$, exchange) it is possible, however, to gain a qualitative understanding of the dependence of the cross section on range, shape, and exchange ratio by application of the f-sum rule. As is well known, the f-sum rule yields a value of the integrated dipole cross section of the deuteron, given by

$$\int^\infty \sigma_d dE=\pi^2e^2\hbar/Mc=\sigma_0.\quad (6)$$

This form implies the assumption of ordinary forces, however. In this case the integrated cross section does not depend on either range or shape of the n-p interaction.

It seems that Fock[7] was the first to point out for atomic problems that the usual form of the f-sum rule needs modification when exchange forces are taken into account. Independently of Fock, Feenberg[8] and Siegert[9] pointed out the same fact in connection with nuclear problems and Way[10] has already applied the modified sum rule to the deuteron photo-effect. For the integrated dipole cross section at exchange ratio x one obtains

$$\sigma_x=\sigma_0\left(1+\frac{2}{3}\frac{M}{\hbar^2}x\int_0^\infty \psi_i^*Vr^2P\psi_i d\tau\right).\quad (7)$$

This formula is in agreement with Way and also with Levinger and Bethe[11] who recently made a detailed application of sum rules to nuclear dipole photo-effects.

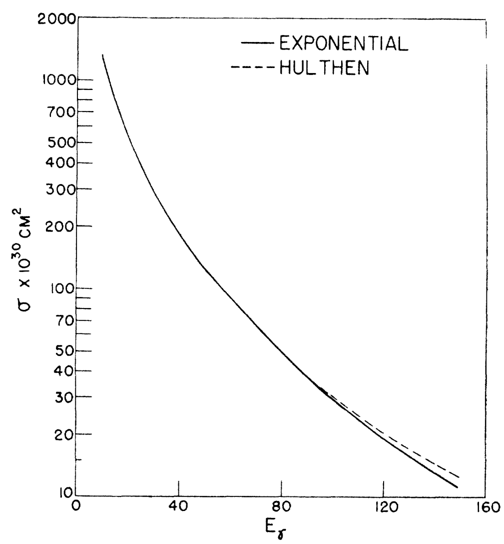

FIG. 1. Total cross sections for $r_0=1.74\times10^{-13}$ cm.

[6] D. J. Hughes, Phys. Rev. **78**, 315 (1950). Hughes, Burgy, and Ringo, Phys. Rev. **77**, 291 (1950).

[7] V. Fock, Zeits. f. Physik **89**, 744 (1934).

[8] E. Feenberg, Phys. Rev. **49**, 328 (1936).

[9] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).

[10] K. Way, Phys. Rev. **51**, 552 (1937).

[11] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950). We are indebted to Dr. Levinger and Professor Bethe for communicating their results to us before publication.

Evaluation of (7) for a square, a Hulthén, and an exponential potential leads to the approximate general form

$$\sigma_x = \sigma_0(1+\zeta\alpha r_0 x), \quad (8)$$

where ζ is a shape factor and r_0 the effective range. This form shows clearly that σ_x increases both with x and r. In particular, for $r_0 = 1.56$ and 1.74 we obtain for a square well

$$\zeta\alpha r_0 = \begin{Bmatrix} 0.40 \\ 0.44 \end{Bmatrix}$$

and for a Hulthén and an exponential potential, which yield about the same value

$$\zeta\alpha r_0 = \begin{Bmatrix} 0.33 \\ 0.37 \end{Bmatrix}.$$

At high energies it seems permissible to conclude that σ_d will also increase with increasing x.[12] An attractive exchange force for the ground (S) state of the deuteron implies a repulsive exchange force for the final (P) state. This repulsion implies, (a) an increase of the energy of a given P state compared to the case $x=0$, when the P state is bound by an attractive ordinary force, and (b) a decrease in the integral (4) for small final energies and an increase in the integral, i.e., in σ_d, for large final energies.

We conclude that at high energies for a given range σ_d for a square well will be larger than that for a Yukawa (Hulthén) or exponential potential, the last two differing but little, and that σ_d increases with increasing x. This last conclusion is certainly not valid for energies less than 30 Mev, as is shown by numerical computations.

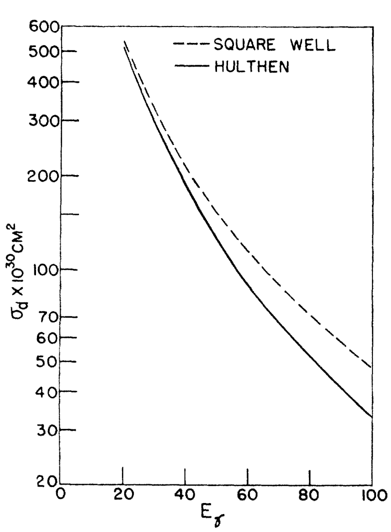

FIG. 2. Total cross sections for $r_0 = 1.56\times10^{-13}$ cm.

[12] The following arguments have already been given by Levinger and Bethe (reference 11).

For quadrupole (even-even) transitions, in contrast with dipole (even-odd) transitions, exchange forces do not modify the matrix elements or the sum rules.

IV. SUMMARY OF CALCULATIONS

The results of calculations based on the assumption of central forces and 50 percent charge exchange are shown in Figs. 1–4. The energy range 10 to 150 Mev is chosen because below 10 Mev one cannot detect any shape effects, and above 150 Mev relativistic effects and interactions other than photo-disintegration become important (see Section V). Total cross sections for the "long-tailed" potentials, including electric dipole, electric and magnetic quadrupole (the magnetic dipole contribution can be neglected) are shown in Fig. 1 for $r_0 = 1.74\times10^{-13}$ cm. Throughout most of the energy range the difference between the Hulthén and the exponential cross sections is quite small, but at the higher end they begin to deviate from each other appreciably, the difference amounting to 13 percent at 150 Mev. It is interesting to note that for energies greater than 80 Mev the cross section for the exponential well always lies below that for the Hulthén well. This agrees with Schiff's theorem[13] on the asymptotic energy dependence, according to which the exponential cross section should fall off more rapidly than that for the Hulthén well by a factor $1/E_\gamma$ at high energies.

Total cross sections for the square-well and Hulthén potentials are compared in Fig. 2 for an effective range of 1.56×10^{-13} cm in the energy range 20 to 100 Mev. The difference between the two curves is quite marked, amounting to about 30 percent at 100 Mev. Consequently, while it would be quite difficult to distinguish between the "long-tailed" wells experimentally, a potential of the square-well type should be recognizable.

The electric and magnetic quadrupole cross sections are plotted in Fig. 3 for the two "long-tailed" potentials. [In computing the electric quadrupole cross sections, it was assumed that the outgoing wave could be treated as free (see Section V).] At low energies both can be neglected as far as their influence on the total cross section is concerned, although the electric quadrupole cross section still has an important influence on the angular distribution. At the higher energies their contributions to the total cross section are about the same: electric quadrupole contributing five percent at 150 Mev and magnetic quadrupole eight percent.

Angular distributions in the center-of-mass system are shown for two energies (150 and 17.5 Mev) in Fig. 4. Distributions for other energies can be obtained very readily by use of Figs. 1 and 2 and Eq. (42). In the 17.5-Mev curve[14] the magnetic dipole contribution, which would yield an isotropic term amounting to

[13] L. I. Schiff, Phys. Rev. **78**, 83 (1950); see also Levinger and Bethe (reference 11).

[14] Angular distributions in the range 4 to 20 Mev have been measured by Fuller. [E. G. Fuller, Phys. Rev. **76**, 576 (1949) and Ph.D. thesis.] We have shown that his forward asymmetry agrees with ours to within the experimental error (see asterisk reference).

about two percent of the 90° cross section, has been subtracted out. The distribution is predominantly of the electric dipole form, but is slightly asymmetric about 90° due to interference between electric dipole and electric quadrupole transitions. At this energy there is no noticeable dependence on shape in the angular distribution, and the angular distribution is given very nearly by the zero-range formula. The distribution normalized to unity at 90° is given by

$$f(\theta)=\sin^2\theta[1+0.264\cos\theta],\text{ at 17.5 Mev} \quad (9)$$

which is approximately

$$f(\theta)=\sin^2\theta[1+2((E_\gamma-\epsilon)/Mc^2)^{\frac{1}{2}}\cos\theta]. \quad (10)$$

The coefficient of $\cos\theta$ in (9) is the same for all three potentials and is larger than that for the zero-range distribution (10) by about three percent.

The angular distribution of photo-protons at 150 Mev is plotted in the solid curve of Fig. 4 for the exponential potential, the angular distributions for the other potentials having much the same form. At this energy the angular distribution has the form

$$f(\theta)=\sin^2\theta[1+1.066\cos\theta+0.2839\cos^2\theta] +0.1880\cos^2\theta. \quad (11)$$

The $\sin^2\theta$ term still dominates, but the forward asymmetry is much larger than in the low energy case. Furthermore, since the magnetic quadrupole cross section is appreciable at this energy there is appreciable scattering in the forward direction.

V. DETAILED CALCULATIONS AND DISCUSSION

A. General Considerations[15]

Møller and Rosenfeld[16] have shown that since the charge density of the virtual meson field has an average value of zero, the exchange moment contribution to the electric dipole and electric quadrupole moments is zero to order v_{nucleon}/c. On the assumption of purely central forces corresponding cross sections are then given by:

$$\sigma_d=\frac{\pi}{3}\frac{e^2}{\hbar c}\frac{M}{\hbar^2}kE_\gamma I_1^2;\quad I_1=\int_0^\infty u_1 r u_i dr, \quad (12)$$

$$\sigma_q=\frac{\pi}{240}\frac{e^2}{\hbar c}\frac{M^2}{\hbar^4}\frac{kE_\gamma^3}{Mc^2}I_2^2;\quad I_2=\int_0^\infty u_2 r^2 u_i dr. \quad (13)$$

The situation with respect to magnetic transitions is not quite so simple since exchange effects will depend upon the specific meson theory assumed. It is true that the magnetic exchange moment vanishes for any stationary state of the deuteron, but the exchange moments of H^3 and He^3 amount to about six percent.[17] The exchange currents are likely to contribute to the photomagnetic cross section, as pointed out by Pais.[18] His original estimate of the magnitude of this contribution on the basis of Møller and Rosenfeld's mixed theory was much too large.[19] Still, the magnitude of this contribution is unknown. The same holds *a fortiori* for the magnetic quadrupole transition. Fortunately, the magnetic dipole cross section, assuming zero exchange contribution, is negligible over our energy range. However, the magnetic quadrupole cross section, again assuming zero exchange contribution, is larger than the electric quadrupole cross section above 100 Mev. For the sake of this comparison we retained the magnetic quadrupole transition, though its inclusion, according to our previous discussion, does not have a sound theoretical basis.

Assuming the exchange moment to be negligible, the magnetic quadrupole cross section for transitions to a final triplet state is given by

$$\sigma_{mq}=\frac{\pi}{18}\frac{e^2}{\hbar c}\left(\frac{\hbar^2}{M}\right)^2\frac{k(\alpha^2+k^2)^3}{(Mc^2)^2}(\mu_p-\mu_n)^2 I_1^2$$

$$=\frac{(\mu_p-\mu_n)^2}{6}\left(\frac{E_\gamma}{Mc^2}\right)^2\sigma_d. \quad (14)$$

The cross section for transitions to singlet states is about one-fiftieth of this amount and can safely be neglected.

The angular distribution of photo-particles in the center-of-mass system is given by[20]

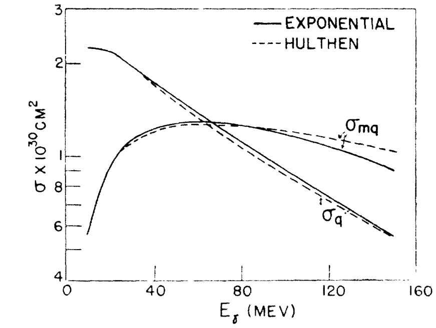

FIG. 3. Electric and magnetic quadrupole cross sections for $r_0=1.74\times10^{-13}$ cm.

[15] In these computations it was assumed that the interaction is purely central. The numerical calculations of T-M. Hu and H. S. W. Massey, Proc. Roy. Soc. **A186**, 135 (1949), indicate for $x=\frac{2}{3}$ a 40 percent increase of σ_d by tensor forces at 28.8 Mev. For $x=\frac{1}{2}$ the percentage increase is smaller, but very likely still appreciable.

[16] C. Møller and J. Rosenfeld, Kgl. Danske Vid. Sels. Mat.-Fys. Medd. **20**, No. 12 (1943).

[17] F. Villars, Phys. Rev. **72**, 256 (1947).

[18] A. Pais, Kgl. Danske Vid. Sels. Math-Fys. Medd. **20**, No. 17 (1943).

[19] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1949), Vol. II, p. 449.

[20] Several authors state that there is no interference between electric dipole and electric quadrupole transitions, as indicated by the above expression, since the final states belong to different isotopic spin states. Were it not for the Pauli principle this would be the case, since cross product terms would vanish on summation over isotopic spins. Since, however, for a given angular momentum and spin one of the two possible isotopic spin states is always

$$\frac{d\sigma}{d\Omega}=\frac{\sin^2\theta}{8\pi}[3\sigma_d+6(5\sigma_q\sigma_d)^{\frac{1}{2}}\cos\delta_1\cos\delta_2\cos\theta$$
$$+15\sigma_q\cos^2\theta]+(3/4\pi)\sigma_{mq}\cos^2\theta. \quad (15)$$

B. Wave Functions for Initial and Final States

The radial wave functions for the ground state satisfy a wave equation of the form

$$u_i''+V(r)u_i-\alpha^2u_i=0. \quad (16)$$

In order to satisfy Eqs. (12)–(14) the function u_i must be normalized to unity,

$$\int u_i^2dr=1. \quad (17)$$

It is more convenient to use a wave function which has the asymptotic form $\exp(-\alpha r)$. Writing

$$u=A\varphi, \quad \text{where} \quad \varphi\xrightarrow[r\to\infty]{}e^{-\alpha r}, \quad (18)$$

we find

$$A^2=2\alpha/(1-\alpha\rho), \quad (19)$$

where

$$2\rho=\int_0^\infty e^{-2\alpha r}-\int_0^\infty \varphi^2dr. \quad (20)$$

It has been shown by Blatt and Jackson that ρ differs from r_0 by about one-half percent, and for all potentials considered here

$$\rho=1.73\times10^{-13} \text{ for } r_0=1.74\times10^{-13},$$
$$\rho=1.55\times10^{-13} \text{ for } r_0=1.56\times10^{-13}.$$

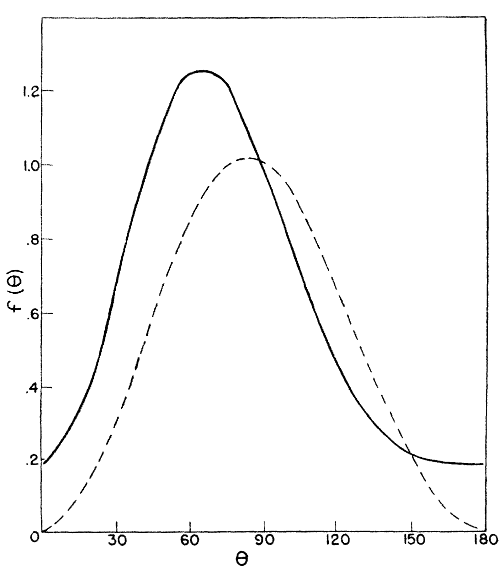

FIG. 4. Angular distributions of photo-protons. The dashed curve is for 17.5-Mev gamma-rays, and the solid curve is for 150-Mev gamma-rays.

forbidden by the Pauli principle, averaging over isotopic spin states is not justified and the cross product terms remain.

The solutions to the wave equation for the three potentials are then:

(a) Hulthén:

$$V(r)=(\beta^2-\alpha^2)\frac{e^{-\beta r}}{e^{-\alpha r}-e^{-\beta r}}, \quad \varphi=e^{-\alpha r}-e^{-\beta r}. \quad (21)$$

(b) Exponential:

$$V(r)=B^2e^{-2\beta r}, \quad \varphi=aJ_p(x) \quad (22)$$

where

$$p=\alpha/\beta; \quad x=(B/\beta)e^{-\beta r}; \quad a=\Gamma(p+1)(B/2\beta)^{-p}. \quad (23)$$

It is found convenient to express (22) in the form of an infinite series

$$\varphi=\sum_0^\infty\frac{(-1)^j(B/2\beta)^{2j}}{j!p_j}e^{-(\alpha+2j\beta)r}, \quad (24)$$

where

$$p_j=(p+1)(p+2)\cdots(p+j).$$

(c) Square well:

$$V(r)=V_0, \quad r<b_t$$
$$V(r)=0, \quad r>b_t$$
$$\varphi=\frac{e^{-\alpha b_t}}{\sin V_0^{\frac{1}{2}}b_t}\sin V_0^{\frac{1}{2}}r, \quad r<b_t, \quad (25)$$
$$\varphi=e^{-\alpha r}, \quad r>b_t. \quad (26)$$

In order to evaluate the integrals I_1 and I_2 we need final state wave functions for the P and D states. On the assumption of $x=0.5$, the nucleons may be treated as free in the P state and the corresponding wave function is simply

$$u_1=rj_1(kr), \quad \delta_1=0. \quad (27)$$

To obtain a general solution of the wave equation for the D state would require a numerical solution for the exponential and Hulthén potentials. A possible approximation would be to assume that in the evaluation of I_2 the important part of the wave function is that outside the range of interaction, and that we can approximate u_2 by

$$u_2=r[\cos\delta_2j_2(kr)-\sin\delta_2n_2(kr)], \quad (28)$$

where δ_2 is given approximately by

$$\sin\delta_2=k\int_0^\infty V(r)j_2^2(kr)r^2dr. \quad (29)$$

It was found that in this approximation the effect of binding in the D state was quite small. Furthermore, the contribution to the integral from inside the well is quite large, and it is not entirely clear that (28) is a much better approximation than the assumption that the outgoing wave can be treated as free. Consequently, the D wave function for the Hulthén and exponential well was taken to be

$$u_2 = r j_2(kr)\,;\quad \delta_2 = 0. \tag{30}$$

[Use of Eqs. (28) and (29) instead of (30) would change σ_q by about 10 percent. Since the asymmetric term in the angular distribution depends on $(\sigma_q)^{\frac{1}{2}}$, the change there is smaller.]

In the case of the square well, the effect of binding is much more pronounced and exact wave functions were used,

$$\begin{aligned} u_2 &= A r j_2(V_0^{\frac{1}{2}} r), & r &< b_t, \\ u_2 &= r[\cos\delta_2 j_2(kr) - \sin\delta_2 n_2(kr)], & r &> b_t. \end{aligned} \tag{31}$$

C. Evaluation of the Cross Sections

1. *Fifty Percent Charge Exchange*

Since over a large part of the energy range considered the cross sections differ very little from the zero-range cross sections, it is convenient to express them in terms of these cross sections. On the assumption of a free outgoing wave in P and D states the expressions for the cross sections take the form

$$\sigma_d = \sigma_B F_1^2/(1-\alpha\rho), \tag{32}$$

where

$$F_1 = \frac{(\alpha^2+k^2)^2}{2k}\int_0^\infty \varphi j_1(kr) r^2 dr.$$

Also

$$\sigma_B = \frac{8\pi}{3}\frac{e^2}{\hbar c}\frac{\hbar^2}{M}\frac{\epsilon^{\frac{1}{2}}(E_\gamma-\epsilon)^{\frac{3}{2}}}{E_\gamma^3}, \tag{33}$$

$$\sigma_q = \sigma_{Bq} F_2^2/(1-\alpha\rho), \tag{34}$$

where

$$F_2 = \frac{(\alpha^2+k^2)^3}{8k^2}\int_0^\infty \varphi j_2(kr) r^3 dr.$$

Finally,

$$\sigma_{Bq} = \frac{(E_\gamma-\epsilon)}{5Mc^2}\sigma_B. \tag{35}$$

Substituting (21) and (24) we obtain for F_1 and F_2:

(a) Hulthén potential:[21]

$$F_1 = 1 - [(\alpha^2+k^2)/(\beta^2+k^2)]^2, \tag{36}$$

$$F_2 = 1 - [(\alpha^2+k^2)/(\beta^2+k^2)]^3. \tag{37}$$

[21] At energies below 15 Mev the factor F_1 is very nearly equal to one for all potentials. Consequently, the electric dipole cross section is given by

$$\sigma_d = \sigma_B/(1-\alpha\rho)$$

which is independent of the shape of the potential. Since the constants of the Yukawa potential are chosen to correspond to $\rho = 1.73\times10^{-13}$ cm, the constant β in the Hulthén potential must be chosen to correspond to this value of ρ, if the two potentials are to yield the same low energy cross section. From (20) one obtains a relation between ρ and β for the Hulthén case

$$\rho = (3\beta-\alpha)/\beta(\beta+\alpha)$$

which can be solved for β yielding $\beta = 1.41\times10^{13}$ cm^{-1}.

(b) Exponential potential:

$$F_1 = 1 - \sum_1^\infty \frac{(-1)^j (B/2\beta)^{2j}}{j!\,p_j}\frac{(\alpha^2+k^2)^2}{[(p+2j)^2\beta^2+k^2]^2}, \tag{38}$$

$$F_2 = 1 - \sum_1^\infty \frac{(-1)^j (B/2\beta)^{2j}(\alpha^2+k^2)^3}{j!\,p_j[(p+2j)^2\beta^2+k^2]^3}. \tag{39}$$

Up to 150 Mev, only two terms ($j=1$, $j=2$) are needed for an accuracy of better than one percent in the total cross sections. Putting in values for α, B, β corresponding to an effective range of 1.74×10^{-13} cm, the series can be written

$$F_1 = 1 - 1.57\left[\frac{\alpha^2+k^2}{(p+2)^2\beta^2+k^2}\right]^2 + 0.146\left[\frac{\alpha^2+k^2}{(p+4)^2\beta^2+k^2}\right]^2, \tag{40}$$

$$F_2 = 1 - 1.57\left[\frac{\alpha^2+k^2}{(p+2)^2\beta^2+k^2}\right]^3 + 0.146\left[\frac{\alpha^2+k^2}{(p+4)^2\beta^2+k^2}\right]^3. \tag{41}$$

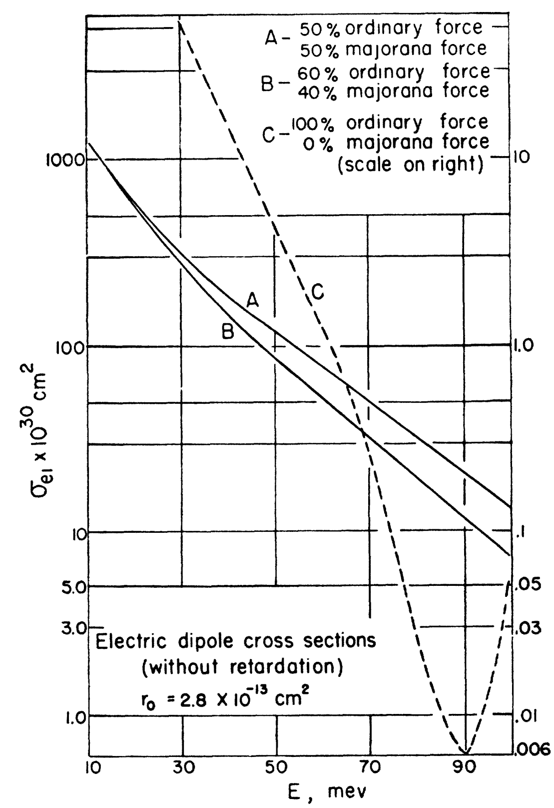

FIG. 5. Electric dipole cross sections for several mixtures.

Taking $\delta_1=\delta_2=0$, we have for the angular distribution

$$d\sigma/d\Omega=(8\pi/3)\sin^2\theta[3\sigma_d+6(5\sigma_d\sigma_q)^{\frac{1}{2}}\cos\theta+15\sigma_q\cos^2\theta]+(3/4\pi)\sigma_{mq}\cos^2\theta. \quad (42)$$

The expressions for the cross sections in the case of the square well are quite a bit more complicated than those for the Hulthén and exponential potentials, and are most simply computed by substitution in recursion formulas.

2. *Effect of Mixture*

Dipole cross sections for several exchange mixtures are plotted in Fig. 5 for a square well of intrinsic range, 2.8×10^{-13} cm. From these curves one can obtain a qualitative idea of how the mixture will affect both total cross sections and angular distributions.

The cross section for $x=0$ (a simple derivation is given in the Appendix) is found to fall off much more rapidly than that for $x=0.5$, and actually goes to zero at 90 Mev. Furthermore, since the quadrupole cross section is unaffected by mixture the angular distributions will be radically different for the two cases. Consequently, it should be very easy to determine experimentally whether the interaction potential has a large percentage of ordinary forces.

The dipole cross section for $x=0.4$, however, is seen to be very nearly equal to that for $x=0.5$ and a rough estimate indicates that all values of x greater than 0.4 yield very nearly the same value of the cross section. Consequently, very careful experiments would have to be made in order to assign an exact value to x.

APPENDIX. DERIVATION OF σ_d FOR ORDINARY FORCES

The matrix element for the dipole transition is proportional to the absolute square of the integral (matrix)

$$x_{01}=\int_0^\infty \psi_i z\psi_1 d\tau. \quad (43)$$

We can transform this integral and evaluate it at once by using the equation of motion

$$M\ddot{x}_{01}=-\left(\frac{\partial V}{\partial x}\right)_{01},\quad \ddot{x}_{01}=-\frac{(E_1-E_0)^2}{M\hbar^2}x_{01}. \quad (44)$$

inserting (44) in (43) we obtain

$$x_{01}=\frac{M\hbar^2}{(E_1-E_0)^2}\int\psi_i\frac{\partial V}{\partial r}\cos\theta\psi_1 d\tau. \quad (45)$$

But for a square well $\partial V/\partial r=\delta(r-b_t)$, therefore

$$\int_0^\infty \psi_i(\partial V/\partial r)\psi_1 r^2 dr=\psi_i(b_t)\psi_1(b_t)b_t^2 \quad (46)$$

giving σ_d in a rather compact form.

This result agrees with that of Breit and Condon[22] which was obtained on the basis of a more involved computation.

[22] G. Breit and E. U. Condon, Phys. Rev. **49**, 904 (1936).

PHYSICAL REVIEW VOLUME 78, NUMBER 6 JUNE 15, 1950

Hyperfine Structure and Nuclear Spins of Tungsten and Tellurium

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(Received March 3, 1950)

By the use of enriched isotopes, hyperfine structure in the optical spectra of Te^{123}, Te^{125}, and W^{183} has been observed. Milligram amounts of the isotopes were used in a modified Schüler hollow-cathode discharge tube. In the visible region, 4000 to 6000A, about a dozen lines of singly ionized tellurium showed hyperfine structure, all of them giving just two components for the odd isotope. Te^{123} and Te^{125} gave identical structures except for a scale factor, the total splitting of the lines of Te^{123} being about 88 percent of that of Te^{125}. In the spectrum of neutral tungsten, all of the lines which show the isotope shift for natural tungsten gave just two components with highly enriched W^{183}. The number of hyperfine-structure components, namely two, gives for Te^{123}, Te^{125}, and W^{183} a nuclear spin of $\frac{1}{2}$. This was also verified by intensity measurements.

I. INTRODUCTION

THE nuclear spins of all the stable isotopes of odd atomic number are now believed to be known, and even the spins of a number of radioactive nuclei have been measured. On the other hand, the spins of about one-third of the stable nuclei of even atomic number and odd atomic weight remain unknown. The reason for the scarcity of data on the spins of even-odd nuclei is in many cases the low abundance of the odd isotopes in the natural element. Enriched isotopes of many elements separated by mass-spectrographic methods have recently become available in milligram amounts through the Atomic Energy Commission. By the use of these enriched isotopes several nuclear spins heretofore not known or not definitely established have been measured recently. Although the spin of any nucleus is a desirable datum, there are some nuclei which are particularly important or interesting from a theoretical point of view. In this investigation the spins of three interesting nuclei—the two odd isotopes of tellurium, Te^{123} and Te^{125}, and the odd isotope of tungsten, W^{183}—are measured by observations on the hyper-

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